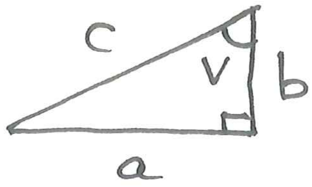
21.09.2018 Faersh

(1)



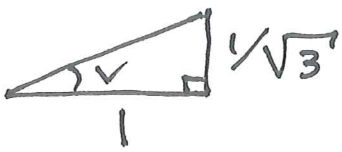
hosliggende katet

motstående katet

$$c \cdot \sin(V) = a$$

$$c \cdot \cos(V) = b$$

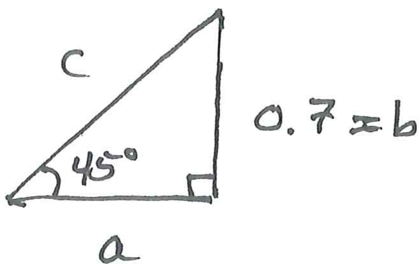
$$\tan(V) = \frac{\sin(V)}{\cos(V)} = \frac{a}{b}$$



$$\tan(V) = \frac{1/\sqrt{3}}{1} = \frac{1}{\sqrt{3}}$$

$$V = \arctan\left(\frac{1}{\sqrt{3}}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$



Likebeina trekant:

$a = b = 0.7$

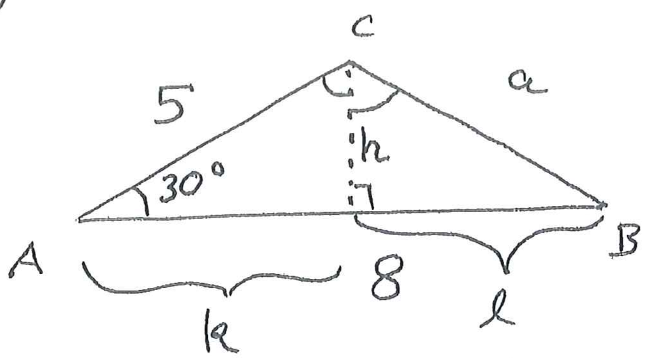
$\left(c = \sqrt{a^2 + b^2} \text{ ved Pytagoras} \right)$

$$\sin(45^\circ) = \frac{b}{c} \quad \text{så} \quad c = \frac{b}{\sin(45^\circ)}$$

$$= \frac{0.7}{1/\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 0.7 \cdot \sqrt{2} = 0.98995\ldots$$

(0.990)

(2)



Hva er lengden a?

Hva er vinklene B og C.

$k + l = 8$

$h = 5 \cdot \sin(30°) = 5 \cdot \frac{1}{2} = 5/2.$

$k = 5 \cdot \cos(30°) = 5 \cdot \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$

$l = 8 - k = 8 - \frac{5\sqrt{3}}{2}$

$\tan(\angle B) = \tan(B) = \frac{h}{l} = \frac{5/2}{8 - 5\sqrt{3}/2} = \frac{5}{16 - 5\sqrt{3}}$

$B = \arctan\left(\frac{h}{l}\right)$

$B = \arctan(0.6812..)$

$= 34.26... \approx 34°$ (2 gyldige siffer)

$\angle C = 180° - 30° - 34° = (180 - 64)°$

Vinkel C er $C = \underline{116°}$

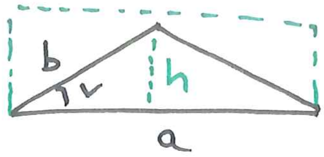
$a = \sqrt{l^2 + h^2}$ eller $a = \frac{h}{\sin(B)}$

$= \sqrt{\left(\frac{5}{2}\right)^2 + \left(8 - \frac{5\sqrt{3}}{2}\right)^2} \approx 4.4$

(hvis dette benyttes bør $B = 34.26°$ brukes heller enn $34°$)

