14 feb. 2012

10.7 Trigonometriske ulikheter

(1) (og 10.1, 10.2 trigonometriske likninger)

Trig. Ulikhet $\quad \sin x > \frac{1}{2}$

Trig likning $\quad \sin x = \frac{1}{2}$

$$\sin(f(x)) = \frac{1}{2} \qquad \text{ex} \quad \sin(2x-3) = \frac{1}{2}.$$

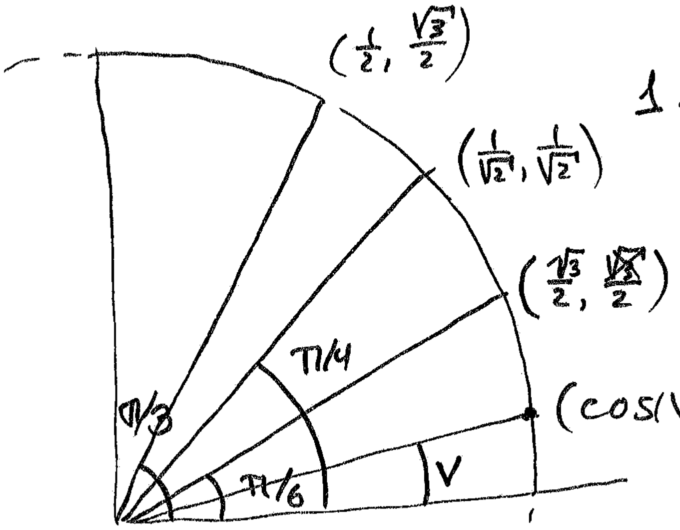
Først : Løs likningen $\sin(u) = \frac{1}{2}$

Deretter : Løs likningen $f(x) = u$ (for løsningene ovenfor).

Det kan av og til være nyttig å bruke trig. identiteter

$$\sin^2 x + \cos^2 x = 1 \qquad \cos 2x = \cos^2 x - \sin^2 x \text{ etc.}$$

$$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\left(\sin x + \cos x\right)$$



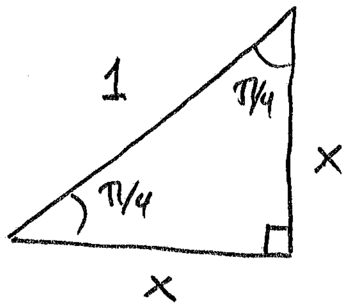
1. kvadrant

$$\frac{1}{2} < \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} < \frac{\sqrt{3}}{2}$$

$$0.50 < 0.707.. < 0.866..$$

Så $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ og $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ etc.

(2)



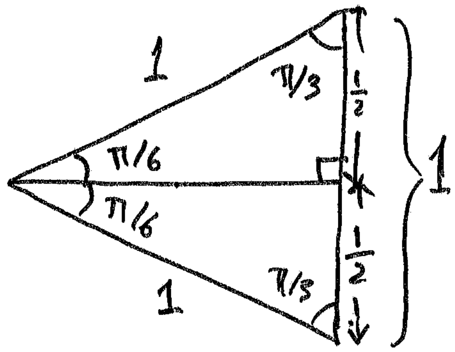
Pytagoras:

$$x^2 + x^2 = 1^2 = 1$$

$$2x^2 = 1$$

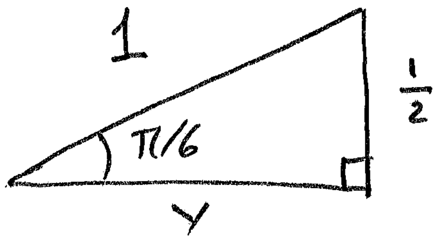
$$x^2 = \frac{1}{2} \qquad x > 0$$

$$\underline{x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}}$$



(30°, 60°, 90°)

Den store trekanten er likesidet (60°, 60°, 60°)



