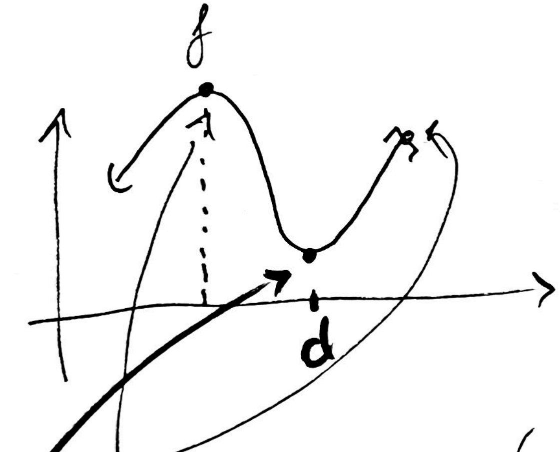
18.01.2021

7.1 Funksjonsdrøfting

Ekstremalpunkt felles betegnelse for maksimums- og minimumspunkt



Toppunkt $(c, f(c))$, c : maksimumspunkt

$f(c)$: maksimalverdi

Bunnpunkt $(d, f(d))$

d : minimumspunktet

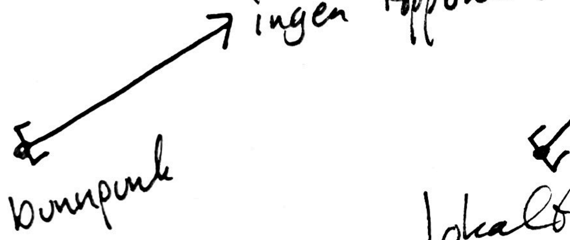
$f(d)$: minimalverdien.

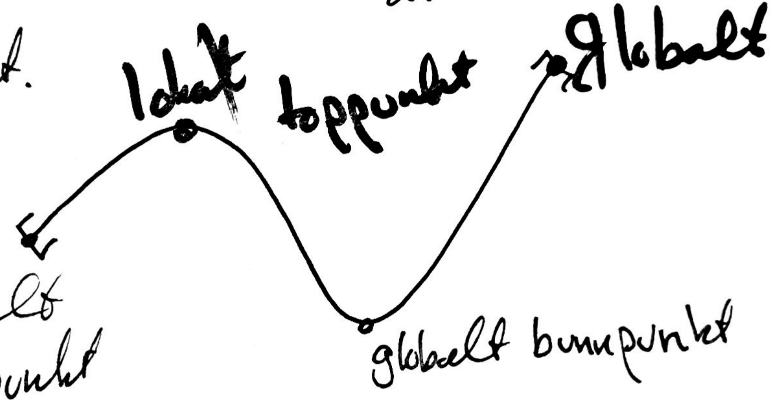
Ekstremalverdier : maksimums- og minimumsverdier.

