Løsningsforslag eksamen i matematikk for forkurset 2018

Oppgave 1

a)

$$\frac{x^2 \cdot (x^2-2x+1) \cdot \ln e^x \cdot \sin\frac{\pi}{4}}{(x^{-1})^{-1} \cdot e^{\ln x} \cdot (x-1) \cdot \sqrt{2}} = \frac{x^2 \cdot (x-1)^2 \cdot x \cdot \frac{\sqrt{2}}{2}}{x \cdot x \cdot (x-1) \cdot \sqrt{2}} = \underline{\underline{\frac{x(x-1)}{2}}}$$

b)

$\sin 3x = -1 \qquad x \epsilon [-\pi, \pi]$

$3x = -\frac{\pi}{2} + k2\pi$

$3x = \pi - \left(-\frac{\pi}{2}\right) + k2\pi = \frac{3\pi}{2} + k2\pi = -\frac{\pi}{2} + k2\pi \quad (Samme\ svar\ som\ likningen\ over)$

$x = -\frac{\pi}{6} + \frac{k2\pi}{3}$

$x = -\frac{\pi}{6} + \frac{(-1)2\pi}{3} = -\frac{5\pi}{6} \qquad k = -1$

$x = -\frac{\pi}{6} + \frac{0 \cdot 2\pi}{3} = -\frac{\pi}{6} \qquad k = 0$

$x = -\frac{\pi}{6} + \frac{1 \cdot 2\pi}{3} = \frac{\pi}{2} \qquad k = 1$

$\underline{\underline{L = \left\{-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}\right\}}}$

c)

$\ln(x+2) - \ln x = 1 \qquad x \in \langle 0, \rightarrow \rangle$

$\ln\frac{x+2}{x} = 1$

$\frac{x+2}{x} = e$

$x + 2 = ex$

$x - ex = -2$

$x(1-e) = -2$

$$x = \frac{-2}{1-e} = \frac{2}{e-1}$$

$$\underline{\underline{L = \left\{\frac{2}{e-1}\right\}}}$$

d)

$$f(x) = \sin(x) \cdot e^x$$

$$f'(x) = e^x \cdot \cos x + e^x \cdot \sin x \underline{\underline{= e^x(\cos x + \sin x)}}$$

$u = \sin x \qquad u' = \cos x$

$v = e^x \qquad v' = e^x$

e)

$$g(x) = \ln(\cos x)$$

$$g' = \frac{-\sin x}{\cos x} \underline{\underline{= -\tan x}}$$

f)

$$\int \frac{4x}{x^2-4} dx = 2\int \frac{2x}{x^2-4} \underline{\underline{= 2\ln|x^2-4| + C}}$$

g)

$$\int \frac{4x}{x-4} dx = 4\int \frac{x-4+4}{x-4} dx = 4\int \left(\frac{x-4}{x-4} + \frac{4}{x-4}\right) dx = 4\int \left(1 + \frac{4}{x-4}\right) dx$$

$$\underline{\underline{= 4(x + 4\ln|x-4|) + C}}$$

$Delbrøkoppspalting\ av\ integranden\ gir$:

$$\frac{4}{x^2-4} = \frac{4}{(x-2)(x+2)} = \frac{1}{x-2} - \frac{1}{x+2}$$

$$\int_3^4 \frac{4}{x^2-4} dx = \int_3^4 \left(\frac{1}{x-2} - \frac{1}{x+2}\right) dx = [\ln|x-2| - \ln|x+2|]\,{}_3^4$$

$$= \ln|4-2| - \ln|4+2| - (\ln|3-2| - \ln|3+2|) = \ln 2 - \ln 6 + \ln 5 = \ln 2 - (\ln 2 \cdot 3) + \ln 5$$

$$= \ln 2 - \ln 2 - \ln 3 + \ln 5 = \underline{\underline{\ln \frac{5}{3}}}$$

i)

$$f(t) = 1 + \frac{t}{\sqrt{7}} + \frac{t^2}{7} + \frac{t^3}{7\sqrt{7}} + \frac{t^4}{49} + \cdots$$

