01.02. 2021

# Kap. 8 Eksponentfunksjoner og logaritmer

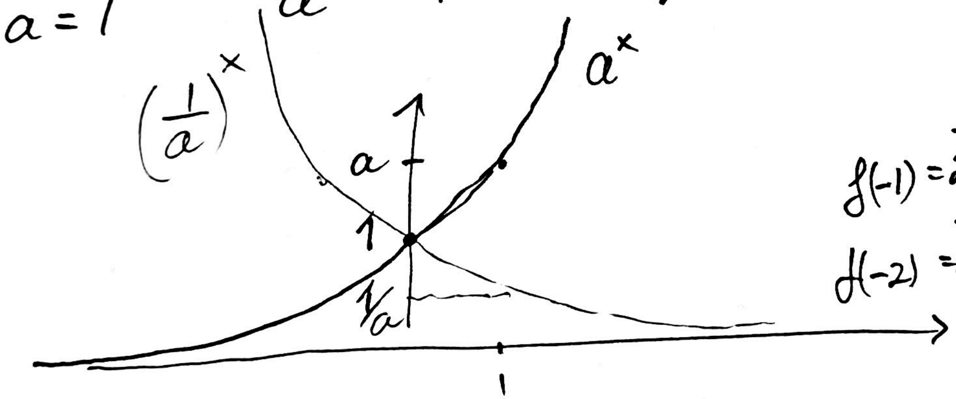
$a^x$ ← eksponent

grunntall → $a$, potens

$f(x) = x^r$ potensfunksjoner

$f(x) = a^x$ eksponentfunksjoner.

($a = 1$ $a^x = 1^x = 1$ for alle $x$, konstant)




$a > 1$

$f(-1) = 2^{-1} = \frac{1}{2}$

$f(-2) = 2^{-2} = \frac{1}{4}$

$a = 2$

$f(1) = 2^1 = 2$

$f(2) = 2^2 = 4$

$f(3) = 2^3 = 8$

$f(4) = 2^4 = 16$

$0 < a < 1$ $a^x = ((a^{-1})^{-1})^x = \left(\frac{1}{a}\right)^{-x}$

[$f(x_2) > f(x_1)$ når $x_2 > x_1$]