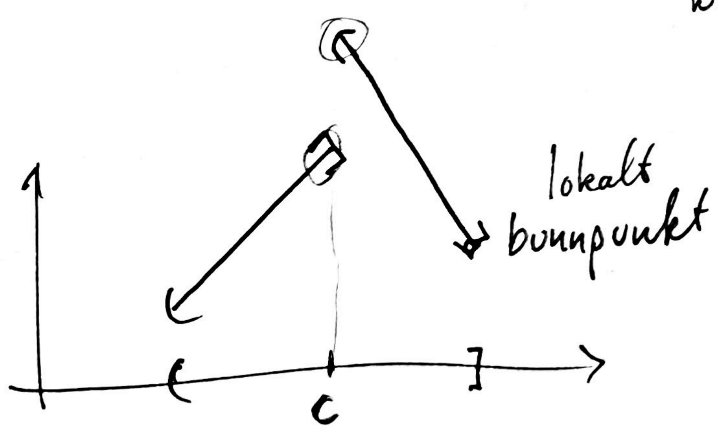
c er et maksimumspunkt for f

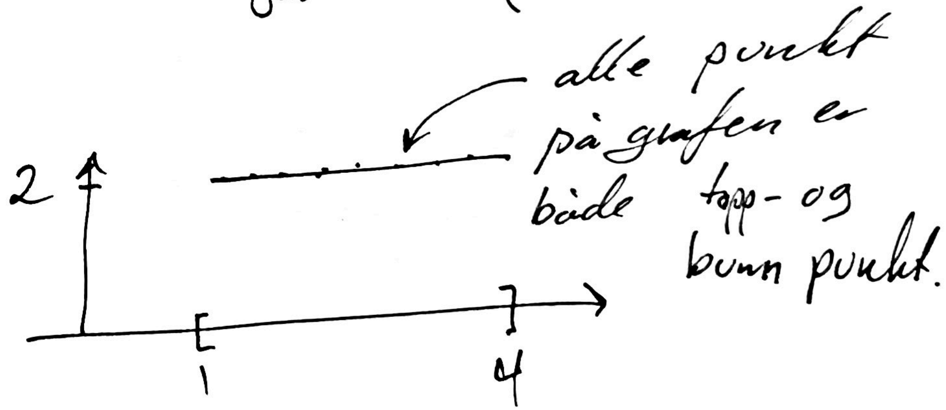
hvis $f(x) \leq f(c)$ $\begin{cases} \text{alle } x \in D_f & \text{: globalt max punkt} \\ \text{i nærheten av } c & \text{: lokalt max punkt.} \end{cases}$

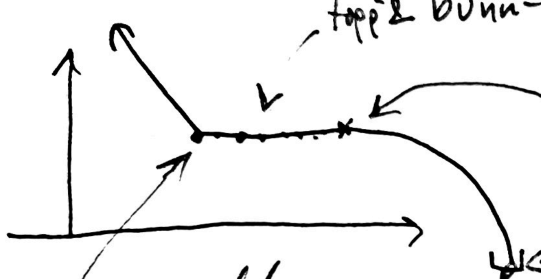
Eks

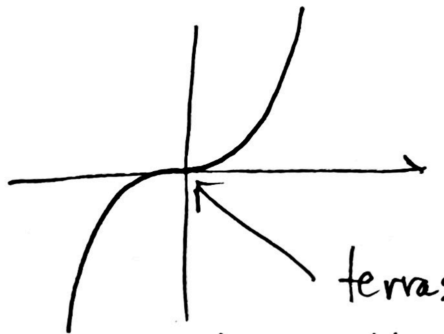








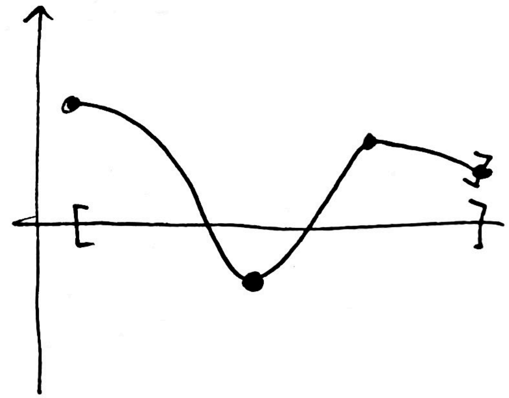


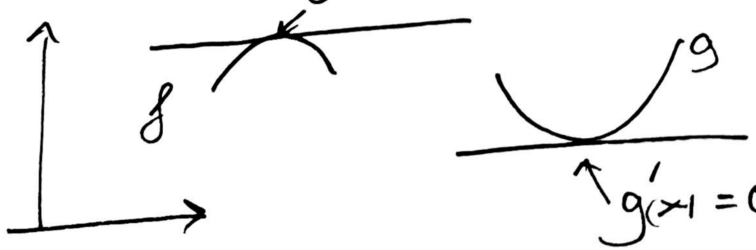


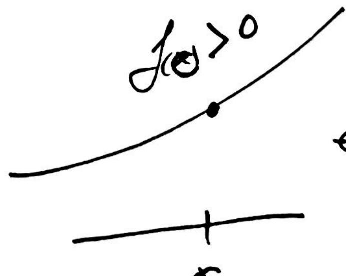
tangentlinjen er horisontal, men ikkje topp- eller bunnpunkt.

Ekstremalverdi Resultatet

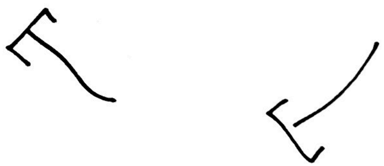
Hvis $f(x)$ på $[a,b]$ (~~begrense~~ lukket intervall) er kontinuerlig, da har $f(x)$ globale topp- og bunnpunkt.



