27.01.2021

7.8 og 7.9 Produktregelen

Produktfunksjonen til $f(x)$ og $g(x)$ er $(f \cdot g)(x) = f(x) \cdot g(x)$

$D_{f \cdot g} = D_f \cap D_g$

$x(1-x)^7 = (x) \cdot ((1-x)^7)$

$\frac{x+3}{\sqrt{1+x^2}} = (x+3) \cdot \left(\frac{1}{\sqrt{1+x^2}}\right) = (x+3)\left((1+x^2)^{-1/2}\right)$

Produktregelen

$$\boxed{(f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)}$$

$(x \cdot x^2)' = (x^3)' = 3x^2$

produktregelen gir: $(x)' \cdot x^2 + x \cdot (x^2)'$

$= 1 \cdot x^2 + x \cdot 2x = x^2 + 2x^2 = \underline{3x^2}$ ✓

$\cancel{(x)'(x^2)' = 1 \cdot 2x = 2x}$

$$(x^s \cdot x^r)' = (x^{s+r})' = (s+r)\,x^{s+r-1}$$

prod. regelen

$$(x^s)' \cdot x^r + x^s \cdot (x^r)'$$

$$= s\,x^{s-1} \cdot x^r + x^s \cdot r\,x^{r-1}$$

$$= s\,x^{s+r-1} + r\,x^{s+r-1}$$

$$= (s+r)\,x^{s+r-1}$$

$$\left(x(1-x)^7\right)' = (x)'\,(1-x)^7 + x\left(\left((1-x)^7\right)'\right)$$

$$= 1 \cdot (1-x)^7 + x\left(7(1-x)^6 \cdot \underbrace{(1-x)'}_{-1}\right)$$

$$= (1-x)^7 + (-7x)(1-x)^6$$

$$= (1-x)^6\left[(1-x) - 7x\right]$$

$$= \underline{(1-8x)(1-x)^6}$$

$$\left(\frac{x+3}{\sqrt{1+x^2}}\right)' = \left((x+3) \cdot (1+x^2)^{-1/2} \right)'$$

$$= (x+3)' \, (1+x^2)^{-1/2} + (x+3) \left((1+x^2)^{-1/2} \right)'$$

$$= 1 \, (1+x^2)^{-1/2} + (x+3) \left(\frac{-1}{2} (1+x^2)^{-3/2} \cdot \underbrace{(1+x^2)'}_{2x} \right)$$

$$= (1+x^2)^{-1/2} - x(x+3) \cdot (1+x^2)^{-3/2}$$

$$= \frac{1}{\sqrt{1+x^2}} - \frac{x(x+3)}{(1+x^2)\sqrt{1+x^2}}$$

$$= \frac{(1+x^2) - x(x+3)}{(1+x^2)\sqrt{1+x^2}}$$

$$= \underline{\underline{\frac{1-3x}{(1+x^2)\sqrt{1+x^2}}}}$$

Vi skal finne de deriverte til $\sqrt{x}$ og $\frac{1}{x}$ ved bruk av prod. regelen.

$$(\sqrt{x}\cdot\sqrt{x})' = (x)' \qquad x \geq 0$$

$$(\sqrt{x})'\cdot\sqrt{x} + \sqrt{x}(\sqrt{x})' = 1$$

$$2\sqrt{x}\,(\sqrt{x})' = 1 \qquad \text{Så } \underline{(\sqrt{x})' = \frac{1}{2\sqrt{x}}} \quad \checkmark$$

$$\frac{1}{x}\cdot x = 1 \qquad x \neq 0$$

$$\left(\frac{1}{x}\right)'\cdot x + \frac{1}{x}\cdot(x)' = (1)' = 0$$

$$\left(\frac{1}{x}\right)'\cdot x = \frac{-1}{x} \qquad \text{Så } \underline{\left(\frac{1}{x}\right)' = \frac{-1}{x^2}} \quad \checkmark$$

$$(f\cdot g)' = f'\cdot g + f\cdot g'$$

oppg Deriver $\underbrace{x^3}_{f}\cdot\underbrace{(2x-4)^5}_{g}$

$$(x^3(2x-4)^5)' = (x^3)'(2x-4)^5 + x^3((2x-4)^5)'$$

$$= 3x^2(2x-4)^5 + x^3 5(2x-4)^4\cdot\overbrace{(2x-4)'}^{2}$$

$$= 3x^2(2x-4)^5 + 10x^3(2x-4)^4$$

$$\left(x^3(2x-4)^5\right)' = x^2(2x-4)^4\left[\underbrace{\underbrace{3(2x-4)}_{6x-12} + 10x}_{16x-12 \,=\, 4(4x-3)}\right]$$

$$= \underline{4x^2(4x-3)(2x-4)^4}$$

$$\left(\frac{x}{(2x-5)^4}\right)' = \left(x \cdot (2x-5)^{-4}\right)'$$

$$= (x)'(2x-5)^{-4} + x\left((2x-5)^{-4}\right)'$$

$$= 1 \cdot (2x-5)^{-4} + x\left(-4(2x-5)^{-5} \cdot \overbrace{(2x-5)'}^{2}\right)$$

$$= (2x-5)^{-4} + (-8x)(2x-5)^{-5}$$

$$= \frac{1}{(2x-5)^4} - \frac{8x}{(2x-5)^5}$$

$$= \frac{2x-5}{(2x-5)^5} - \frac{8x}{(2x-5)^5}$$

$$= \underline{\frac{-(6x+5)}{(2x-5)^5}}$$

Kvotientregelen $\quad \left(\frac{f}{g}\right)' = \frac{f'\cdot g - f\cdot g'}{g^2}$

