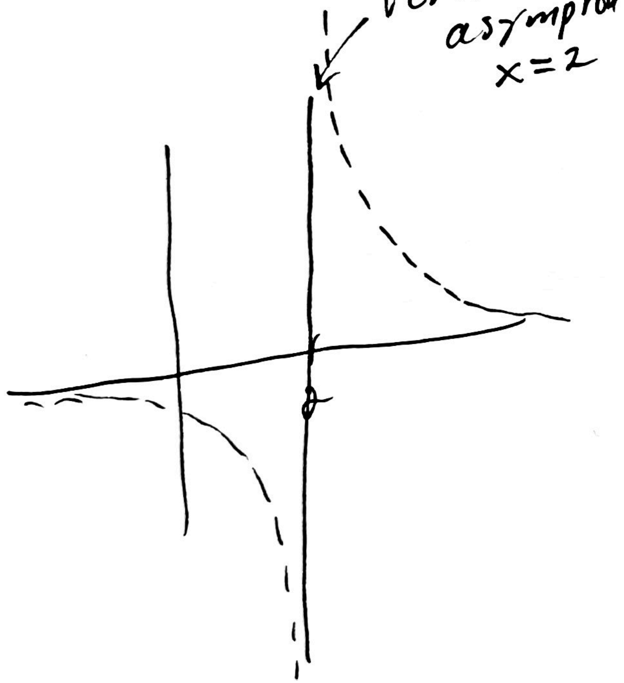
6.01.2021

6.3 Vertikale asymptoter

$f(x) = \frac{1}{x-2} \qquad x \neq 2$



$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{1}{x-2} = \infty$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{1}{x-2} = -\infty$

En funksjon $f(x)$ har en vertikal asymptote i $x=a$ hvis

$\lim_{x\to a^+} f(x) = +\infty$ (eller $-\infty$)

eller $\lim_{x\to a^-} f(x) = +\infty$ (eller $-\infty$)

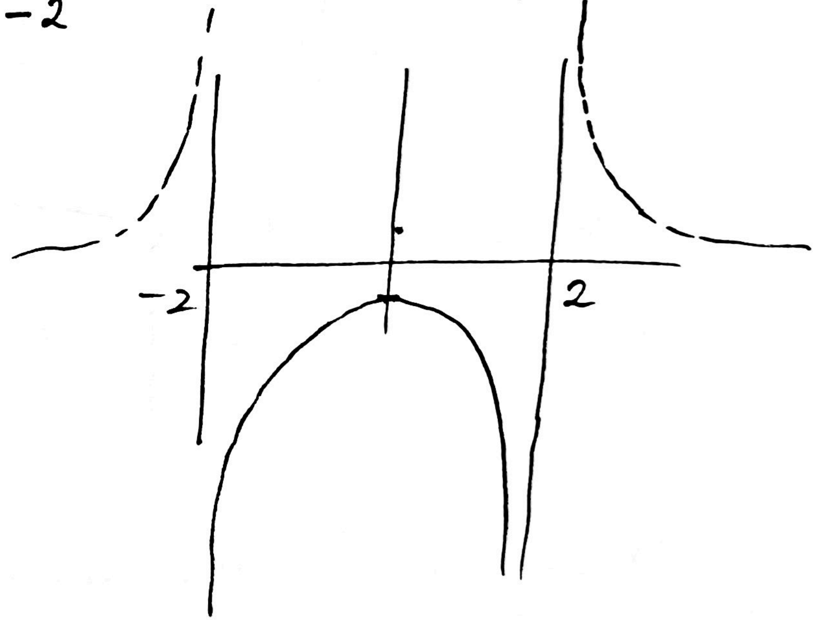
$$g(x) = \frac{1}{x^2-4} \qquad x \neq -2, 2$$