$$k = \frac{a_2}{a_1} = \frac{\frac{t}{\sqrt{7}}}{1} = \frac{t}{\sqrt{7}}$$

$$-1 < k < 1$$

$$-1 < \frac{t}{\sqrt{7}} < 1$$

$-\sqrt{7} < t < \sqrt{7}$ eller $\underline{\underline{t \in \langle -\sqrt{7}, \sqrt{7} \rangle}}$

$$s(t) = \frac{a_1}{1-k} = \frac{1}{1 - \frac{t}{\sqrt{7}}} = \underline{\underline{\frac{\sqrt{7}}{\sqrt{7} - t}}}$$

Oppgave 2

a)

$$D_f = R \setminus \{0\}$$

$$f(x) = 0$$

$$\frac{4x^2 + 1}{x} = 0$$

$$4x^2 + 1 = 0$$

$Denne\ likningen\ har\ ingen\ reelle\ løsninger\ og\ derfor\ har\ f(x)\ ingen\ nullpunkter.$

b)

$Vertikal\ asymptote: Nevner\ li\quad 0.$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{4x^2 + 1}{x} = \pm\infty \qquad \underline{\underline{Vertikal\ asymptote\ x = 0}}$$

$$f(x) = \frac{4x^2 + 1}{x} = \frac{4x^2}{x} + \frac{1}{x} = 4x + \frac{1}{x}$$

$linja\ y = 4x\ er\ en\ skråasymptote\ fordi:$

$$\lim_{x \to \pm\infty} f(x) - y = \lim_{x \to \pm\infty} 4x + \frac{1}{x} - 4x = \lim_{x \to \pm\infty} \frac{1}{x} = 0 \qquad \underline{\underline{Skråasymptote\ y = 4x}}$$

c)

$$f(x) = \frac{4x^2+1}{x}$$

$u = 4x^2 + 1 \qquad u' = 8x$

$v = x \qquad v' = 1$

$$f'(x) = \frac{8x \cdot x - (4x^2+1) \cdot 1}{x^2} = \frac{8x^2 - 4x^2 - 1}{x^2} = \underline{\underline{\frac{4x^2-1}{x^2}}}$$