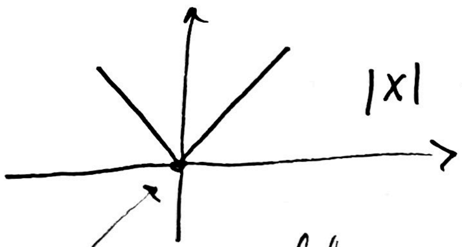




kan ikke ha ekstremalpunkt i c.



kan ha topp og bunnpunkt i endepunkt.



bunnpunkt.

$$|x| = \begin{cases} x & x>0 \\ -x & x<0 \end{cases}$$

$$(|x|)' = \begin{cases} 1 & x>0 \\ -1 & x<0 \end{cases}$$

ikke deriverbar i $x=0$.

KRITISKE PUNKT til $f(x)$ er alle $x \in D_f$ slik at:

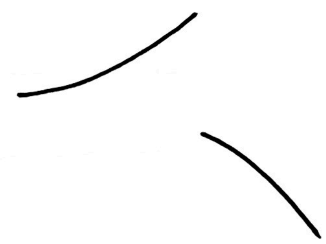
- Endepunkt
- $f'(x)$ eksisterer ikkje
- $f'(x) = 0$

Ekstremal punkt $\subset$ Kritiske punkt.

Tilstrekkeleg å lete blant de kritiske punktene.

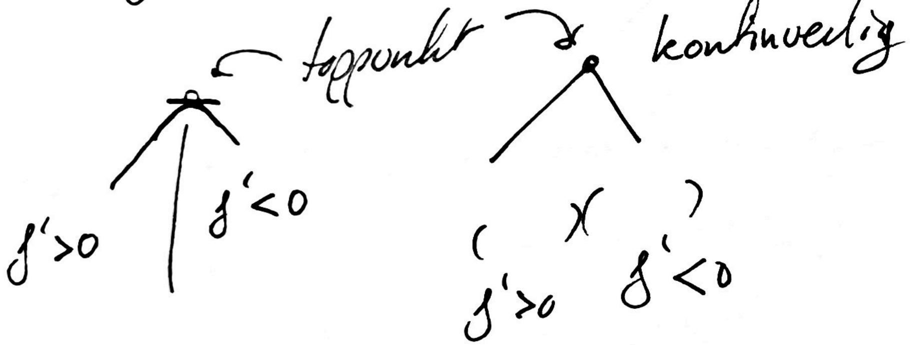
En funksjon er voksende, stigende, økende hvis $f(x_2) \geq f(x_1)$ for $x_2 > x_1$

minkende, synkende, avtagende hvis $f(x_2) \leq f(x_1)$



$f'(x) \geq 0 \Rightarrow$ stigende

$f'(x) \leq 0 \Rightarrow$ avtagend



eks. $f(x) = \frac{x^3}{3} + x^2 + 2$ $\quad D_f = \mathbb{R}$

$f'(x) = \frac{1}{3}(x^3)' + 2x + (2)' = x^2 + 2x$ deriverbar for alle x

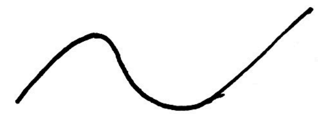
$= x(x+2)$

Kritiske punkt = punkt x hvor $f'(x) = x(x+2) = 0$

$\{-2, 0\}$.

Ser på fortegn til $f'(x)$:

	-2		0	
x	- - - - -	- - -	0	————
$x+2$	- - - - 0	————		————
$f'(x)$	—— 0	- - - -	0	————



så $x=-2$ er et maksimumspunkt

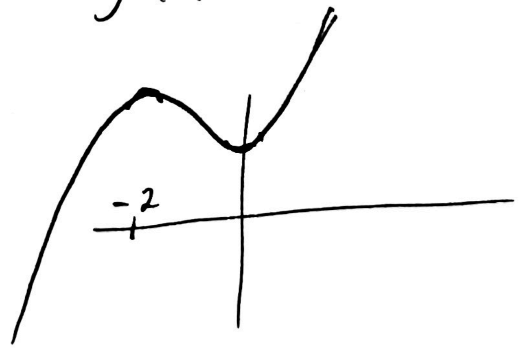
$f(-2) = \frac{(-2)^3}{3} + (-2)^2 + 2 = 6 - \frac{8}{3} = \frac{10}{3}$ maksimumsverdi

~ 3.33

$(-2, \frac{10}{3})$ toppunktet.