$$= \frac{1}{(x-2)(x+2)}$$

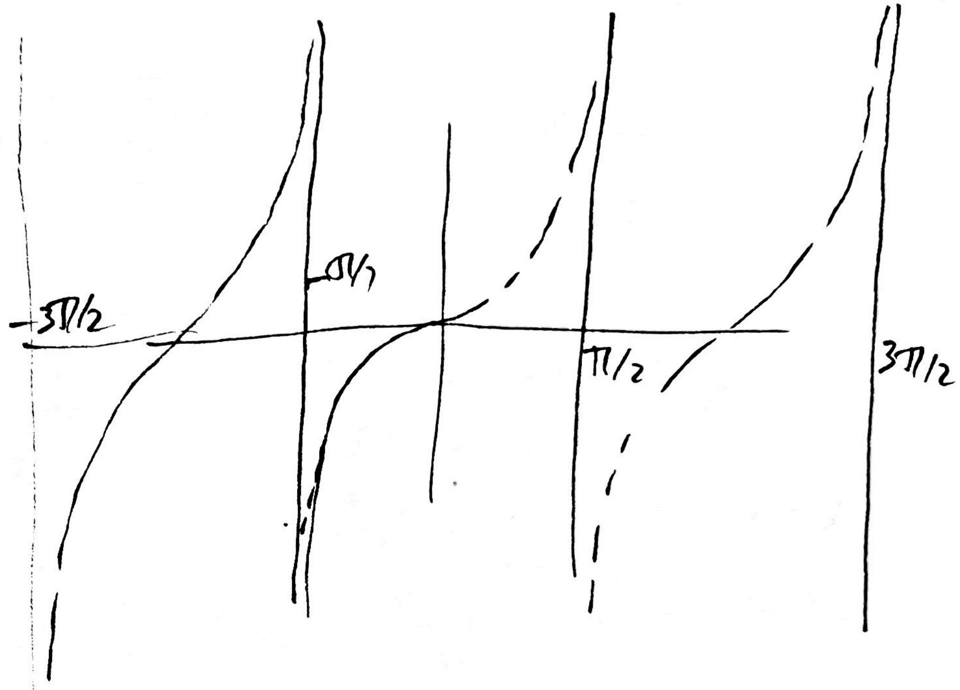
$\lim_{x \to 2^-} g(x) = -\infty$ $\qquad$ $\lim_{x \to 2^+} g(x) = +\infty$

$\lim_{x \to -2^-} g(x) = \infty$ $\qquad$ $\lim_{x \to -2^+} g(x) = -\infty$

To vertikale asymptoter



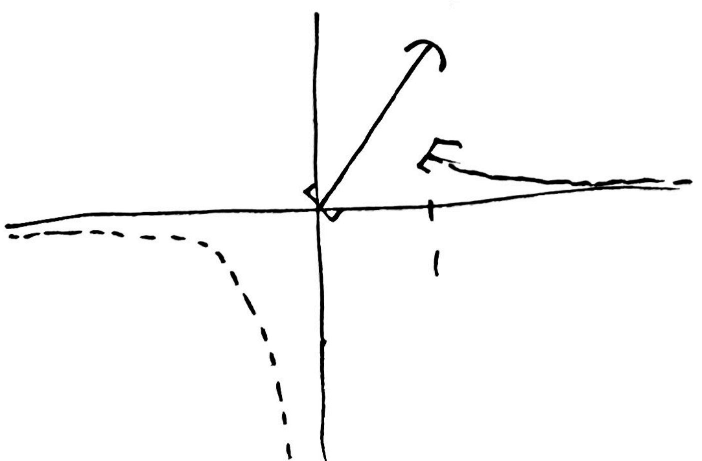
$$\tan x = \frac{\sin x}{\cos x}$$



Har uendelig mange vertikale asymptoter.

oppg. Finn de vertikale asymptotene til

$$f(x) = \begin{cases} \frac{1}{x} & x < 0 \\ 2x & 0 \leq x < 1 \\ \frac{1}{x+1} & x \geq 1 \end{cases}.$$

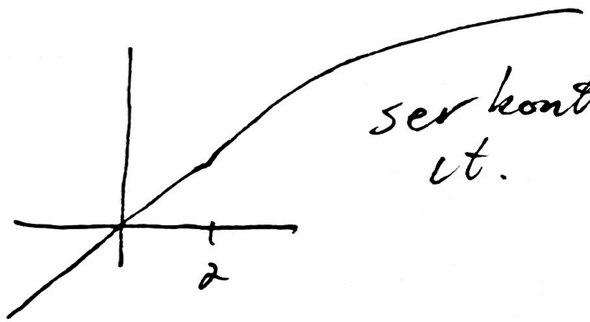


vertikal asymptote i $x = 0$.

$\left(\lim_{x \to 0^-} f(x) = -\infty\right)$

6.22 $f(x) = \begin{cases} \frac{7}{10}x & x < 2 \\ \sqrt{x} & x \geq 2 \end{cases}$

a)



ser kont. ut.

Zoomer vi inn ser vi at grafen hopper opp ved $x=2$. Hopp-diskontinuitet.

b) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{7}{10}x = \frac{7}{10}\cdot 2 = \underline{1.4}$ ulike!

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \sqrt{x} = \sqrt{2} \sim 1.41421\ldots$

$= f(2)$ Hopp-diskontinuitet.

6.31 c) $f(x) = \dfrac{x^2+x-2}{x-1}$

nullpunkt til nevneren $x-1$ er $x=1$.