$a > 1$ så er $a^x$ strengt voksende

$0 < a < 1$ ——— strengt avtagende

} injektiv

$0 < a$ $a \neq 1$

$a^x$ er injektiv så den har en inversfunksjon: "logaritmen med basis a av x" =

$\log_a x$

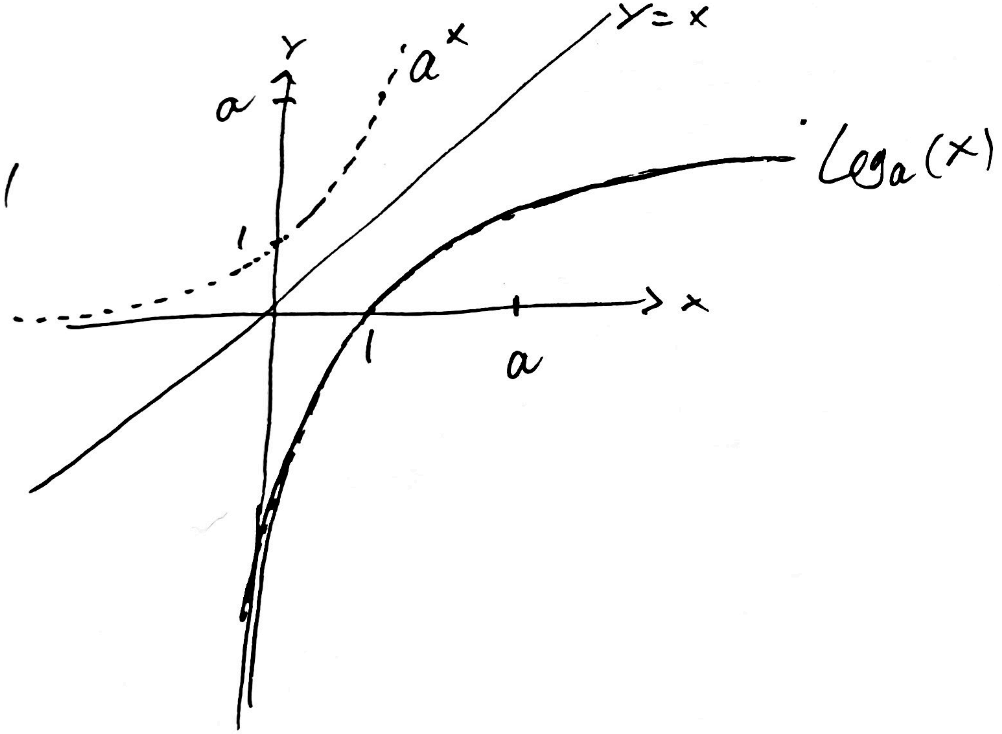
$a^{\log_a x} = x$

$x > 0$

$\log_a(a^x) = x$

$x \in \mathbb{R}$

$a > 1$




$\log_{10} = \log$
er vanlig å bruke

$\log_{10} 10 = 1$

$\log_{10} 100 = 2$ (siden $10^2 = 100$)

$\log_{10}\left(\frac{1}{1000}\right) = -3$ $\left(10^{-3} = \frac{1}{1000}\right)$

$\log(\sqrt{10}) = 1/2$ $\left(\sqrt{10} = 10^{1/2}\right)$

$a > 0$

Minner om eksponentreglene:

$a^{x+y} = a^x \cdot a^y$

$(a^x)^y = a^{x \cdot y}$ for alle $x$ og $y$.

$$a^{\log_a(x \cdot y)} = x \cdot y = \big(\underbrace{a^{\log_a x}}_{x}\big)\big(\underbrace{a^{\log_a y}}_{y}\big)$$

$$= a^{\log_a x + \log_a y}$$

Så

$$\boxed{\log_a(x \cdot y) = \log_a(x) + \log_a(y)}$$

LOGARITMEREGLENE

logaritme tar produkt til sum.

( $\log_a(x+y)$ ? kan ikke forenkles )

$$x^r = \underline{a^{\log_a(x^r)}} = x^r = \left(a^{\log_a x}\right)^r = \underline{a^{r \log_a x}}$$

så

$$\log_a x^r = r \log_a x$$

$5^x = 10$ hva må x være?

Tar logaritme på begge sider.

$\underbrace{\text{Log } 5^x} = \text{Log } 10$

$x \cdot \text{Log} 5 = \text{Log } 10$

$x = \frac{\text{Log } 10}{\text{Log } 5}$

oppg. Finn x slik at $9^x = 2$

$\text{Log } 9^x = \text{Log } 2$

$x \text{Log } 9 = \text{Log } 2$

$x = \frac{\text{Log } 2}{\text{Log } 9} = 0.31546\ldots$

$$\log_a(x) = \frac{\log_{10}(x)}{\log_{10}(a)} \quad \text{for } x > 0$$

$$= \left(\frac{1}{\log_{10}(a)}\right) \cdot \log_{10}(x)$$

$x = a^{\log_a(x)}$ setter inn: $a = 10^{\log_{10}(a)}$

$$x = \left(10^{\log_{10}(a)}\right)^{\log_a(x)} = 10^{\log_{10}(a) \cdot \log_a(x)}$$

$$= 10^{\log_{10}(x)}$$

Dette fører til $\log_{10}(a) \cdot \log_a x = \log_{10}(x)$

deler med $\log_{10}(a)$

gir $\underline{\log_a(x) = \frac{1}{\log_{10}(a)} \cdot \log_{10}(x)}$

Tilsvarende for eksponentfunksjonen: $a^x = (\underbrace{10^{\log_{10}(a)}}_{a})^x$

$$\underline{a^x = 10^{\log_{10}(a) \cdot x}}$$

$$2^x = 10^{\log_{10}(2)\cdot x} = 10^{0.30103\cdot x}$$

$$\log_2(x) = \frac{\log_{10} x}{\log_{10}(2)} = \frac{\log_{10}(x)}{0.30103..}$$

$$a^{(\log_a x)} = x$$

$\log_2 8 = 3$ siden $2^3 = 8$

$\log_3\left(\frac{1}{9}\right) = -2$ siden $3^{-2} = (3^2)^{-1} = \frac{1}{3^2} = \frac{1}{9}$ ✓

$\left(2^{-2} = (2^2)^{-1} = \frac{1}{2^2} = \frac{1}{4}\right)$

$\log_2\left(\frac{-1}{4}\right) =$ eksisterer ikkje

$\left(\log_a x \text{ er bare definert for } x > 0\right)$

$\log_{81}(9) = \frac{1}{2}$

fordi $81^{1/2} = \sqrt{81} = 9$.

