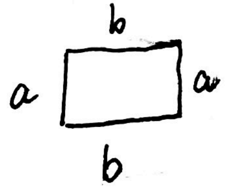
7.3-4 Optimering



omkrets $\mathcal{O} = 2(a+b)$ fast

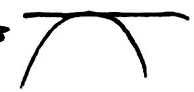
Hvordan bør a, b velges slik at

Arealet $A = a \cdot b$ blir størst mulig?

Anta $\mathcal{O} = 1 = 2(a+b)$ dette gir b som en funksjon av a.

$b = \frac{1}{2} - a.$

$A = a \cdot b = a\left(\frac{1}{2} - a\right) = -a^2 + \frac{1}{2}a.$ parabel

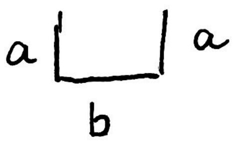


$A'(a) = -2a + \frac{1}{2} = 0$ $\qquad A'(a) = 0$

$a = \frac{1}{4}$

og da $b = \frac{1}{2} - a = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$

Arealet er størst når $\underline{a = b}$.



$A = a \cdot b$

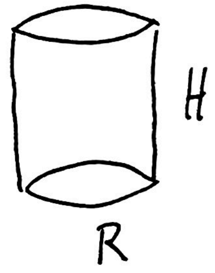
$\mathcal{O} = 2a + b$

$2a = c$

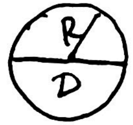
$A = \frac{1}{2} c \cdot b$ størst når $c = b$

$\mathcal{O} = c + b$ Fast

$\underline{2a = b}$



diameter $D = 2R$



Sylinder kuttet opp

$V = \pi R^2 \cdot H$

$A = 2\pi R \cdot H + n\pi R^2$

$n=0$ hul

$n=1$ bunn uten topp

$n=2$ bunn og topp

n annet: ulik "vekting".

V fast (0.33L, 0.5L ...)

Minimere A ("kostnad")

$V = \pi R^2 \cdot H$ gir $H = \frac{V}{\pi} \cdot \frac{1}{R^2}$

setter dette inn i A:

$$A(R) = 2\pi \cdot R\left(\frac{V}{\pi}\frac{1}{R^2}\right) + n \cdot \pi R^2$$

$$= 2V \cdot \frac{1}{R} + n \cdot \pi \cdot R^2$$

$$A'(R) = 2V\left(\frac{-1}{R^2}\right) + n \cdot \pi(2R) = 0$$

$$\frac{2V}{R^2} = (2n\pi)R \Rightarrow R^3 = \frac{2V}{2 \cdot n \cdot \pi} = \underline{\frac{V}{n \cdot \pi}}$$

Setter inn $V = \pi R^2 \cdot H$ i $\frac{2V}{R^2} = 2n\pi R$

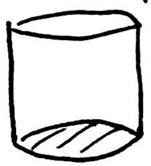
$$\frac{\pi R^2 H}{R^2} = 2n\pi R \Rightarrow H = 2nR$$

$$\boxed{\frac{H}{R} = 2n}$$

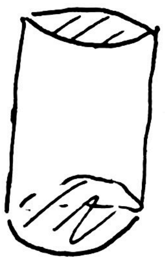
$2R = D$

$$\boxed{\frac{H}{D} = n}$$

åpen sylinder

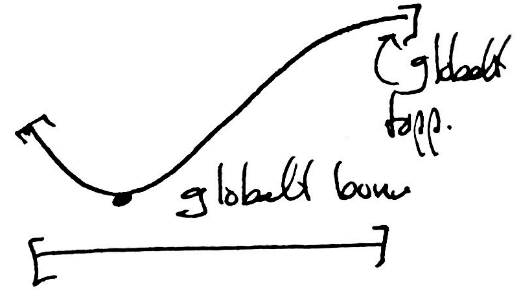
$n=1$  $H = D$

Lukka sylinder

$n=2$  $H = 2D$

(Brusbokser $\frac{H}{D} \sim 1.86$)