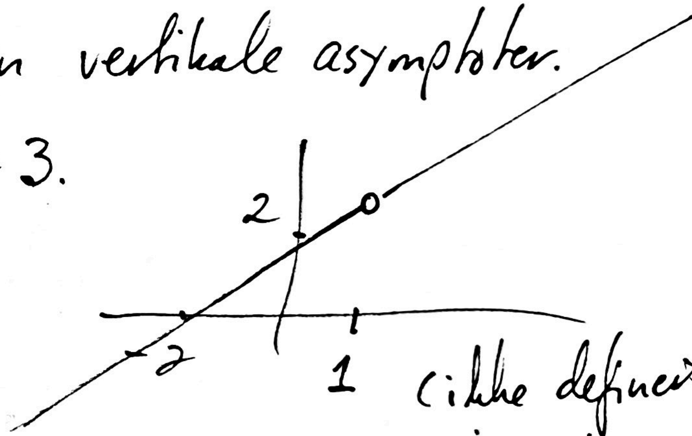
$$\lim_{x\to 1} x^2+x-2 = 1^2+1-2 = 0.$$

$$x^2+x-2 = (x-1)(x+2)$$

$$f(x) = \frac{(x-1)(x+2)}{x-1} \qquad x \neq 1$$

$$= x+2 \qquad \text{ingen vertikale asymptoter.}$$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} x+2 = 3.$$



(ikke definert i $x=1$)

6.33 $f(x) = \frac{x-1}{x^2-x-6}$

a) Finn nullpunkt
b) — Vertikale asymptoter
c) skisse.

a) $x-1=0$ når $x=1$. nullpunktet er $x = 1$.
nevneren: $1^2-1-6 = -6 \neq 0$. eller $(1,0)$.

b) Nevneren: $x^2-x-6 = (x-3)(x+2)$ er lik 0 når $x=-2$ og $x=3$.
telleren er da ulik 0.
Vertikale asymptoter til $f(x)$ er $\underline{x=-2}$ og $\underline{x=3}$

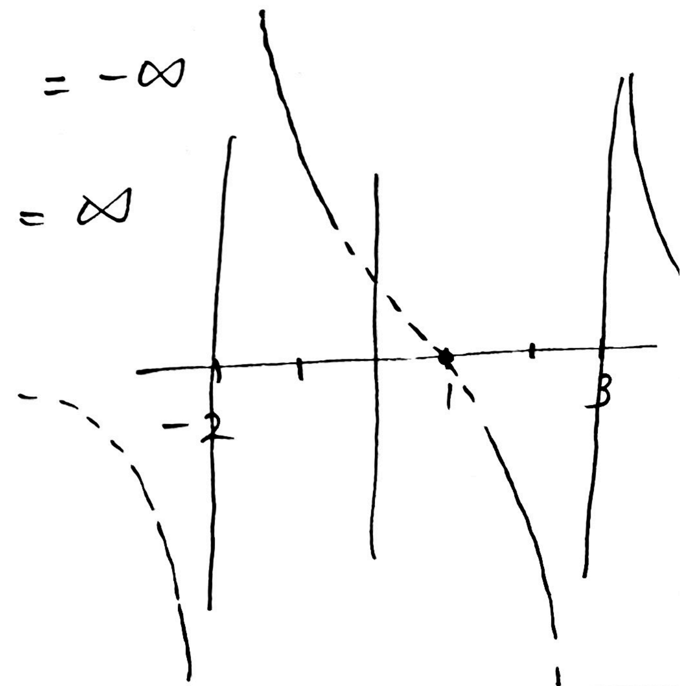
$\lim\limits_{x\to -2^-} \frac{x-1}{(x-3)(x+2)} = -\infty$

$\lim\limits_{x\to -2^+} f(x) = \infty$

$\lim\limits_{x\to 3^-} \frac{x-1}{(x-3)(x+2)} = -\infty$

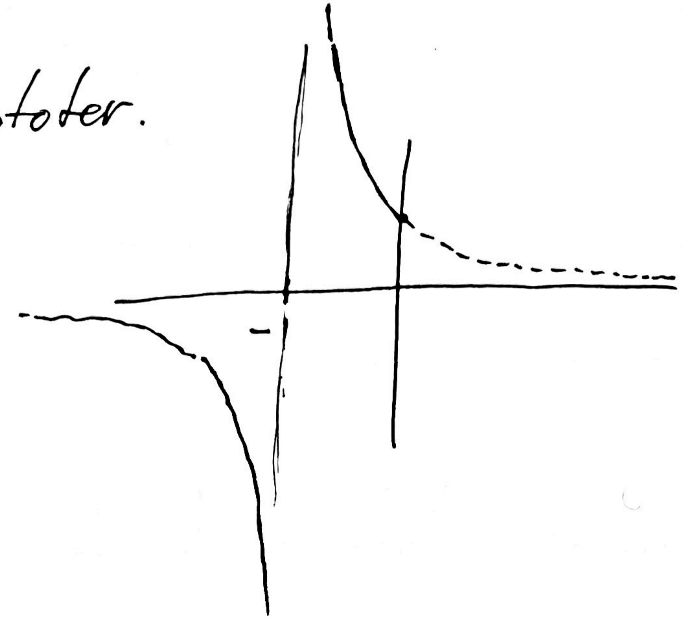
$\lim\limits_{x\to 3^+} f(x) = \infty$

c)