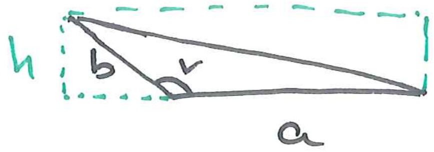
Arealsetningen

③



$h = b\sin(v)$

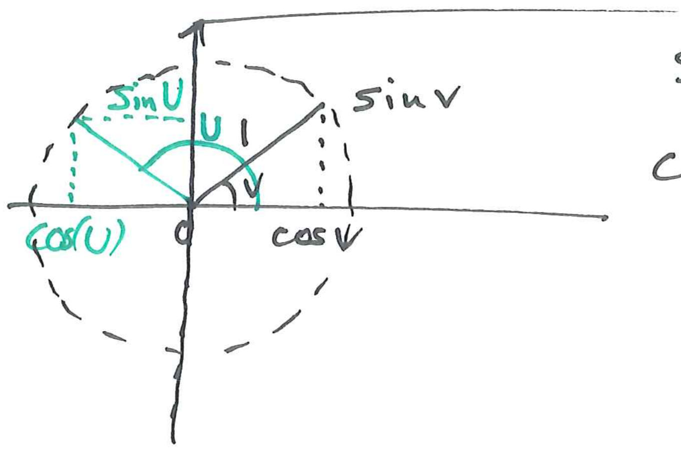
Arealet til trekanten er $A = \dfrac{a \cdot h}{2}$



utvider $\sin(v)$

slik at formelen $A = \dfrac{a \cdot b\sin(v)}{2}$

er gyldig for alle $0° < v < 180°$.



$\sin(180° - v) = \sin(v)$

$\cos(180° - v) = -\cos(v)$

Det er to vinkler mellom 0° og 180°

slik at $\sin(v) = 1/2$

30° og $180° - 30° = 150°$

Trekant: to av sidene har lengde

(4) 12cm og 2dm, vinkelen mellom de to sidene er 30°.

Hva er arealet i cm^2, dm^2?

$$A = a \cdot b \sin(V) / 2$$

$$= 12cm \cdot 2dm \cdot \underbrace{\sin(30°)}_{1/2} \cdot \frac{1}{2}$$

$$= 6\ cm \cdot dm$$

$$= 6\ cm\ (10 cm) = \underline{60 cm^2}$$

$$= 6\left(\frac{1}{10} dm\right) \cdot 1dm = \frac{6}{10} dm^2$$

$$\left(\begin{array}{l} 1cm = \frac{1}{100} m \\ 1dm = \frac{1}{10} m \\ 10cm = 1dm \end{array}\right)$$

$$(10\ cm)^2 = (1dm)^2$$

$$\underline{100 cm^2 = 1 dm^2}$$

$$(100 cm)^2 = (1m)^2$$

$$\underline{10\,000\ cm^2 = 1m^2}$$

En trekant har areal $3dm^2$ og to av sidene har lengde 2dm og 6dm

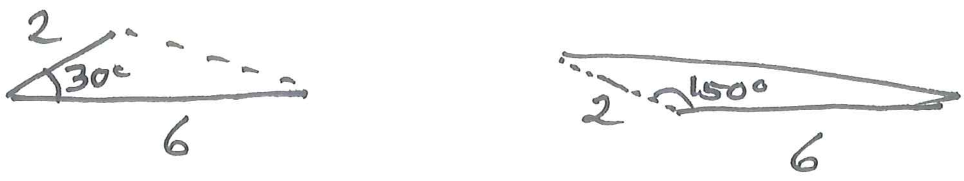
Hva er vinkelen mellom sidene?

$$A = a \cdot b \cdot \sin(V) / 2$$

så $$\sin(V) = \frac{2A}{a \cdot b} = \frac{2 \cdot 3dm^2}{2dm \cdot 6dm} = \frac{1}{2}$$

$$V = 30° \quad \text{eller} \quad V = 180° - 30° = 150°$$

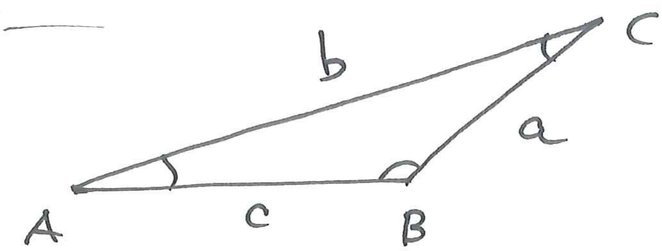
5



Trekanten må være én av disse to!