eks. $\quad \left(\frac{1}{x}\right)' \qquad f=1\ ,\ g=x$

$$\left(\frac{1}{x}\right)' = \frac{(1)'\cdot x - 1\cdot(x)'}{x^2} = \frac{-1}{x^2} \quad \checkmark$$

$$\left(\frac{x}{(2x-5)^4}\right)' = \frac{(x)'(2x-5)^4 - x\left((2x-5)^4\right)'}{\left((2x-5)^4\right)^2}$$

$$= \frac{(2x-5)^4 - x\,4(2x-5)'(2x-5)^3}{\left((2x-5)^4\right)^2}$$

$$= \frac{1}{(2x-5)^4} - \frac{8x}{(2x-5)^5} \quad \text{(som tidligere)}$$

Bruk fremgangsmåten du liker best.

oppgave $\quad \left(\frac{2x-1}{x^2-3x}\right)' = \frac{(2x-1)'(x^2-3x) - (2x-1)(x^2-3x)'}{(x^2-3x)^2}$

$$= \frac{2(x^2-3x) - (2x-1)(2x-3)}{(x^2-3x)^2}$$

$$\left(\frac{2x-1}{x^2-3x}\right)' = \frac{2x^2-6x-(4x^2-8x+3)}{(x^2-3x)^2}$$

$$= \underline{\underline{\frac{-2x^2+2x-3}{(x^2-3x)^2}}}$$

Kvotientregelen følger fra produktregelen:

$$\left(\frac{f}{g}\right)' = \left(f\cdot\frac{1}{g}\right)' \overset{\text{prod regel}}{=} f'\cdot\frac{1}{g} + f\cdot\left(\frac{1}{g}\right)'$$

Vi har $\left(\frac{1}{g(x)}\right)' = \left((g(x))^{-1}\right)' \overset{\text{kjerne regel}}{=} -1\cdot(g(x))^{-2}\cdot g'(x)$

setter dette inn:

$$\left(\frac{f}{g}\right)' = \frac{f'}{g} + f\cdot\left(\frac{-1}{g^2}\cdot g'\right)$$

$$= \frac{f'\cdot g}{g^2} + \frac{f\cdot(-g')}{g^2}$$

$$= \underline{\underline{\frac{f'\cdot g - f\cdot g'}{g^2}}}$$

eks $h(x) = \frac{(x^2+3)^5 \cdot x}{(4-3x)^4} = (x^2+3)^5 \cdot x \cdot (4-3x)^{-4}$

tre faktorer.

$$h'(x) = \underbrace{\left((x^2+3)^5 \cdot x\right)'}_{((x^2+3)^5)' \cdot x + (x^2+3)^5 \cdot (x)'} (4-3x)^{-4} + \left((x^2+3)^5 \cdot x\right)\left((4-3x)^{-4}\right)'$$

$$= \left(\left(5(x^2+3)^4 \cdot 2x\right) \cdot x + (x^2+3)^5 \cdot 1\right)(4-3x)^{-4} + \left((x^2+3)^5 \cdot x\right)\left(-4(4-3x)^{-5}(-3)\right)$$

$$= \left(10x^2(x^2+3)^4 + (x^2+3)^5\right)(4-3x)^{-4} + 12x(x^2+3)^5(4-3x)^{-5}$$

$$= (x^2+3)^4(10x^2 + x^2+3)(4-3x)^{-4} \quad \text{— } \| \text{ —}$$

$$= \frac{(x^2+3)^4(11x^2+3)}{(4-3x)^4} + \frac{12x(x^2+3)^5}{(4-3x)^5}$$

$$= \frac{(x^2+3)^4}{(4-3x)^5}\left[\underbrace{(11x^2+3)(4-3x) + 12x(x^2+3)}_{-33x^3+44x^2-9x+12+12x^3+36x}\right]$$

$$= \frac{(x^2+3)^4(-21x^3+44x^2+27x+12)}{(4-3x)^5}$$

bevis av produktregelen

$$(f\cdot g)'(x) = \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\,g(x+h) - f(x)\,g(x+h) + f(x)\,g(x+h) - f(x)\,g(x)}{h}$$

$$= \lim_{h\to 0} \frac{\big(f(x+h) - f(x)\big)\,g(x+h) + f(x)\big(g(x+h) - g(x)\big)}{h}$$

$$= \left(\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}\right) \lim_{h\to 0} g(x+h) + f(x) \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x)\cdot g(x) + f(x)\cdot g'(x) \qquad \square$$

$$\left(\frac{\sqrt{x-2}}{x}\right)' = \left((x-2)^{1/2}\cdot x^{-1}\right)' = \left((x-2)^{1/2}\right)'\cdot x^{-1} + (x-2)^{1/2}\cdot\left(x^{-1}\right)'$$

$$= \frac{1}{2}(x-2)^{-1/2}\cdot 1\cdot x^{-1} + (x-2)^{1/2}\cdot\left(-x^{-2}\right)$$

$$= \frac{1}{2}\cdot\frac{1}{x\sqrt{x-2}} + \frac{-\sqrt{x-2}}{x^2} = \frac{1}{x^2\sqrt{x-2}}\left(\frac{x}{2} - (x-2)\right)$$