15.01.2020

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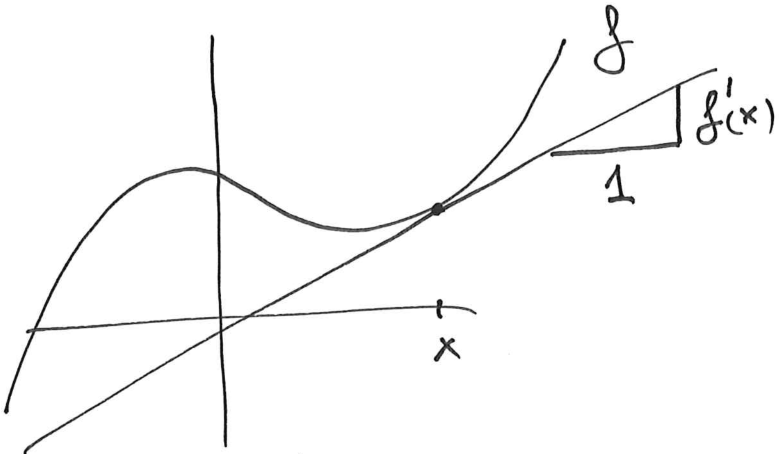
Derivasjon

Den deriverte til $f(x)$

$$\frac{d}{dx}f(x) = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\left(= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \right)$$

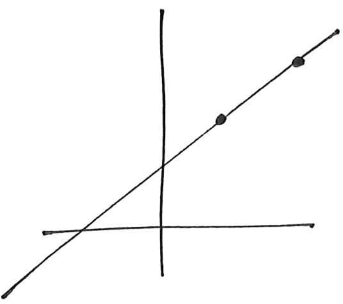


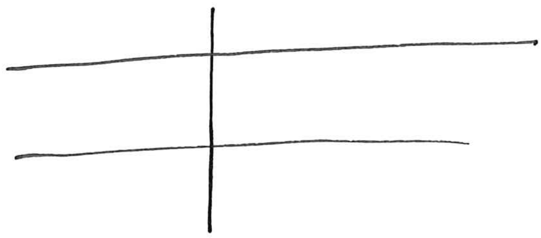
stigningstallet til tangentlinjen (gjennom $(x, f(x))$)

tangentlinje

Eks $\quad f(x) = ax + b \qquad a, b \in \mathbb{R}$

$f'(x) = a$ for alle x





$y(x) = b$

$y'(x) = 0$

Oppg. Finn den deriverte til

1) $f(x) = -3x + \pi$ $\qquad$ 2) $g(t) = \frac{2t+3}{5}$

1) $f'(x) = -3$

2) $g(t) = \left(\frac{2}{5}\right)t + \frac{3}{5}$

$g'(t) = \frac{2}{5}$.

(2) $f(x) = k \cdot x^2$

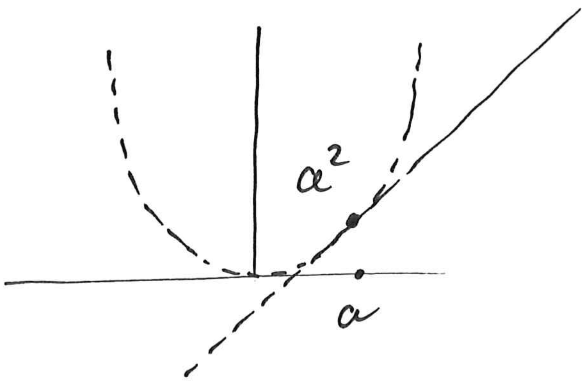
$$f'(x) = \lim_{h \to 0} \frac{k(x+h)^2 - kx^2}{h}$$

$$= k \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= k \lim_{h \to 0} \frac{2xh + h^2}{h} = k \lim_{h \to 0} (2x+h) = \underline{2kx}$$

$(x^2)' = 2x$ $\qquad$ $(kx^2)' = 2kx$

Finn tangentlinjen til $y = x^2$ i (a, a^2)



Tangenlinjen er $\quad y = y'(a)(x-a) + a^2$

$$\underline{y(x) = 2a(x-a) + a^2}$$

$a = 0 \qquad y = 0 \qquad$ (x-aksen)