$x=0$ er et minimumspunkt

minimumsverdien $f(0) = 2$, bunnpunkt i $(0, 2)$



$g(x) = |x|$ $D_f = [-2, 3]$

$-2, 3$ endepunkt

Kritiske punkt $\{-2, 0, 3\}$.

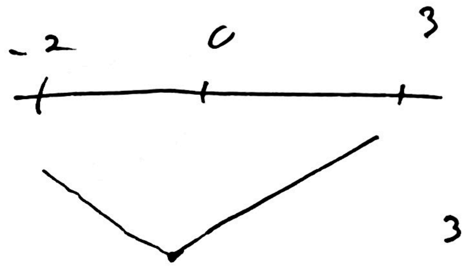
$g(x)$ er kont. i $x=0$

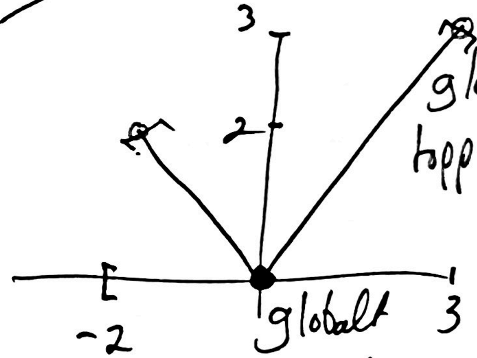
Så minimums~~...~~ punkt $x=0$.

maksimumspunkt i $x=-2$ og $x=3$.

$$g'(x) = \begin{cases} 1 & x>0 \\ -1 & x<0 \end{cases}$$

eksisterer ikkje for $x=0$



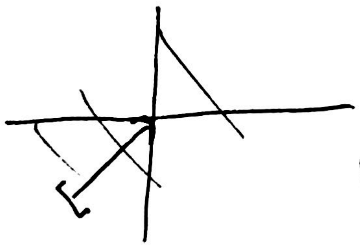


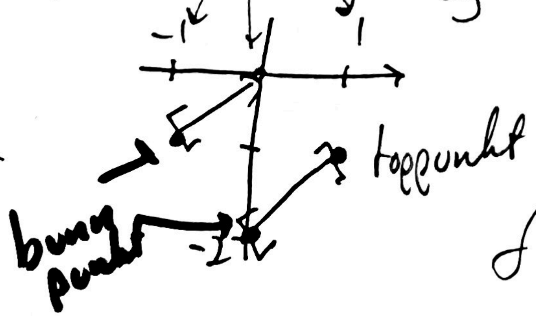
$$f(x) = \begin{cases} x & -1 \le x < 0 \\ x-2 & 0 \le x \le 1 \end{cases}$$

$D_f = [-1, 1]$ kritiske punkt. Finn ekstremalpunkt og avgjør hva $f(x)$ stiger og synker.

$$f'(x) = \begin{cases} 1 & -1 \le x < 0 \\ 1 & 0 < x \le 1 \end{cases}$$

eksisterer ikkje i $x=0$





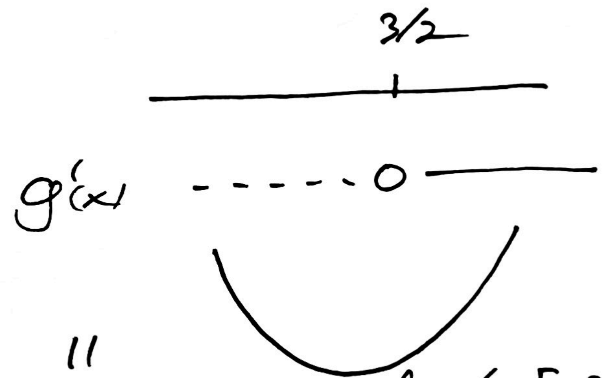
$f(x)$ har ikkje globalt toppunkt.