Pytagoras:

$$1^2 = \left(\frac{1}{2}\right)^2 + y^2$$

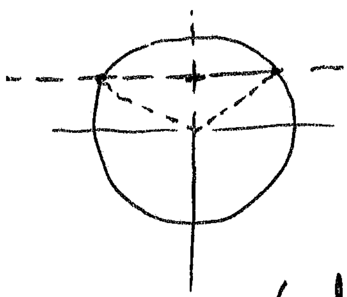
$$1 = \frac{1}{4} + y^2$$

$$y^2 = 1 - \frac{1}{4} = \frac{3}{4} \qquad y > 0$$

$$y = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

(3)

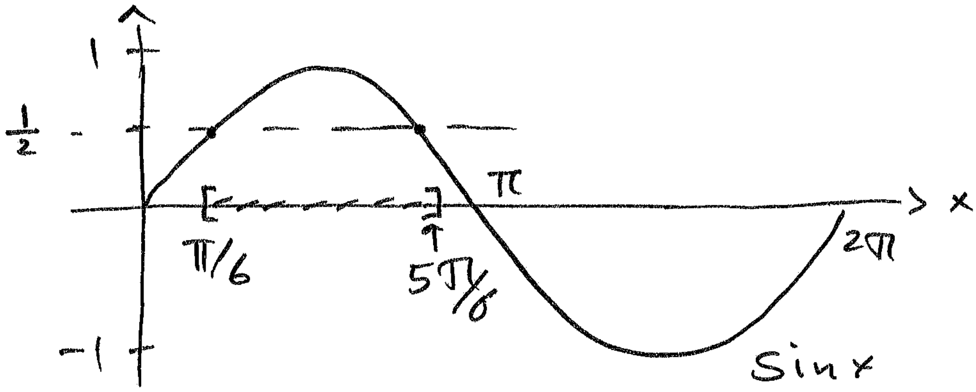
ex $\quad \sin x \geq \frac{1}{2} \qquad x \in [0, 2\pi]$



$\left(\sin x = \frac{1}{2} \ : \ x = \frac{\pi}{6}, \ \pi - \frac{\pi}{6} = \frac{5\pi}{6} \right)$

Løsningen er $\quad x \in \left[\frac{\pi}{6}, \frac{5\pi}{6} \right]$

(alternativ skrive måte $\quad \frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$)



Løsningen er $\quad x \in \left[\frac{\pi}{6}, \frac{5\pi}{6} \right]$.

ex $\quad \cos x \cdot \sin x \geq \frac{1}{4} \qquad \times 2$ på begge side $(2 > 0)$

$$\underbrace{2\cos x \cdot \sin x} \geq \frac{1}{2}$$

$$\sin(2x) \geq \frac{1}{2}$$

La $\quad u = 2x \qquad \sin(u) \geq \frac{1}{2}$

Fra eksempelet ovenfor er løsningene :

$$u \in \left[\frac{\pi}{6} + 2\pi \cdot n, \ \frac{5\pi}{6} + 2\pi \cdot n \right] \qquad n \text{ heltall.}$$

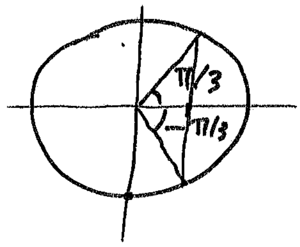
$u = 2x \quad$ så $\quad x = \frac{u}{2}$.

Løsningene av ulikheten $\quad \cos x \cdot \sin x \geq \frac{1}{4}$ er

(Unionen av) $\quad \underline{\left[\frac{\pi}{12} + \pi \cdot n, \ \frac{5\pi}{12} + \pi \cdot n \right]} \qquad n$ heltall oppg. Løs ulikheten

4 $\cos\left(\frac{x}{2}\right) > \frac{1}{2}$ $x \in [-\pi, \pi]$.

