tirs. 24. januar

9.8 Produktregelen

① Hvis f og g er deriverbare i x, så er $(f \cdot g)(x) = f(x) \cdot g(x)$ deriverbar i x og

$$(f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Produktfunksjonen $f \cdot g$ er gitt ved $(f \cdot g)(x) = f(x) g(x)$

prikk

Definisjonsmengden består av alle x som er i definisjonsmengden til både f og g.

Eksempel $f(x) = x$ $g(x) = \sqrt{x}$

$$(f \cdot g)(x) = x \cdot \sqrt{x} \quad \left(= x \cdot x^{1/2} = x^{3/2} \right)$$

Deriverer $f \cdot g$ ved å bruke produktregelen:

$$(x \cdot \sqrt{x})' = (x)' \cdot \sqrt{x} + (x) \cdot (\sqrt{x})'$$

$$= 1 \cdot \sqrt{x} + x \cdot \left(\frac{1}{2} \cdot x^{-1/2}\right)$$

$$= \sqrt{x} + x \cdot \frac{1}{2\sqrt{x}}$$

$$= \sqrt{x} + \frac{1}{2} \cdot \sqrt{x} = \underline{\frac{3}{2}\sqrt{x}}$$

Produkt av to potenser:

$f(x) = x^s$ $g(x) = x^r$ r, s reelle tall.

$$(f \cdot g)' = f' g + f \cdot g'$$

$$(x^s \cdot x^r)' = (x^s)' \cdot x^r + x^s (x^r)'$$

→

(2)

$$= s \cdot x^{s-1} \cdot x^{r} + x^{s} \cdot r \cdot x^{r-1}$$

$$= s \cdot x^{s+r-1} + r x^{s+r-1}$$

$$= \underline{(s+r)\, x^{s+r-1}}.$$

(som forventet siden $f \cdot g(x) = x^{s+r}$.)

Eks. Deriver $(2+x)^4 (3-x)^5$.

Dette er lik $f(x) \cdot g(x)$ når

$f(x) = (2+x)^4$ og $g(x) = (3-x)^5$.

$$f'(x) = 4(2+x)^3 (2+x)' = 4(2+x)^3 \cdot 1 = 4(2+x)^3$$

$$g'(x) = 5(3-x)^4 (3-x)' = 5(3-x)^4 (-1) = -5(3-x)^4$$

$$\left((2+x)^4 \cdot (3-x)^5\right)' = \left((2+x)^4\right)' (3-x)^5 + (2+x)^4 \left((3-x)^5\right)'$$

$$= 4(2+x)^3 (3-x)^5 + (2+x)^4 \left(-5(3-x)^4\right)$$

$$= (2+x)^3 (3-x)^4 \left[\underbrace{4(3-x) - 5(2+x)}_{-9x+2}\right]$$

$$= \underline{(2+x)^3 (3-x)^4 (-9x+2)}$$

oppg Deriver $(1-2x)^3(13+5x)^7$

(3) La $f(x) = (1-2x)^3$ og $g(x) = (13+5x)^7$

$$f'(x) = 3(1-2x)^2 \cdot (1-2x)'$$
$$= 3(1-2x)^2(-2)$$
$$= -6(1-2x)^2$$

$$g'(x) = 7(13+5x)^6(13+5x)'$$
$$= 7(13+5x)^6(+5)$$
$$= 35(13+5x)^6$$

$$\left((1-2x)^3(13+5x)^7\right)' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$= -6(1-2x)^2(13+5x)^7 + (1-2x)^3\left(35(13+5x)^6\right)$$

$$= (1-2x)^2(13+5x)^6\left[\underbrace{\underbrace{-6(13+5x) + 35(1-2x)}_{-100x\ +35\ -65\ -13}}_{-100x\ -43}\right]$$

$$= \underline{(1-2x)^2(13+5x)^6(-100x-43)}$$

Eks Deriver $\frac{1}{x} \cdot \left(1+\sqrt[3]{x}\right)^{2,9} = f \cdot g(x)$

(4) La $f(x) = \frac{1}{x} = x^{-1}$ og $g(x) = \left(1+\sqrt[3]{x}\right)^{2,9}$

$$f'(x) = -1 \cdot x^{-2} = \frac{-1}{x^2}$$

$$g'(x) = 2{,}9\left(1+\sqrt[3]{x}\right)^{2,9-1}\left(1+\sqrt[3]{x}\right)'$$

$$= 2{,}9\left(1+\sqrt[3]{x}\right)^{1,9}\left(\frac{1}{3} \cdot x^{1/3-1}\right)$$