$a = 1 \qquad y = 2(x-1) + 1 = 2x - 1$

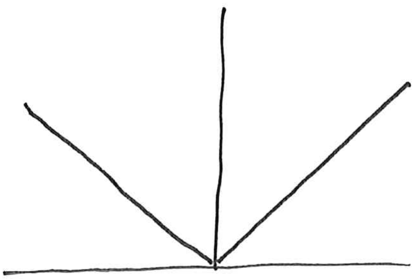
$a = -1 \qquad y = -2(x-(-1)) + (-1)^2 = -2x - 1$

(Likning for en linje med stigningstall a gjennom (x_0, y_0) er $\quad y = a(x - x_0) + y_0$)

③ Hvis $f(x)$ er deriverbar i a så er $f(x)$ også kontinuerlig i a.

Eksempel $\quad f(x) = |x| \quad$ kontinuerlig for alle x

$$= \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



"knekk punkt"

$$f'(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

Ikkje deriverbar i 0.

$$\lim_{\Delta x \to 0^+} \frac{\Delta f}{\Delta x} = +1 \,, \quad \lim_{\Delta x \to 0^-} \frac{\Delta f}{\Delta x} = -1$$

så $\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$ eksisterer ikkje

Derivasjon er lineær.

$$(k \cdot f(x))' = k \cdot f'(x) \qquad k \text{ konstant}$$

$$(\underbrace{f(x) + g(x)}_{(f+g)(x)})' = f'(x) + g'(x)$$

Følger fra grensesetningene

$$(f+g)'(x) = \lim_{h \to 0} \frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h}$$

$$= \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right) =$$

④

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

$$= f'(x) + g'(x)$$

Tilsvarende for $k \cdot f(x)$.

Eks. $f(x) = 3x^2 - 7x + 5$

$$f'(x) = (3x^2)' + (-7x)' + (5)'$$

$$= 3(x^2)' - 7(x)' + (5)'$$

$$= 3(2x) - 7(1) + 0$$

$$= \underline{6 \cdot x - 7}$$

oppg Finn den deriverte til

1) $f(x) = -3x^2 + 2$ 2) $g(x) = \frac{(x+1)(x-1)}{3}$

1) $f'(x) = -3(x^2)' + 2' = -3(2x) = -6x$

2) $g(x) = \frac{(x+1)(x-1)}{3} = \frac{1}{3}(x^2-1)$

$g'(x) = \frac{1}{3}(x^2-1)' = \frac{2x}{3}$.

$$\left(g'(x) \neq \frac{1}{3}(x+1)'(x-1)' = \frac{1}{3} \cdot 1 \cdot 1 = \frac{1}{3} \right)$$

$$(a(x) \cdot b(x))' \neq a'(x) \cdot b'(x)$$

PRODUKT REGELEN

$$(a(x) \cdot b(x))' = a(x) \cdot b'(x) + a'(x) \cdot b(x)$$

kjerneregelen

$$((x-1)(x+1))' = (x-1)'(x+1) + (x-1)(x+1)' = (x+1) + (x-1)$$
$$= 2x$$

⑤ $(x^n)' = n x^{n-1}$ n naturlig tall

bevis: $x^n - 1 = (x-1)(\underbrace{x^{n-1} + x^{n-2} + \cdots + x + 1}_{\text{geometrisk rekke}})$

La $x = \frac{b}{a}$ gang med a^n på begge sider av likhetstegnet:

$$b^n - a^n = (b-a)(b^{n-1} + b^{n-2}a + \cdots + ba^{n-2} + a^{n-1})$$

så
$$(x+h)^n - x^n = (\overbrace{x+h-x}^{h})((x+h)^{n-1} + (x+h)^{n-2}x + \cdots x^{n-1})$$

så $$(x^n)' = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h\to 0}\Big((x+h)^{n-1} + (x+h)^{n-2}x + \cdots (x+h)x^{n-2} + x^{n-1}\Big)$$