Finn ekstemalpunkt og bestem monotoniegenskapene
(= egenskapene stigende, synkende)
til $g(x) = x^2 - 3x + 1$ $\quad |x| \leq 2$
($x \in [-2, 2]$)

$g'(x) = 2x - 3$ deriverbar for alle x

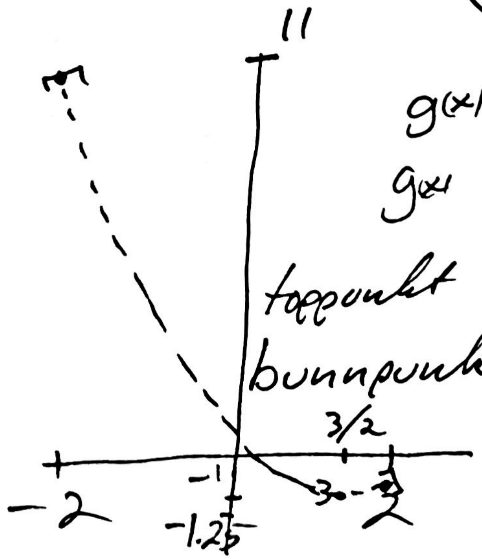
$g'(x) = 0$: $\quad 2x - 3 = 0$ så $x = 3/2$
$= 1.5.$

Kritiske punkt: $\{-2, 3/2, 2\}$.



$$g(3/2) = \left(\frac{3}{2}\right)^2 - 3\cdot\frac{3}{2} + 1$$
$$= \frac{9}{4} - \frac{9}{2} + 1 = 1 - \frac{9}{4} = \frac{4-9}{4}$$
$$= \frac{-5}{4} = -1.25$$

$g(-2) = (-2)^2 - 3(-2) + 1 = 11$
$g(2) = 2^2 - 3 \cdot 2 + 1 = -1$



$g(x)$ synkende $[-2, 1.5]$
$g(x)$ stigende $[1.5, 2]$
toppunkt $(-2, 11)$ globalt, $(2, -1)$
bunnpunkt $(3/2, -5/4)$ globalt

Finn ekstremalpunkt og skisser grafen til $\langle -\infty, 0\rangle \cup \langle 0, \infty\rangle$

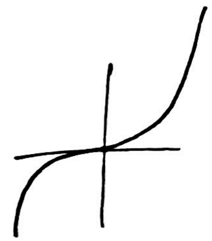
$$f(x) = \frac{16}{x} + x^2 - 1. \qquad x \neq 0$$

$$f'(x) = 16 \cdot \left(\frac{1}{x}\right)' + 2x + 0$$

$$= \frac{-16}{x^2} + 2x = \frac{2x^3 - 16}{x^2} \qquad x \neq 0$$

$$f'(x) = 0 \iff \frac{-16 + 2x^3}{x^2} = 0 \iff x^3 = 8$$

$$\iff \underline{x = 2}$$



Kritiske punkt: $x = 2$.

$$f'(x) = \frac{2x^3 - 16}{x^2} \quad \begin{matrix} > 0 & x > 2 \\ < 0 & x < 2 \end{matrix}$$

$f(x)$ avtagende $x < 0$

$0 < x < 2$

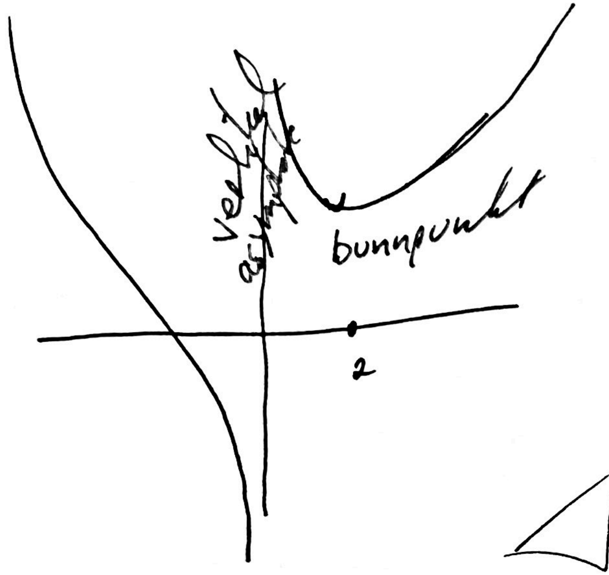
stigende $x > 2$

f avtar mot $x = 2$ fra venstre, stiger fra $x = 2$ fra høyre $\Big\}$ $x = 2$ minimumspunkt

$$\lim_{x \to 0^-} f(x) = -\infty$$

$$\lim_{x \to 0^+} f(x) = \infty$$

$$f(2) = \frac{16}{2} + 2^2 - 1 = 11$$



Oppgave Finn ekstremalpunkt og bestem monotoniegenskapene til

$$g(x) = x^4 + 4x^3 + 4x^2 - 1.$$

$$g'(x) = (x^4)' + 4(x^3)' + 4(x^2)' - (1)'$$
$$= 4x^3 + 4\cdot 3x^2 + 4\cdot 2x$$
$$= 4x(x^2+3x+2)$$
$$= 4x(x+2)(x+1)$$