$$f'(x) = 0$$

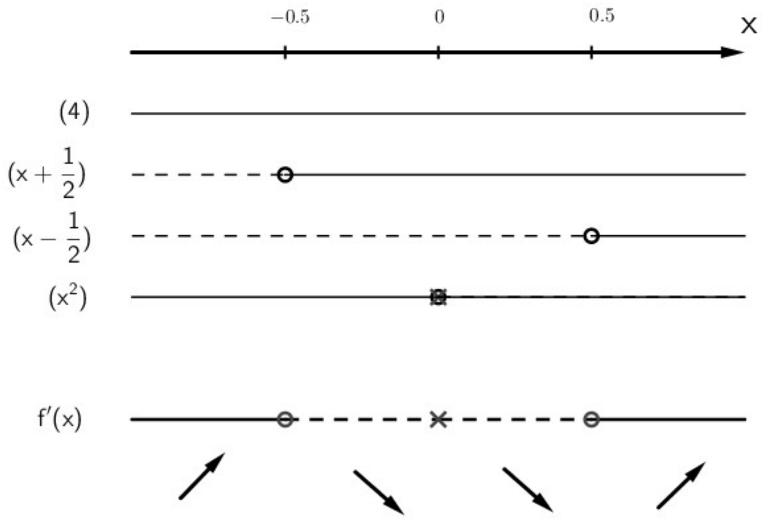
$$\frac{4x^2-1}{x^2} = 0$$

$$4x^2 - 1 = 0$$

$$x^2 = \frac{1}{4}$$

$$x = \pm\frac{1}{2}$$

$$f'(x) = \frac{4x^2-1}{x^2} = \frac{4(x-\frac{1}{2})(x+\frac{1}{2})}{x^2}$$



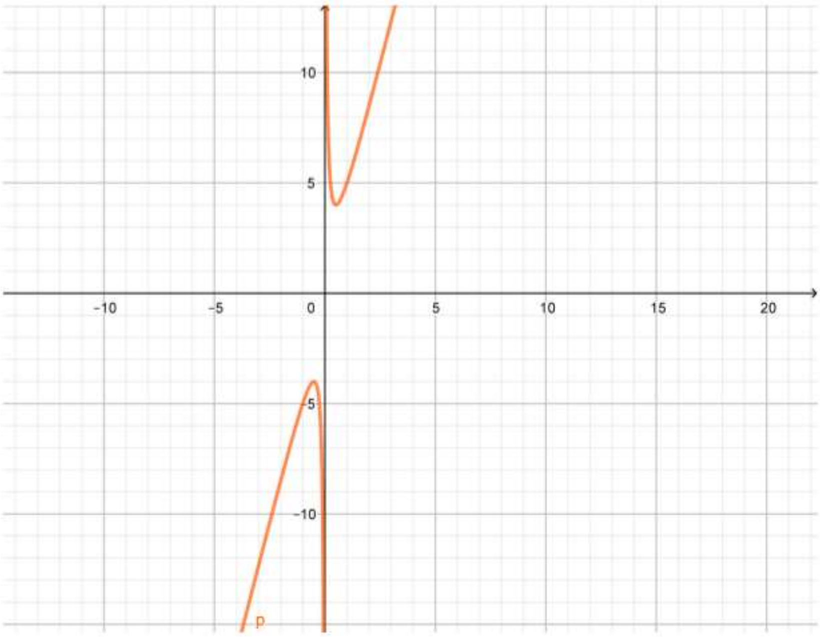
$$f\left(\frac{1}{2}\right) = \frac{4\left(\frac{1}{2}\right)^2 + 1}{\frac{1}{2}} = 4$$

$$f\left(-\frac{1}{2}\right) = \frac{4(-\frac{1}{2})^2 + 1}{-\frac{1}{2}} = -4$$

$$\underline{\underline{Toppunkt\ \left(-\frac{1}{2}, -4\right) \qquad Bunnpunkt\ \left(\frac{1}{2}, 4\right)}}$$

$$\underline{\underline{Funksjonen\ stiger\ for\ x \in \langle \leftarrow, -\frac{1}{2}\rangle \cup \langle \frac{1}{2}, \rightarrow\rangle}}$$

$$\underline{\underline{Funksjonen\ synker\ for\ x \in \langle -\frac{1}{2}, 0\rangle \cup \langle 0, \frac{1}{2}\rangle}}$$



$$\underline{\underline{Fra\ figuren\ ser\ vi\ at\ V_f = R \setminus \langle -4,4\rangle}}$$

Oppgave 3

a)

$$\vec{u} \cdot \vec{v} = [-2,1,-1] \cdot [1,-2,3] = (-2) \cdot 1 + 1 \cdot (-2) + (-1) \cdot 3 \underline{\underline{= -7}}$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \overrightarrow{e_x} & \overrightarrow{e_y} & \overrightarrow{e_z} \\ -2 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix} = \overrightarrow{e_x} + 5\overrightarrow{e_y} + 3\overrightarrow{e_z} \underline{\underline{= [1,5,3]}}$$

b)

$$|\vec{u}| = \sqrt{(-2)^2 + 1^2 + (-1)^2} = \sqrt{6}$$

$$|\vec{v}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} = \frac{-7}{\sqrt{6}\cdot\sqrt{14}} \;\Rightarrow\; \underline{\underline{\theta = 140°}}$$

c)

$\mathit{Normalvektor\ til\ planet\ er}\ \vec{u}\times\vec{v} = [1,5,3]\ \ \mathit{og\ planet\ går\ igjennom}\ O(0,0,0)$

$1(x-0) + 5(y-0) + 3(z-0) = 0$

$\underline{\underline{\mathit{Likning\ for\ planet}\ \alpha\ \mathit{blir}\ \ x + 5y + 3z = 0}}$

d)