Sinussetningen

(Bruker arealsetningen for hver av de 3 vinklene)



Arealet til trekanten er lik

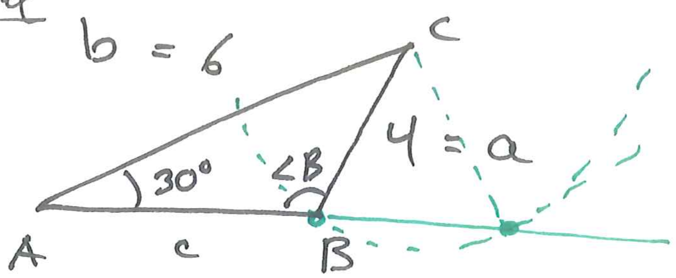
$$A = \frac{1}{2} b \cdot c \cdot \sin(\angle A) = \frac{1}{2} a \cdot c \sin(\angle B)$$
$$= \frac{1}{2} a \cdot b \sin(\angle C)$$

Ganger med $2/(abc)$:

$$\boxed{\frac{\sin(\angle A)}{a} = \frac{\sin(\angle B)}{b} = \frac{\sin(\angle C)}{c}}$$

Eksempel

(6)



Hva er $\angle B$

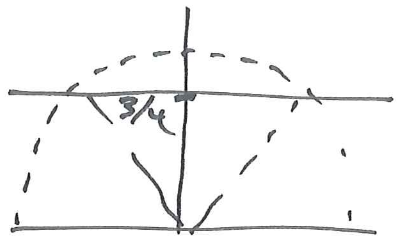
$\angle C$

og c

$$\frac{\sin(\angle A)}{a} = \frac{\sin(30^\circ)}{4} = \frac{1/2}{4} = \frac{1}{8}$$

sinussetningen gir $\frac{1}{8} = \frac{\sin(\angle C)}{c} = \frac{\sin(\angle B)}{b}$

$$\sin(\angle B) = b \cdot \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$$



$\angle B = \underline{48.59^\circ}$

og $\angle B = 180^\circ - 48.59 = \underline{131.41^\circ}$

to løsninger!

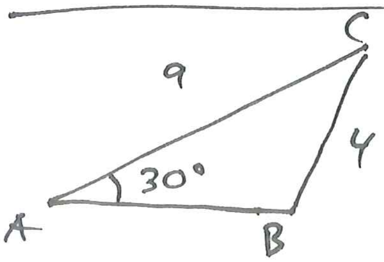
summen av vinklene er 180°:

Tilfelle I $\quad \angle B = 48.59^\circ, \quad \angle C = 180^\circ - 30^\circ - 48.59 = \underline{101.41^\circ}$

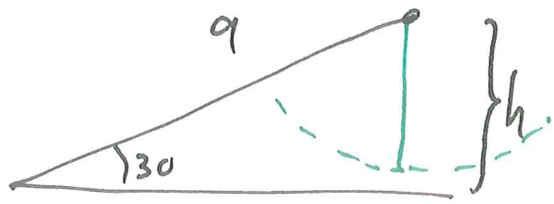
$$c = 8\sin(\angle C) = 7.84$$

Tilfelle II $\quad \angle B = 131.41^\circ \quad \angle C = 180^\circ - 30^\circ - 131.41^\circ = \underline{18.59^\circ}$

$$c = 8 \cdot \sin(\angle C) = 2.55$$



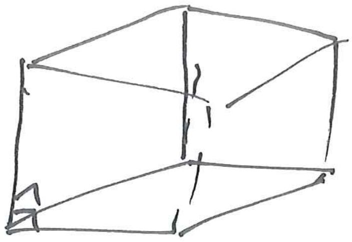
Det finnes ikke en slik trekant!