Kritiske punkt: $g'(x) = 0$ når $\underline{x = 0, -1 \text{ og } -2}$, ingen endepunkt. $g'(x)$ eksisterer for alle x

$g(0) = -1$, $g(-1) = 1-4+4-1 = 0$

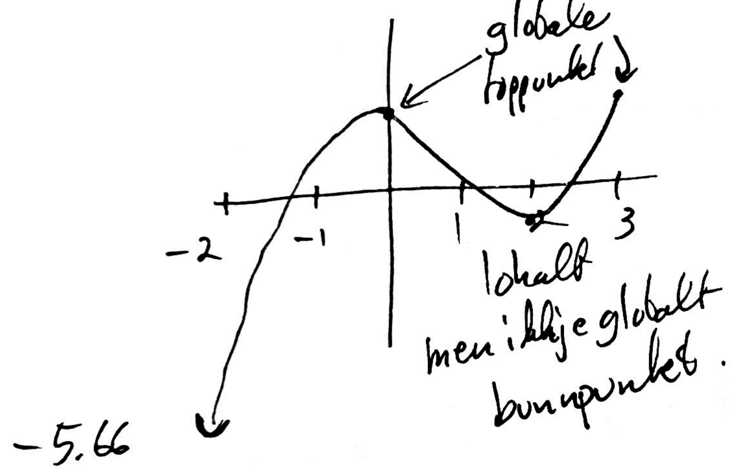
$g(-2) = 16 - 32 + 16 - 1 = -1$

	-2	-1	0
$4x$	- - -	- - -	0 ———
$x+2$	0 ———		
$x+1$	- - -	0 ———	
$g'(x)$	- - - 0	——— 0	- - - 0 ———

maksimumspunkt $x=0$ og $x=3$.

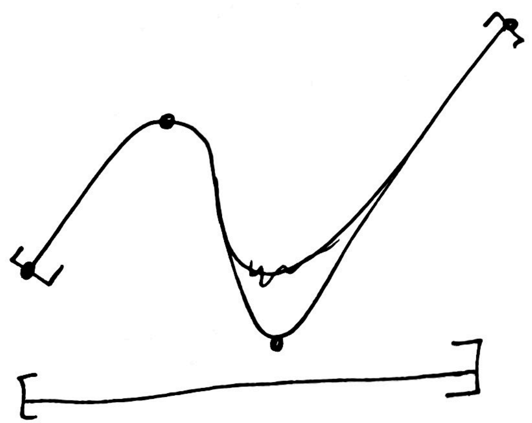
minimumspunkt $x=2$

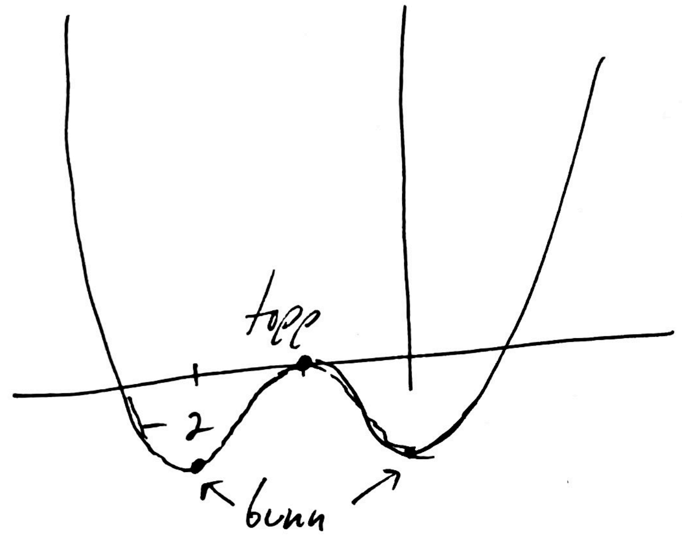
$f(0)=1$ $\quad f(3) = \frac{3^3}{3} - 3^2 + 1 = 1$ $\quad f(2) = \frac{2^3}{3} - 4 + 1 = \frac{8}{3} - 3 = \frac{-1}{3}$.