6.4 Horisontale asymptoter.

$f(x) = \frac{1}{x+1} \quad x \neq -1$



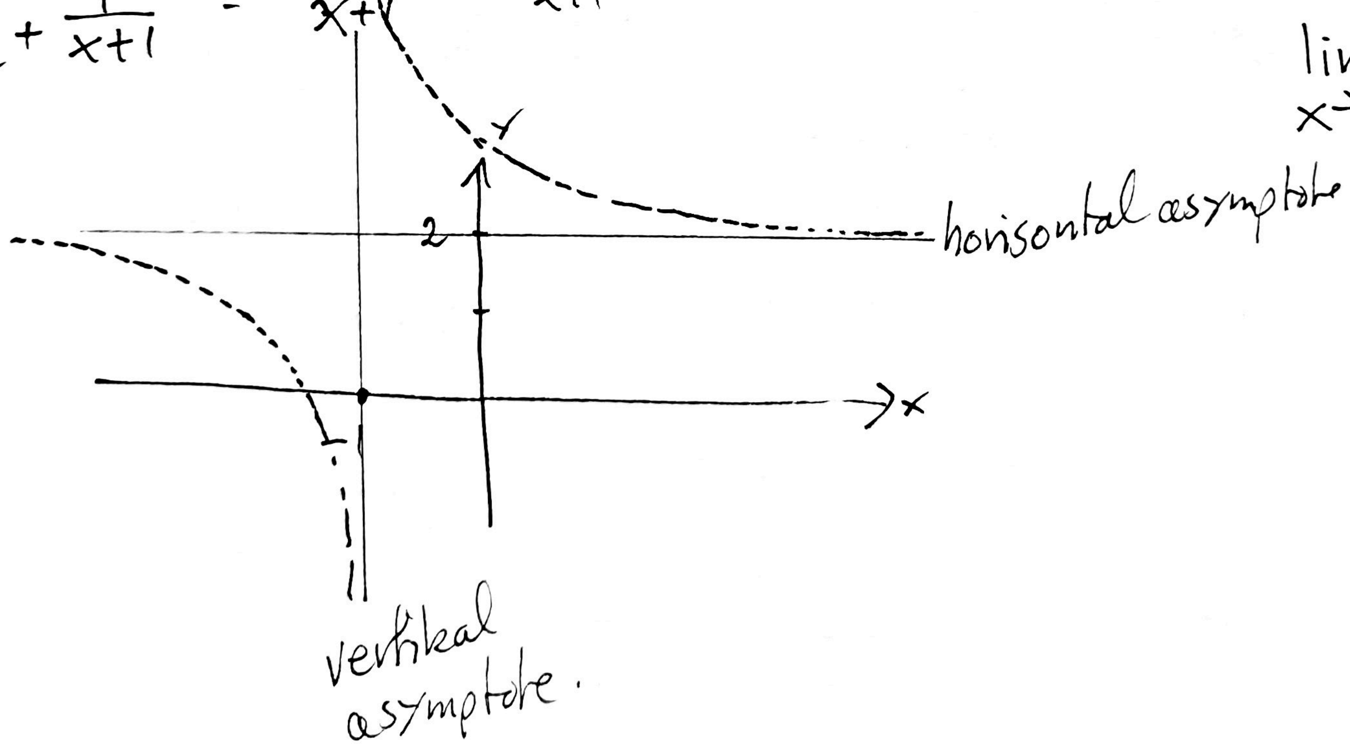
$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = 0$

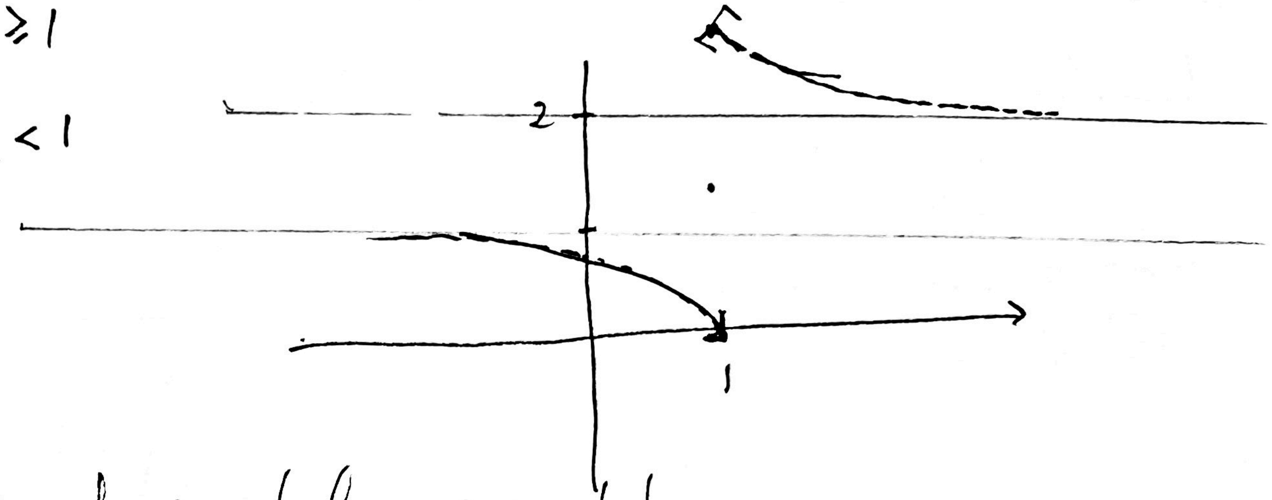
$g(x) = 2 + \frac{1}{x+1} = \frac{2(x+1)}{x+1} + \frac{1}{x+1} = \frac{2x+3}{x+1}$