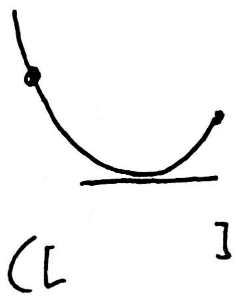
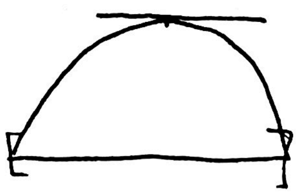
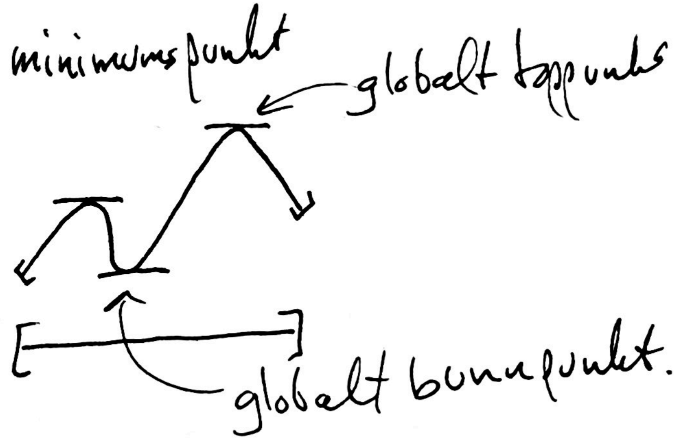
$n=0$? H minst mulig.



Ekstremalverdi setningen

La f være en kontinuerlig funksjon på et lukket, begrenset intervall $[a, b]$.

Da vil $f(x)$ ha global maksimum og minimumspunkt

Eks Vare med pris P per enhet.

x antall varer vi forventer å selge.

$$x = 1000 - k \cdot P$$

Fortjeneste $F = P \cdot X = P(1000 - k \cdot p)$

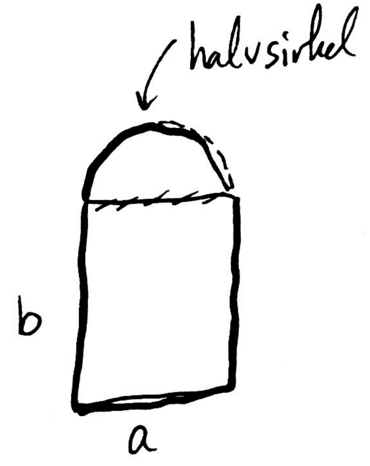
$$= -kP^2 + 1000 \cdot p$$

$$F'(p) = -2kp + 1000 = 0$$

$$p = \frac{1000}{2k}.$$

optimal pris. (anta $k \sim 1$)

oppgave



Fast omkrets

Finn forholdet mellom a og b slik at arealet blir størst mulig.

$$O = a + 2b + \frac{1}{2} 2\pi \cdot \frac{a}{2} = a\left(1 + \frac{\pi}{2}\right) + 2b$$

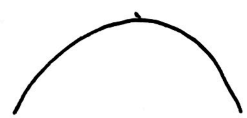
$$A = a \cdot b + \frac{1}{2}\pi\left(\frac{a}{2}\right)^2 = a \cdot b + \frac{\pi}{8} a^2.$$

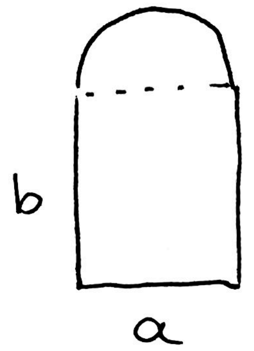
O fast

$$b = \frac{1}{2}\left(O - a\left(1 + \frac{\pi}{2}\right)\right)$$

$$A = a \cdot \frac{1}{2}\left(O - a\left(1 + \frac{\pi}{2}\right)\right) + \frac{\pi}{8} a^2$$

$$= a^2 \underbrace{\left(\frac{\pi}{8} - \frac{1}{2}\left(1 + \frac{\pi}{2}\right)\right)}_{\underbrace{\frac{\pi}{8} - \frac{\pi}{4} - \frac{1}{2}}_{-\frac{\pi}{8} - \frac{1}{2}}} + \frac{O}{2} \cdot a$$





radius $r = \frac{a}{2}$

$O = a + 2b + \frac{1}{2} 2\pi\left(\frac{a}{2}\right) = \left(1+\frac{\pi}{2}\right)a + 2b$

omkrets er fast

Vi ønsker å maksimere arealet

$A = a \cdot b + \frac{1}{2}\pi\left(\frac{a}{2}\right)^2 = a \cdot b + \frac{\pi}{8}a^2.$