La $u = \frac{x}{2}$ $u \in [-\frac{\pi}{2}, \frac{\pi}{2}]$



$\cos(u) > \frac{1}{2}$

Løsningen er $\frac{-\pi}{3} < u < \frac{\pi}{3}$

$\left(\frac{-\pi}{3} < \frac{x}{2} < \frac{\pi}{3}\right) \cdot 2$

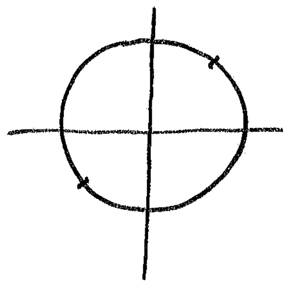
$\underline{\frac{-2\pi}{3} < x < \frac{2\pi}{3}}$

Løsningen til ulikheten $\cos\left(\frac{x}{2}\right) > \frac{1}{2}$ $x \in [-\pi, \pi]$

er $\underline{x \in \left\langle \frac{-2\pi}{3}, \frac{2\pi}{3} \right\rangle}$.

(eller $\left(-\frac{2\pi}{3}, \frac{2\pi}{3}\right)$)

ex $\sin x - \cos x > 0$ $x \in [0, 2\pi]$



$\sin x > \cos x$

Fra enhetsirkelen er løsningen

$\underline{x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)}$

Alternativt: Anta $\cos x \neq 0$ $\left(x \neq \frac{\pi}{2}, \frac{3\pi}{2}\right)$

Deler med $\cos x$: $\tan x > 1$ $\cos x > 0$

$\tan x < 1$ $\cos x < 0$.

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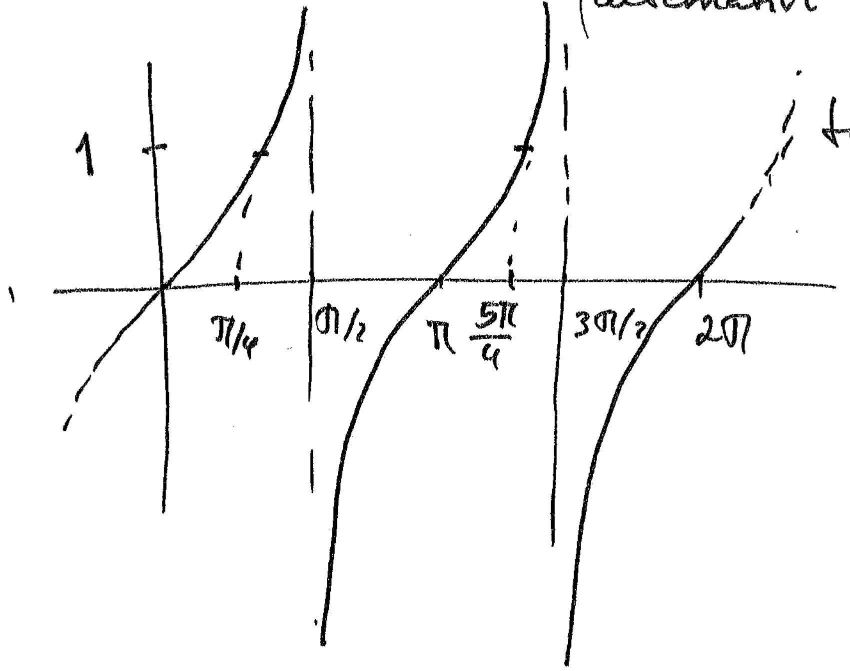
ex $\tan x > 1$ $\quad x \in [0, 2\pi] \setminus \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

(alternativt: $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi]$)



Fra grafen ser vi at løsningene er

$(\frac{\pi}{4}, \frac{\pi}{2}) \cup (\frac{5\pi}{4}, \frac{3\pi}{2})$

Tilsvarende er løsningen til $\tan x < 1$ (samme område)

$x \in [0, \frac{\pi}{4}) \cup (\frac{\pi}{2}, \frac{5\pi}{4}) \cup (3\pi/2, 2\pi]$.

Vi relaterer dette til foregående eksempel.

Løsningen til $\tan x > 1$ (når $\cos x > 0$) og

$\tan x < 1$ (når $\cos x < 0$) er:

$(\frac{\pi}{4}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{5\pi}{4})$

I tillegg har $\sin x - \cos x > 0$ løsningen $x = \frac{\pi}{2}$ når $\cos x = 0$.