$\lim_{x \to \infty} g(x) = 2.$

$\lim_{x \to -\infty} g(x) = 2$



$$h(x) = \begin{cases} \frac{1}{x} + 2 & x \geq 1 \\ \frac{1}{x-2} + 1 & x < 1 \end{cases}$$



Det finnes 0, 1 eller 2 horisontale asymptoter.

(degree $\overset{\text{eng}}{=}$ grad)

Rasjonal funksjon. $R(x) = \frac{p(x)}{q(x)}$

$\deg p < \deg q$ Da vil $\lim\limits_{x \to +\infty \, (-\infty)} R(x) = 0$

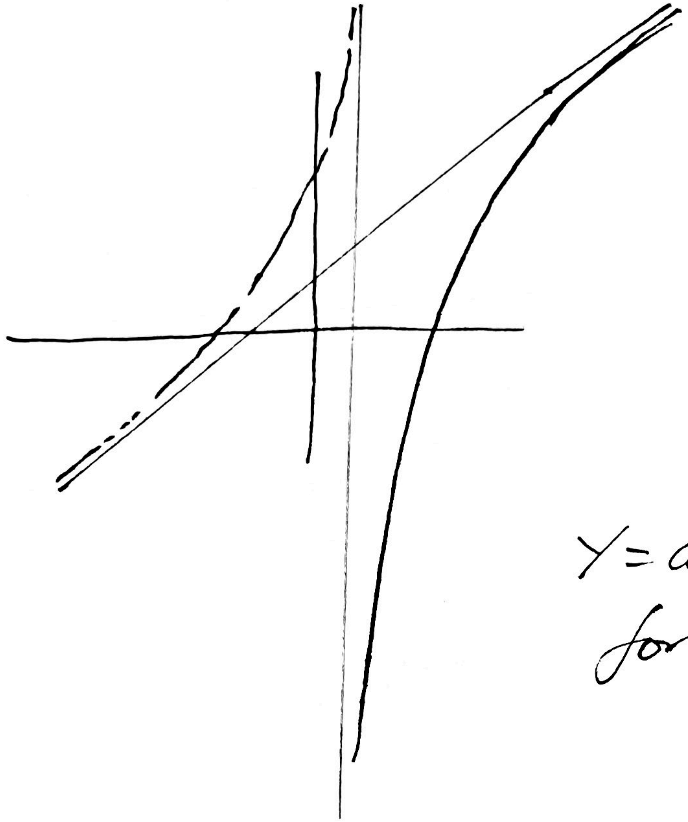
Da er x-aksen en horisontal asymptote.

$\deg p = \deg q$: Én horisontal akse (forskjellig fra x-aksen)

$$\frac{x^3+2}{4x^3+x^2-1} = \frac{1}{4} + \frac{r(x)}{4x^3+x^2-1} \quad (\deg r(x) < 3)$$

$$= \frac{1 + 2/x^3}{4 + 1/x - 1/x^3} \to \frac{1}{4} \text{ når } x \to \infty \text{ eller } x \to -\infty$$

Skrå asymptoter



Linje (ikke vertikal)