$\mathit{Planet}\ \beta\ \mathit{er\ utspent\ av}\ \vec{u} = [-2,1,-1]\ \mathit{og}\ \ \vec{v} = [1,-2,3]\ \mathit{og\ går\ igjennom}\ P(2,3,-2)$

$$\underline{\underline{\mathit{Parameterframstillingen\ blir\ da}: \alpha: \begin{cases} x = 2 - 2s + t \\ y = 3 + s - 2t \\ z = -2 - s + 3t \end{cases}}}$$

e)

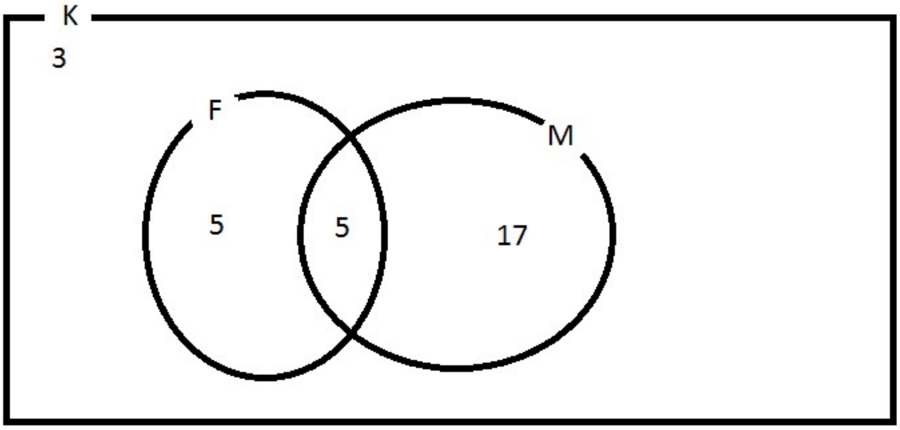
$\overrightarrow{QP} = [2-4, 3-1, -2-(-3)] = [-2,2,1]$

$\mathit{Linjen}\ l\ \mathit{går\ gjennom\ punktet}\ Q(4,1,-3)$

$$\underline{\underline{\mathit{Parameterframstilling\ for\ linjen}\ l\ \mathit{blir}: \quad l: \begin{cases} x = 4 - 2t \\ y = 1 + 2t \\ z = -3 + t \end{cases}}}$$

Oppgave 4

a)



b)

$$P(F) = \frac{10}{30} = \underline{\underline{\frac{1}{3}}}$$

$$P(F \cap M) = \frac{5}{30} = \underline{\underline{\frac{1}{6}}}$$

$$P(F \cup M) = \frac{5+5+17}{30} = \frac{27}{30} = \underline{\underline{\frac{9}{10}}}$$

$$P(F|M) = \underline{\underline{\frac{5}{22}}}$$

$$P(M|\overline{F}) = \underline{\underline{\frac{17}{20}}}$$

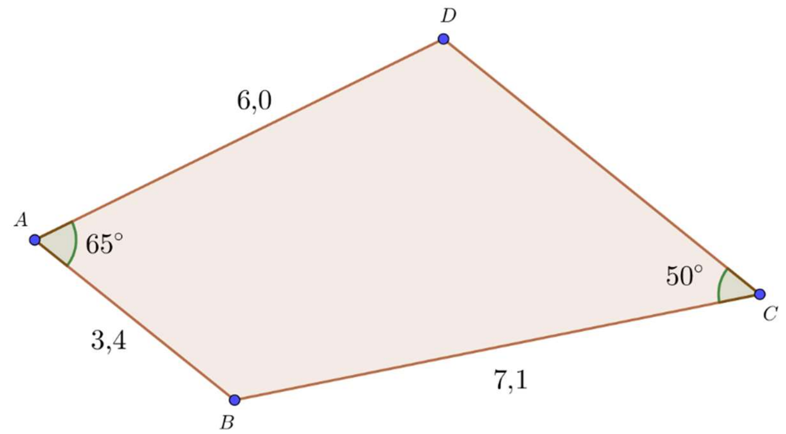
c)

Antall måter det går å velge 5 viner av 7 mulige på er $\binom{7}{5} = \frac{7!}{(7-5)! \cdot 5!} = 21$

Hvis det 〚er 22 deltakere må minst to velge de samme vinene.