$f(-2) = \frac{(-2)^3}{3} - (-2)^2 + 1 = \frac{-8}{3} - 3$

$= \frac{-8}{3} - 4 + 1 = -\frac{17}{3} < -5$





minimumspunktene er $\underline{-2 \text{ og } 0}$

maksimumspunktet er $\underline{-1}$.

$-2 < x \leq 3$

$(x \in (-2, 3].)$

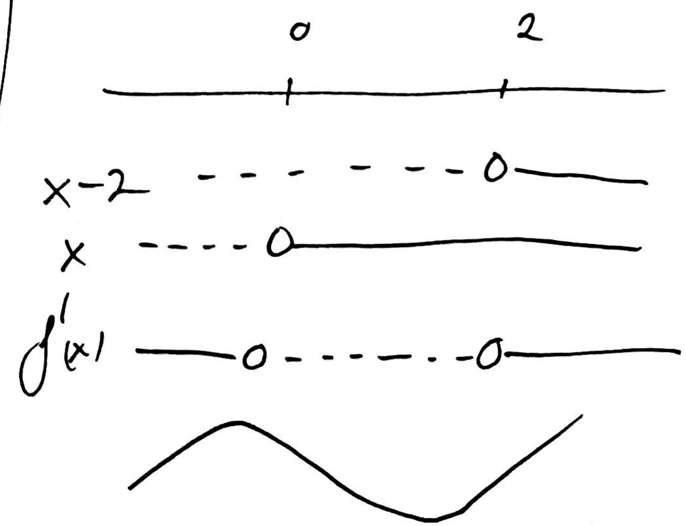
oppgave. $f(x) = \frac{x^3}{3} - x^2 + 1$

Finn ekstemalpunktene.

$f'(x) = x^2 - 2x(+1)' = x^2 - 2x = x(x-2)$

$f'(x) = 0$; $x = \underline{0 \text{ og } 2}$

Kritiske punkt $\{\underline{0, 2, 3}\}$



$$\left((-x)^2\right)' = \underset{2u}{2(-x)} \cdot \underset{u'}{(-x)'} = 2(-x)(-1) = \underline{2x}$$

$$\left(\underbrace{(2x+3)}^2\right)' = 2(2x+3) \cdot (2x+3)'$$

$$= 2 \cdot (2x+3) \cdot 2 = \underline{4(2x+3)}$$

$2^0 = 1 \quad a^0 = 1$

$$(2^0)' = 0$$
$$(a^0)' = 0$$
$$(x^0)' = 0$$
$$(0x^{0-1} = 0 \ldots)$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$(x^{1/2})' = \frac{1}{2} \cdot x^{1/2 - 1} = \frac{1}{2} x^{-1/2} = \frac{1}{2} \frac{1}{x^{1/2}}$$

$$= \underline{\frac{1}{2\sqrt{x}}}$$

$$\sqrt[3]{x^2} = (x^2)^{1/3} = x^{2/3}$$

$$(x^n)' = n \cdot x^{n-1}$$

$$(x^{2/3})' = \frac{2}{3} x^{\frac{2}{3}-1} = \frac{2}{3} x^{-1/3}$$

$$\left(\sqrt[3]{x^2}\right)' = \frac{2}{3\sqrt[3]{x}}$$

$$\left(6\sqrt{2x^3}\right)'$$

$$\left(\underbrace{6\cdot\sqrt{2}\sqrt{x^3}}\right)' = 6\sqrt{2}\left(\left(x^3\right)^{1/2}\right)'$$

$$= 6\sqrt{2}\left(x^{3/2}\right)' = 6\sqrt{2}\,\underbrace{\left(\frac{3}{2}x^{\frac{3}{2}-1}\right)}_{\frac{3}{2}\sqrt{x}}$$

$$= \frac{6\cdot 3}{2}\sqrt{2}\;x^{1/2}$$

$$= \underline{9\sqrt{2}\sqrt{x}}$$

Via kjerneregelen: $\left(\sqrt{x^3}\right)' = \frac{1}{2\sqrt{x^3}} \cdot (x^3)'$

$$= \frac{1}{2\sqrt{x^3}} \cdot 3x^2$$

$$= \frac{3}{2}\frac{x^2}{\sqrt{x^3}} = \frac{3}{2}\frac{x^2}{x\sqrt{x}} = \frac{3}{2}\frac{x}{\sqrt{x}} = \underline{\frac{3}{2}\sqrt{x}}$$