$y = ax + b$

b snittet med y-aksen

a stigningstallet

$y = ax + b$ er en skrå asymptote for $f(x)$ hvis

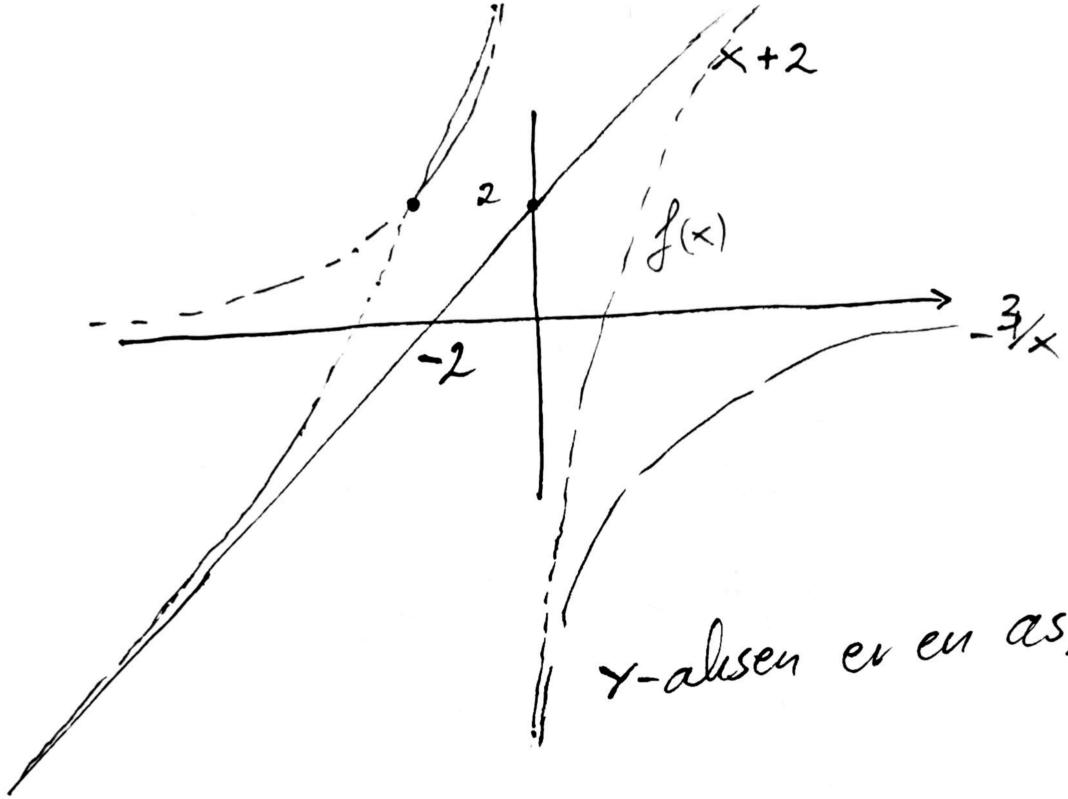
$$\lim_{\substack{x \to \infty \\ (\text{eller} \to -\infty)}} \left(f(x) - (ax+b) \right) = 0$$

$$f(x) = \frac{x^2 + 2x - 3}{x} = x + 2 + \frac{-3}{x}$$

$$\left(f(x) - (x+2) \right) = \frac{-3}{x}$$

Så $\lim_{\substack{x \to \infty \\ (\to -\infty)}} \left(f(x) - (x+2) \right) = 0$

Så $y = x+2$ er en skrå asymptote for $f(x)$.



$$f(x) = x+2 + \frac{-3}{x}$$

$y = x+2$
skrå asymptote.

$$\lim_{x\to 0^+} f(x) = -\infty$$
$$\lim_{x\to 0^-} f(x) = \infty$$

y-aksen er en asymptote.

$$R(x) = \frac{p(x)}{q(x)} \qquad \deg(p(x)) = 1 + \deg q(x)$$

polynomdivisjon $\qquad R(x) = ax + b + \frac{r(x)}{q(x)}$

$\deg r < \deg q$

Har en skrå asymptote for alle rasjonale uttrykk hvor graden til teller er ēn mer enn graden til nevner.

Finn asymptoter til a) $\frac{2x+3}{x+1}$

b) $\frac{x^2}{x+1}$

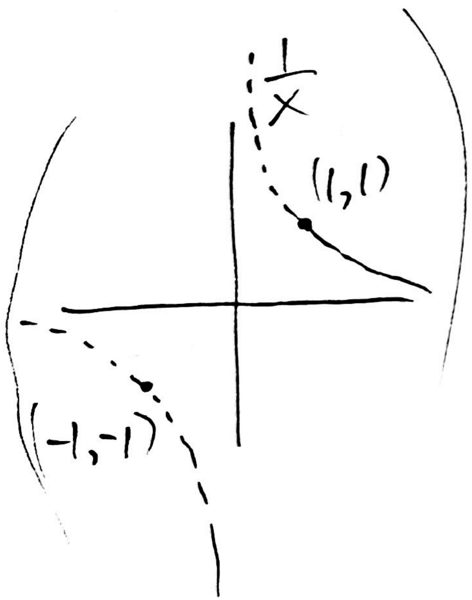
a) $f(x) = \frac{2x+3}{x+1} = \frac{2+3/x}{1+1/x}$