$$= 2{,}9\left(1+\sqrt[3]{x}\right)^{1,9} \cdot \frac{1}{3}\, x^{-2/3}$$

$$= \frac{2{,}9}{3}\left(1+\sqrt[3]{x}\right)^{1,9} \frac{1}{\sqrt[3]{x^2}}.$$

$$\left(\frac{1}{x}\left(1+\sqrt[3]{x}\right)^{2,9}\right)' = \left(f \cdot g(x)\right)' = f'(x)\, g(x) + f(x) \cdot g'(x)$$

$$= \frac{-1}{x^2}\left(1+\sqrt[3]{x}\right)^{2,9} + \frac{1}{x}\left(\frac{2{,}9}{3}\left(1+\sqrt[3]{x}\right)^{1,9}\frac{1}{\sqrt[3]{x^2}}\right.$$

$$= \underline{\frac{\left(1+\sqrt[3]{x}\right)^{1,9}}{x^2}\left[-\left(1+\sqrt[3]{x}\right) + \frac{2{,}9}{3}\sqrt[3]{x}\right]}$$

Oppg 5 Deriver $(\sqrt{x}-x)^5(1+x^2) = f \cdot g(x)$

$$f(x) = (\sqrt{x}-x)^5 \qquad g(x) = 1+x^2$$

$$g'(x) = (1+x^2)' = (1)' + (x^2)'$$
$$= 0 + 2x = \underline{2x}$$

$$f'(x) = 5(\sqrt{x}-x)^4(\sqrt{x}-x)'$$
$$= 5(\sqrt{x}-x)^4\big(\underbrace{x^{1/2}-x}_{\left(\frac{1}{2}x^{\frac{1}{2}-1}-1\right)}\big)'$$
$$= 5(\sqrt{x}-x)^4\left(\frac{1}{2\sqrt{x}}-1\right)$$

$$\frac{d}{dx}\left((\sqrt{x}-x)^5(1+x^2)\right) = (f\cdot g)'(x)$$
$$= f'(x)\cdot g(x) + f(x)\cdot g'(x)$$
$$= 5(\sqrt{x}-x)^4\left(\frac{1}{2\sqrt{x}}-1\right)(1+x^2) + (\sqrt{x}-x)^5\cdot 2x$$
$$= (\sqrt{x}-x)^4\left[5\left(\frac{1}{2\sqrt{x}}-1\right)(1+x^2) + (\sqrt{x}-x)\cdot 2x\right]$$
$$= \frac{(\sqrt{x}-x)^4}{2\sqrt{x}}\left[5(1-2\sqrt{x})(1+x^2) + 2(\sqrt{x}-x)x\cdot 2\sqrt{x}\right]$$
$$= \frac{(\sqrt{x}-x)^4}{2\sqrt{x}}\left[5(1-2\sqrt{x}+x^2-2x^2\sqrt{x}) + (4x^2-4x^2\sqrt{x})\right]$$
$$= \frac{(\sqrt{x}-x)^4}{2\sqrt{x}}\left[5-10\sqrt{x}+9x^2-14x^2\sqrt{x}\right]$$

$$(f\cdot g)'(x) = \lim_{\Delta x\to 0} \frac{(f\cdot g)(x+\Delta x) - f(x)\,g(x)}{\Delta x}$$

$f(x+\Delta x) - f(x) = \Delta f$ tilsvarende for Δg

$$\lim_{\Delta x\to 0} \frac{(f(x)+\Delta f)(g(x)+\Delta g) - f(x)\cdot g(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{f(x)\,g(x) + f(x)\cdot\Delta g + \Delta f\cdot g(x) + \Delta f\cdot\Delta g - f(x)g(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{f(x)\cdot\Delta g + \Delta f\cdot g(x) + \Delta f\,\Delta g}{\Delta x}$$

$$= \lim_{\Delta x\to 0} f(x)\cdot\frac{\Delta g}{\Delta x} + \frac{\Delta f}{\Delta x}\cdot g + \frac{\Delta f\,\Delta g}{\Delta x}$$

$$= f(x)\lim_{\Delta x\to 0}\frac{\Delta g}{\Delta x} + \left(\lim_{\Delta x\to 0}\frac{\Delta f}{\Delta x}\right) g + \lim_{\Delta x\to 0}\frac{\Delta f}{\Delta x}\cdot\Delta g$$

$$= f(x)\,g'(x) + f'(x)\cdot g(x) + 0$$

Derfor er

$$(f\cdot g)'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x).$$