$$= \underline{n \cdot x^{n-1}}$$

TIL ORIENTERING

Eks $(x^4)' = 4x^3$

$(3x)' = (3 \cdot x^1)' = 3(x^1)' = 3 \cdot (1x^0)$
$= 3$

$\Big((5)' = (5x^0)' = 5(0 \cdot x^{-1}) = 0 \ldots\Big)$

$(13x^7)' = 13(x^7)' = 13(7x^6)$

$13 \cdot 7 \cdot x^6 = (10+3)(10-3)\, x^6$
$= \underline{91 \cdot x^6}$

6

$$p(x) = 5x^6 - 7\cdot x^5 + 3x^2 - 2$$

$$p'(x) = 5(x^6)' + (-7)(x^5)' + 3(x^2)' + (-2)'$$

$$= 5(6x^5) + (-7)(5x^4) + 3(2x) + 0$$

$$p'(x) = \underline{30x^5 - 35x^4 + 6x}$$

oppg. $p(x) = -7x^{11} + 5x^8 + 3x^3 - 12$

Finn $p'(x)$.

$$p'(x) = -7(x^{11})' + 5(x^8)' + 3(x^3)' + (-12)'$$

$$= -7(11\cdot x^{10}) + 5(8x^7) + 3(3x^2) + 0$$

$$= -77x^{10} + 40x^7 + 9x^2$$

Eks. $f(x) = x^3 + 3x^2 - 2$

For hvilke x har tangentlinjen stigningstall lik 9.

stigningstallet til tangentlinjen i x er $f'(x)$

så $f'(x) = 9$

$$f'(x) = (x^3)' + 3(x^2)' + (-2)' = 3x^2 + 6x$$

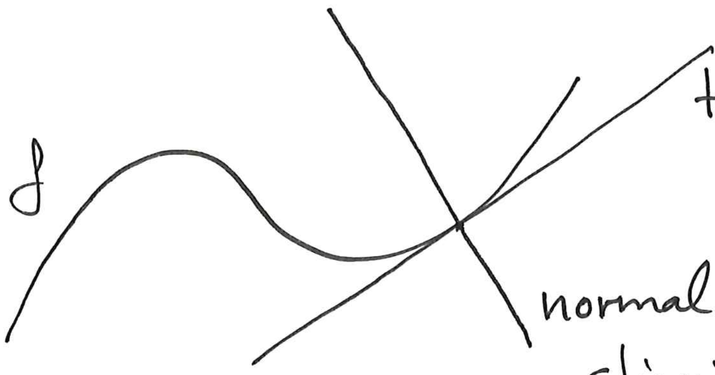
$$f'(x) = 3x^2 + 6x = 9$$

$$\Leftrightarrow \quad x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

Løsningene er $x = -3$ og $x = 1$

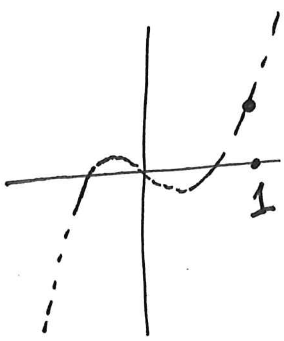
⑦



hvis $f'(a) = 0$, er normallinjen vertikal

oppg. Finn likningen for både tangent og normallinje til

$f(x) = 2x^3 - x$ i $x = 1$



$$f'(x) = 2(x^3)' - (x)'$$
$$= 2(3x^2) - 1$$
$$= 6x^2 - 1 .$$

$f'(1) = 6 \cdot 1^2 - 1 = 5$

går gjennom $(1, f(1)) = (1, 1)$

Tangentlinjen $y = 5(x-1) + 1$

$\underline{y = 5x - 4}$

Normallinjen stigningstall $\frac{-1}{f'(1)} = \frac{-1}{5}$

$$y = \frac{-1}{5}(x-1) + 1$$
$$\underline{y = -\frac{x}{5} + \frac{6}{5}}$$

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$$\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$$

$$(x^{-1})' = (-1)\cdot x^{-1-1} = -1\cdot x^{-2} = \frac{-1}{x^2}$$

Fra definisjonen av den deriverte

$$\left(\frac{1}{x}\right)' = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{x-(x+h)}{(x+h)\cdot x}\right)}{h} = \lim_{h\to 0} \frac{1}{\cancel{h}}\left(\frac{-h}{(x+h)\cdot x}\right)$$

$$= \lim_{h\to 0} \frac{-1}{(x+h)\cdot x} = \underline{\frac{-1}{x^2}}$$ ✓