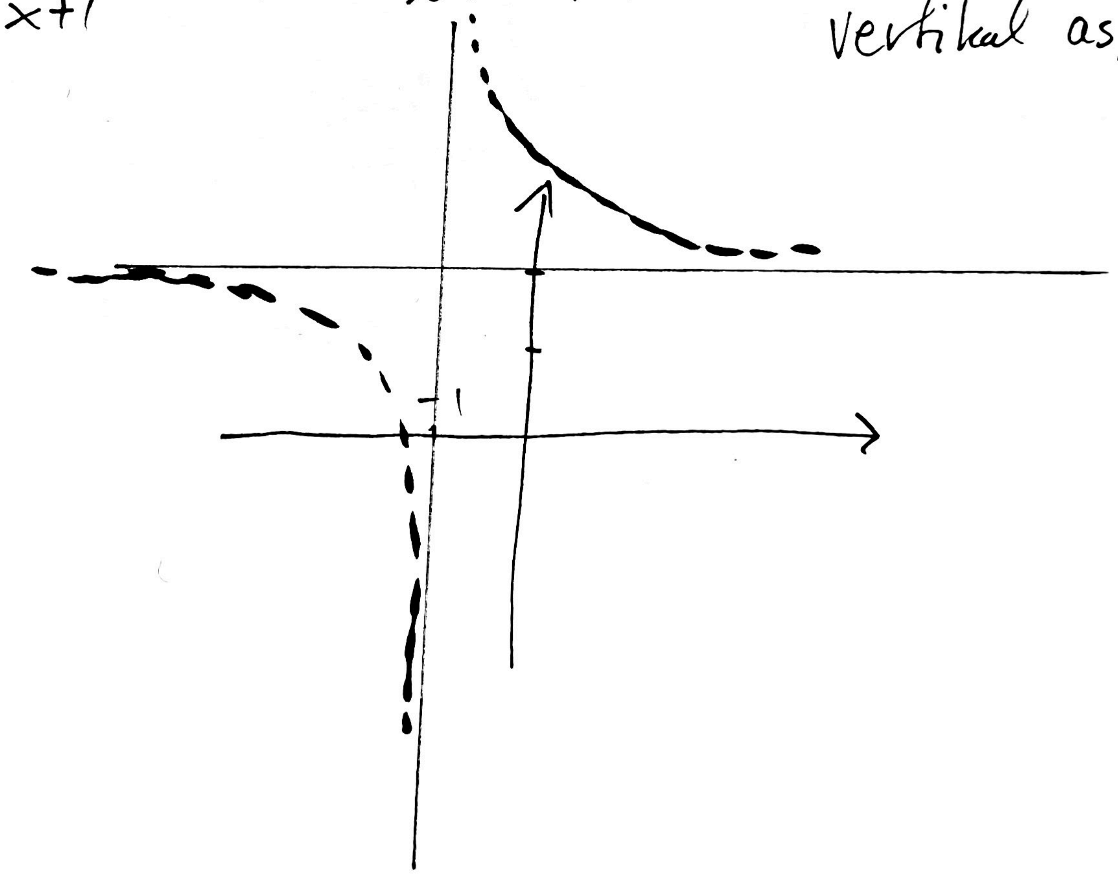
$= \frac{(2x+2)+1}{x+1} = 2 + \frac{1}{x+1}$

horisontal asymptote

$\lim_{\substack{x \to \infty \\ (-\infty)}} f(x) = 2.$ $\therefore y=2$

vertikal asymptote $x=-1$





b) $g(x) = \dfrac{x^2}{x+1}$

$= x - 1 + \dfrac{1}{x+1}$

$y = x-1$ skrå asymptote

$x = -1$ vertikal asymptote.

pol. div.