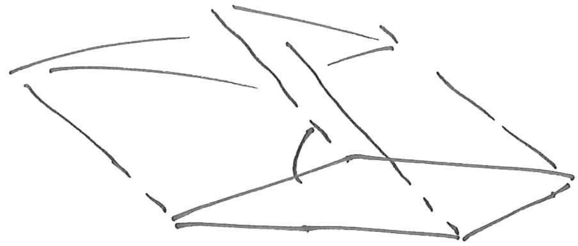
$$h = 9\sin(30^\circ) = \frac{9}{2} = 4.5 > 4$$

Prismer

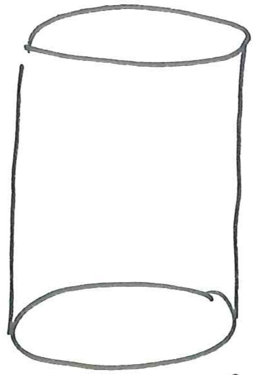
(7)



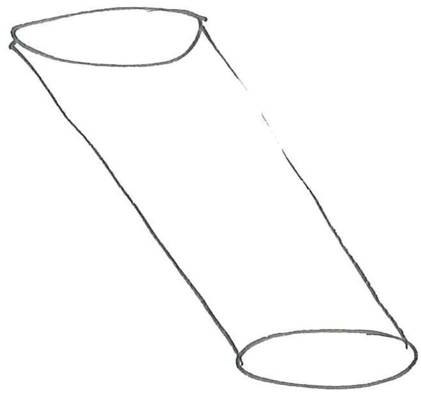
Rett prisme



skeiv prisme



Rett sylinder



skeiv sylinder

Volum = areal grunnflate × høyde

Eks $a = 10$ cm $b = 20$ cm

$c = 30$ cm lengden på sidene

i et rett prisme.

$V = 10\,cm \cdot 20\,cm \cdot 30\,cm = 6000\,cm^3$

$1\,dm \cdot 2\,dm \cdot 3\,dm = 6\,dm^3$

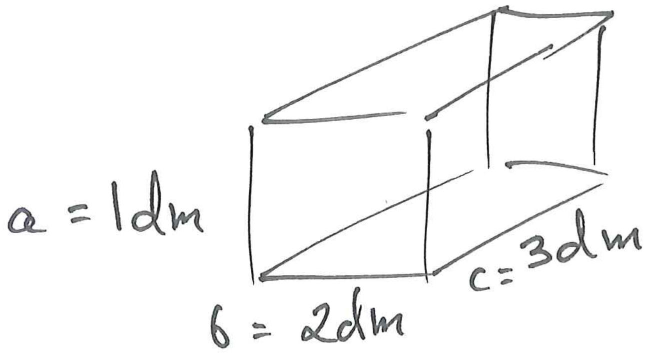
(8)

$$(1\,dm)^3 = (10\,cm)^3$$
$$1\,dm^3 = 10^3\,cm^3 = 1000\,cm^3$$

$$1\text{ liter} = 1\,dm^3 = 1000\,cm^3$$

$$1\,m^3 = (10\,dm)^3 = 1000\,dm^3$$
$$= 1000\,L$$

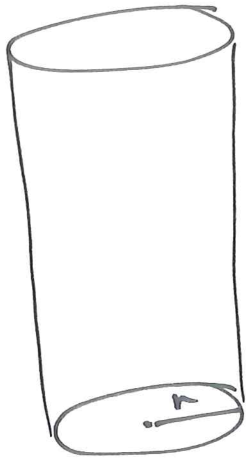
$$1\,m^3 = (100\,cm)^3$$
$$= (10^2\,cm)^3 = (10^2)^3 \cdot cm^3$$
$$= 10^6\,cm^3 = \underline{1\,000\,000\,cm^3}$$



Hva er overflatearealet?

$$A = 2a \cdot b + 2bc + 2a \cdot c$$
$$= 2[1 \cdot 2\,dm^2 + 2 \cdot 3\,dm^2 + 1 \cdot 3\,dm^2]$$
$$= 2(2+6+3)\,dm^2 = 22\,dm^2$$

(9)

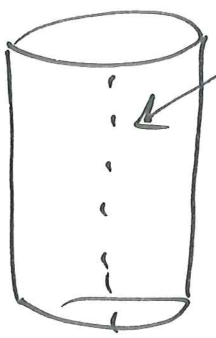


grunnflaten har areal πr^2

Volum : $V = \pi r^2 \cdot h$

Overflate areal :

$$A = 2(\pi r^2) + 2\pi r h = \underline{2\pi r (r + h)}$$



kutter og bretter ut