Derfor blir løsningen til $\sin x - \cos x > 0$ $\quad x \in [0, 2\pi]$

$(\frac{\pi}{4}, \frac{5\pi}{4})$

oppg. Løs ulikheten

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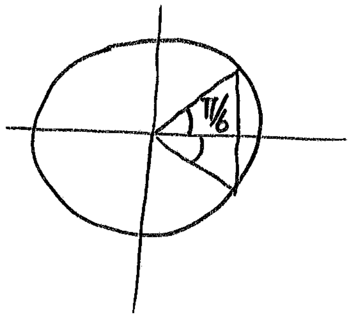
$$\cos(\pi x - \pi) \leq \frac{\sqrt{3}}{2} \qquad x \in [0, 2].$$

$$u = \pi x - \pi \qquad u \in [-\pi, \pi]$$

$$\cos(u) \leq \sqrt{3}/2$$



Løsningen er

$$u \in [-\pi, -\frac{\pi}{6}] \cup [\frac{\pi}{6}, \pi]$$

Finner x-verdiene (løsningene)

1) $$-\pi \leq \pi x - \pi \leq \frac{-\pi}{6} \quad | +\pi$$

$$0 \leq \pi x \leq \pi - \frac{\pi}{6} \quad | \text{ deler med } \pi > 0$$

$$\underline{0 \leq x \leq \frac{5}{6}}.$$

2) $$\frac{\pi}{6} \leq \pi x - \pi \leq \pi \quad | +\pi$$

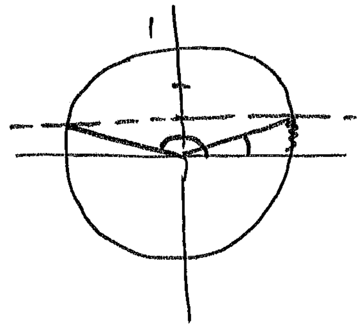
$$\pi + \frac{\pi}{6} \leq \pi \cdot x \leq 2\pi \quad | \text{ deler med } \pi > 0$$

$$\underline{\frac{7}{6} \leq x \leq 2}$$

Løsningene er $\underline{\underline{x \in [0, \frac{5}{6}] \cup [\frac{7}{6}, 2]}}$

eks. $\sin x < \frac{1}{3}$ $x \in [0, 2\pi]$.

(7)



$\sin x = \frac{1}{3}$.

Løsning: $v = \arcsin \frac{1}{3} = 0.339836\ldots$

$(19,47\ldots°)$

Løsningen til ulikheten $\sin x < \frac{1}{3}$ $x \in [0, 2\pi]$

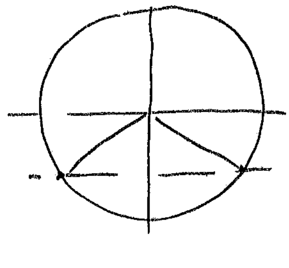
er $\underline{\underline{x \in [0, v) \cup (\pi - v, 2\pi]}}$

(setter inn tallverdier : $x \in [0, 0.3398\ldots) \cup (2{,}8017\ldots, 2\pi]$.)

eks/oppg. $\sqrt{2}\sin(x - \frac{\pi}{4}) + 1 > 0$ $x \in [0, 2\pi]$

$\sqrt{2}\sin(x - \frac{\pi}{4}) > -1$

$\sin(x - \frac{\pi}{4}) > \frac{-1}{\sqrt{2}}$



$u = x - \frac{\pi}{4}$ $\frac{-\pi}{4} \le u \le \frac{7\pi}{4}$ (def. mengde)

$\sin u > \frac{-1}{\sqrt{2}}$ har løsning : $u \in (\frac{-\pi}{4}, \frac{5\pi}{4})$.

$-\frac{\pi}{4} < x - \frac{\pi}{4} < \frac{5\pi}{4}$ legger til $\frac{\pi}{4}$.

$0 < x < \frac{6\pi}{4} = \frac{3\pi}{2}$.

Løsningen er $\underline{\underline{x \in (0, \frac{3\pi}{2})}}$