$x^2 : x+1 = x - 1 + \dfrac{1}{x+1}$

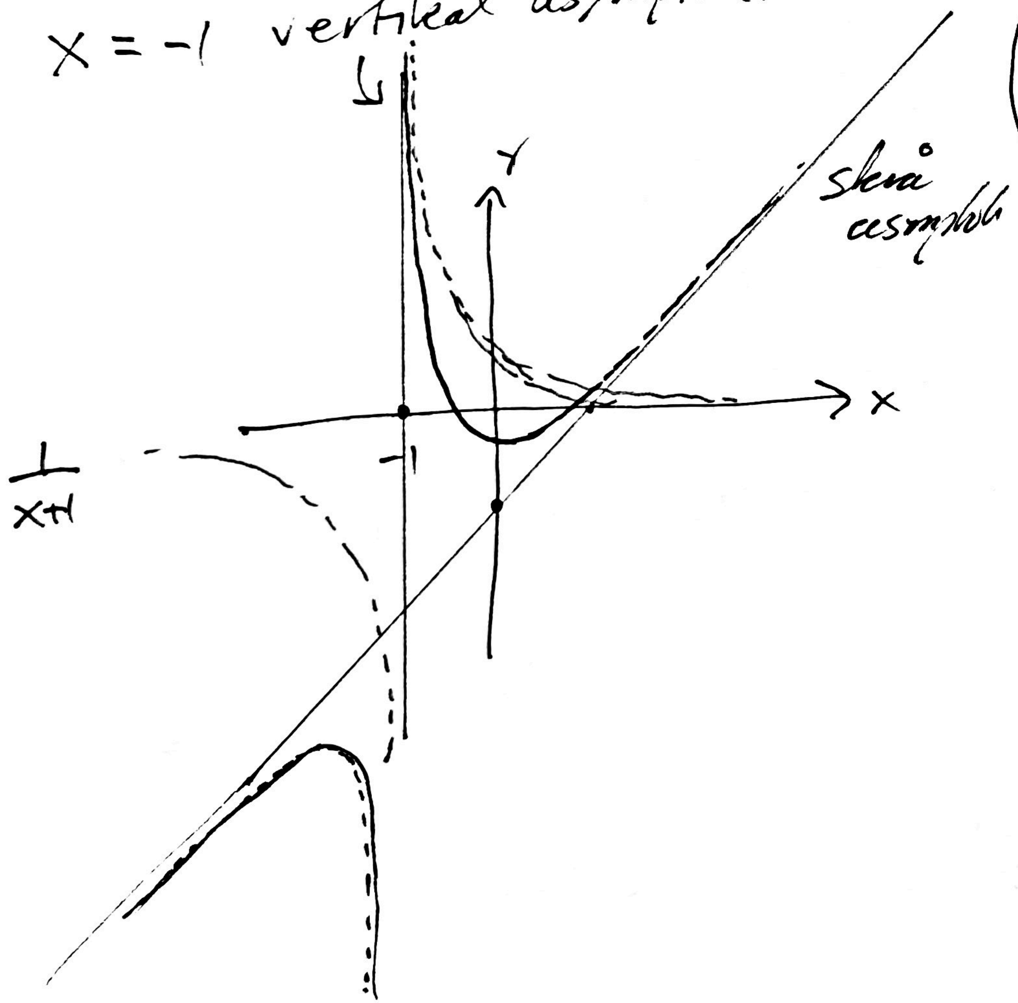
$\underline{x^2+x}$

$-x$

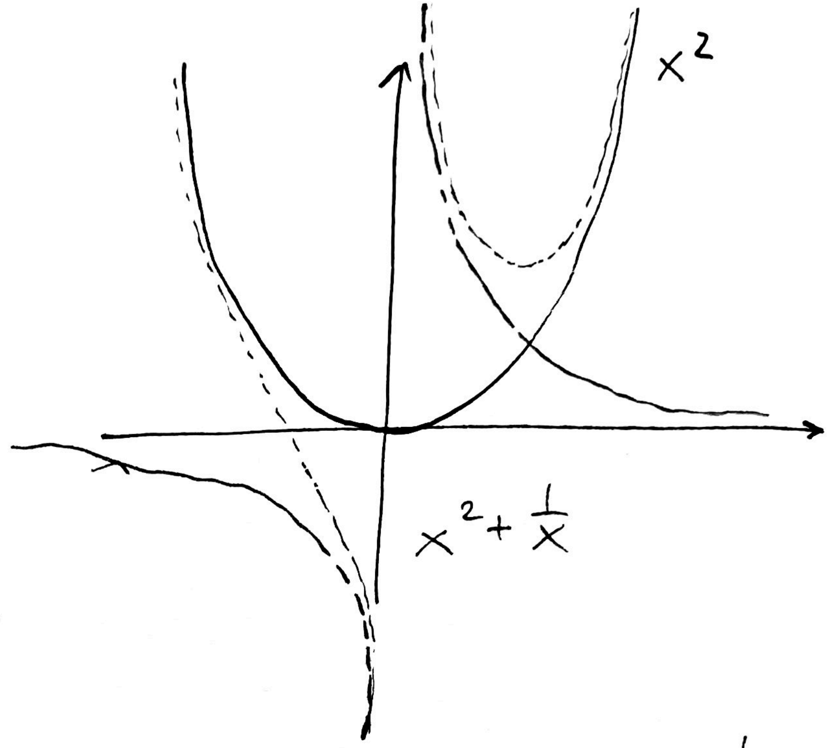
$\underline{-x-1}$

1

$\left(\dfrac{x^2-1+1}{x+1} = \dfrac{(x+1)(x-1)+1}{x+1} = x-1+\dfrac{1}{x+1} \right)$



$y = x^2 + \frac{1}{x}$



oppg. Finn asymptoter til

$$f(x) = \begin{cases} 2x-1 & x \leq 2 \\ \dfrac{3x-4}{x-3} & x > 2. \end{cases}$$

$y = 2x - 1$ skrå asymptote

$f(x) - (2x-1) = 0 \qquad x \leq 2$

(så $\lim_{x \to -\infty} f(x) - (2x-1) = 0 \ldots$)

$$\frac{3x-4}{x-3} = \frac{3(x-3)+5}{x-3} = 3 + \frac{5}{x-3}$$

$\lim_{x \to \infty} f(x) = 3$ horisontal asymptote $y = 3$.

$x = 3$ vertikal asymptote.