O fast gir: $b = \frac{1}{2}\left(O - \left(1+\frac{\pi}{2}\right)a\right)$

$A(a) = a \cdot \frac{1}{2} \cdot \left(O - \left(1+\frac{\pi}{2}\right)a\right) + \frac{\pi}{8}a^2$

$= \frac{O}{2}a - \left(\frac{1}{2} + \frac{\pi}{8}\right)a^2$

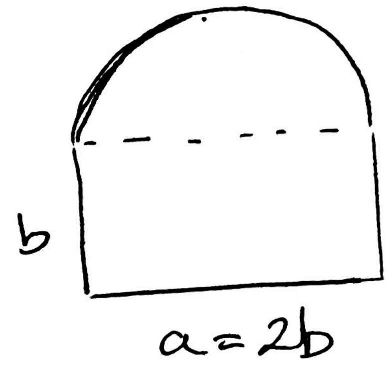
$A'(a) = \frac{O}{2} - \left(\frac{1}{2} + \frac{\pi}{8}\right) \cdot 2a = 0$

(A går mot 0 når $a \to 0$ eller $b \to 0$)

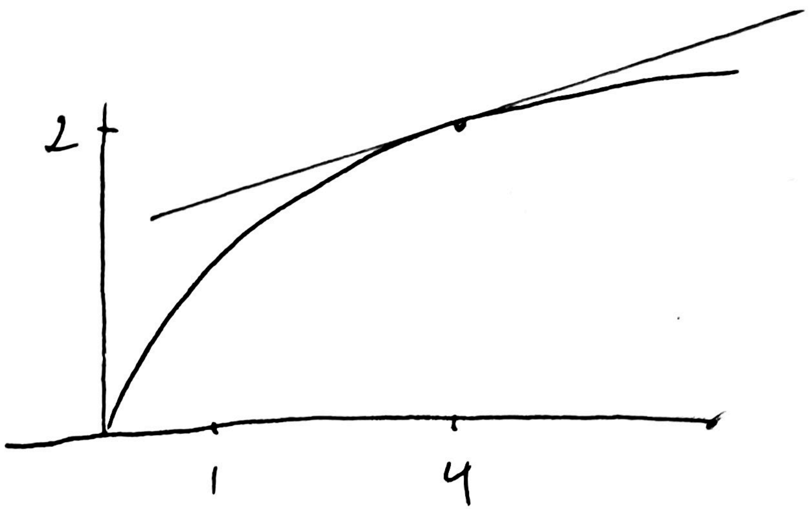
maksimalpunkt: $a = \frac{O/2}{1+\pi/4}$.

Da er $b = \frac{1}{2}\left(O - \frac{(1+\pi/2)}{(1+\pi/4)} \cdot \frac{O}{2}\right) = \frac{O}{2}\left(1 - \frac{(1+\pi/2)}{2(1+\pi/4)}\right)$

$\frac{b}{a} = \left(1+\frac{\pi}{4}\right) - \frac{1}{2}\left(1+\frac{\pi}{2}\right) = \underline{\frac{1}{2}}$



gir størst areal



Tangen linje til å tilnerme funksjoner

$$f(c+h) \sim f(c) + f'(c)(h)$$

verdiene på tangentlinjen

$a > 0$

$$\sqrt{a^2+h} \cong \sqrt{a^2} + \frac{1}{2\sqrt{a^2}} \cdot h$$

$$\underline{\sqrt{a^2+h} \sim a + \frac{h}{2a}}$$

$a = 2$:

$$\sqrt{4+h} \sim 2 + \frac{h}{4}$$

geogebra:

$h = 0.1$

$\sqrt{4.1} \sim 2 + \frac{0.1}{4} = 2.025$ (2.02485)

$h = -1$

$\sqrt{3} \sim 2 + \frac{-1}{4} = 1.75$ (1.73205)

$\sqrt{4.4} \sim 2 + \frac{0.4}{4} = 2.1$ (2.09762)