<u>Det maksimale antall deltakere blir da 21</u>

Oppgave 5



Trekk linjestykket mellom B og D. Trekanten T_1 til venstre for dette linjestykket

har areal $A_{T_1} = \dfrac{6{,}0 \cdot 3{,}4 \cdot \sin 65°}{2} \approx 9{,}2$

Lengden BD får vi fra cosinussetningen $BD = \sqrt{6{,}0^2 + 3{,}4^2 - 2 \cdot 6{,}0 \cdot 3{,}4 \cdot \cos 65°} \approx 5{,}5$

Vi kan nå finne vinkelen α som spennes opp av DB og DC ved hjelp av sinussetningen

$\alpha = \sin^{-1}\left(\dfrac{7{,}1 \cdot \sin 50°}{5{,}5}\right) \approx 81° \lor \alpha = 180° - 81° = 99°$

Vinkelen β som spennes opp mellom BD og BC er da

$$\beta = 180° - 81° - 50° = 49° \lor \beta = 180° - 99° - 50° = 31°$$

Arealet til trekanten T_2 til høyre for BD blir

$$A_{T_2} = \frac{5{,}5 \cdot 7{,}1 \cdot \sin 49°}{2} \approx 14{,}7 \lor A_{T_2} = \frac{5{,}5 \cdot 7{,}1 \cdot \sin 31°}{2} \approx 10{,}1$$

$A = A_{T_1} + A_{T_2} = 9{,}2 + 14{,}7 \underline{\underline{= 23{,}9}}$

eller

$A = A_{T_1} + A_{T_2} = 9{,}2 + 10{,}1 \underline{\underline{= 19{,}3}}$

Oppgave 6

$y'(x) = (1 - x)y(x)$

$\dfrac{dy}{dx} = (1 - x)y$

$$\frac{dy}{y} = (1-x)dx$$

$$\int \frac{dy}{y} = \int (1-x)dx$$

$$\ln|y| = x - \frac{1}{2}x^2 + C_1$$

$$|y| = e^{x-\frac{1}{2}x^2+C_1}$$

$$|y| = e^{x-\frac{1}{2}x^2} \cdot e^{C_1}$$

$$y = \pm e^{C_1} \cdot e^{x-\frac{1}{2}x^2} = \pm C \cdot e^{x-\frac{1}{2}x^2}$$

Siden starttempraturen er positiv og $e^{x-\frac{1}{2}x^2} > 0$ kan vi se bort i fra den negative løsningen.

$$y(0) = C \cdot e^{0-\frac{1}{2}\cdot 0^2} = 10 \quad \Rightarrow \quad C = 10$$

$$\underline{\underline{y = 10 \cdot e^{x-\frac{1}{2}x^2}}}$$

b)

Største verdi av x får vi for $y(0)$ eller når $y' = 0$

$$y' = 10(1-x)e^{x-\frac{1}{2}x^2} = 0$$

$1 - x = 0 \qquad \vee \qquad e^{x-\frac{1}{2}x^2} = 0$

$x = 1 \qquad\qquad\qquad \emptyset$

$$y(1) = 10 \cdot e^{1-\frac{1}{2}\cdot 1^2} = 10 \cdot e^{\frac{1}{2}} > 10$$

$\underline{\underline{\text{Fortegnsskjema eller den dobbeltderiverte viser at } y(1) = 10 \cdot e^{\frac{1}{2}} \text{ er maksverdi for } y.}}$

c)

$\lim\limits_{x\to\infty}\left(10 \cdot e^{x-\frac{1}{2}x^2}\right) = \lim\limits_{x\to\infty} \frac{10 \cdot e^x}{e^{\frac{1}{2}x^2}} = 0 \qquad siden\ e^{\frac{1}{2}x^2}$ dominerer fullstendig over e^x for store x.

$\underline{\underline{Det\ betyr\ at\ y(x) \to 0\ når\ x \to \infty}}$