$$\log_a\left(\frac{1}{x}\right) = \log_a\left(x^{-1}\right) = -\log_a(x)$$

$$\log_2\left(\frac{1}{7}\right) = -\log_2(7) \;..$$

$$\log\left(\frac{x}{y}\right) = \log x + \log\left(\frac{1}{y}\right) \qquad x, y > 0$$

$$= \log(x) - \log(y)$$

oppg. Løs

$$5^x = 3 \quad : \quad x \log 5 = \operatorname{Log}(5^x) = \operatorname{Log} 3$$

$$x = \frac{\operatorname{Log} 3}{\operatorname{Log} 5} \sim 0.68261$$

$$2 \cdot 5^x = 2(5^x) = 3$$

deler med 2: $5^x = 3/2$

gir $x = \frac{\log(3/2)}{\log(5)} = \frac{\operatorname{Log}(3) - \operatorname{Log}(2)}{\operatorname{Log}(5)}$

$$= 0.25193...$$

$$\operatorname{Log}(2 \cdot 5^x) = \log 3$$

$$\operatorname{Log} 2 + \log 5^x = \log 3$$

$$x \log 5 = \log 3 - \log 2$$

$$x = \frac{\log 3 - \log 2}{\log 5}.$$ (som tidligere)

1) $4^x = 3^x$

2) $4^x = 2 \cdot 3^x = 2 \cdot (3^x)$

3) $4^x = 2 \cdot 3^{2x+1}$

1) "ser at $x=0$":

$$\frac{4^x}{3^x} = 1 \iff \left(\frac{4}{3}\right)^x = 1 \quad \text{så } x=0.$$

$$\log(4^x) = \log(3^x)$$

$$x \log 4 = x \log 3$$

$$x(\underbrace{\log 4 - \log 3}_{\neq 0}) = 0 \quad \text{så } x=0.$$

2)

$$\log(4^x) = \log(2 \cdot 3^x) = \log 2 + \underbrace{\log(3^x)}_{x \log 3}$$

$$x(\log 4 - \log 3) = \log 2$$

$$x = \frac{\log 2}{\log(4/3)} \sim \underline{2.40942}$$

3) $4^x = 2 \cdot 3^{2x+1} = 2 \cdot 3^{2x} \cdot 3^1 = 6\,(3^2)^x$

$= 6 \cdot 9^x$

$\left(\frac{4}{9}\right)^x = 6$ så $x = \frac{\log(6)}{\log(4/9)}$

* Ta log på begge sider:

$\log(4^x) = \log(2 \cdot 3^{2x+1})$

$= \log 2 + \log(3^{2x+1})$

$x \log 4 = \log 2 + (2x+1)\log 3$

$x(\log 4 - \underbrace{2\log 3}_{\log 3^2}) = \underbrace{\log 2 + \log 3}_{\log(2 \cdot 3)}$

$x(\log 4 - \log 9) = \log 6$

så $x = \frac{\log 6}{\log(4/9)} \sim -2.20951$

Vi har penger i banken.
Renten er 10% per år.
Hvor lenge må pengene
stå i banken for at pengemengden
skal dobles?

$$P_0 (1 + 10\%)^r = P_r$$

$$(1,1)^r = \frac{P_r}{P_0}$$

Finn $r$ slik at $P_r = 2P_0$

$$(1.1)^r = \frac{P_r}{P_0} = 2$$

Løsning $r = \dfrac{\log 2}{\log(1.1)} = 7.27\ldots$ år.

benytter $\frac{1}{4} = \left(\frac{1}{2}\right)^2$

$$\left(\frac{1}{4}\right)^x - 2\left(\frac{1}{2}\right)^x - 3 = 0$$

$$\left(\left(\frac{1}{2}\right)^2\right)^x - 2\left(\frac{1}{2}\right)^x - 3 = 0$$

$$\left(\left(\frac{1}{2}\right)^x\right)^2 - 2\left(\frac{1}{2}\right)^x - 3 = 0$$

2-grads likning i $\left(\frac{1}{2}\right)^x$.

$$U^2 - 2U - 3 = 0 \qquad U = \left(\frac{1}{2}\right)^x$$

$$(U - 3)(U + 1) = 0$$

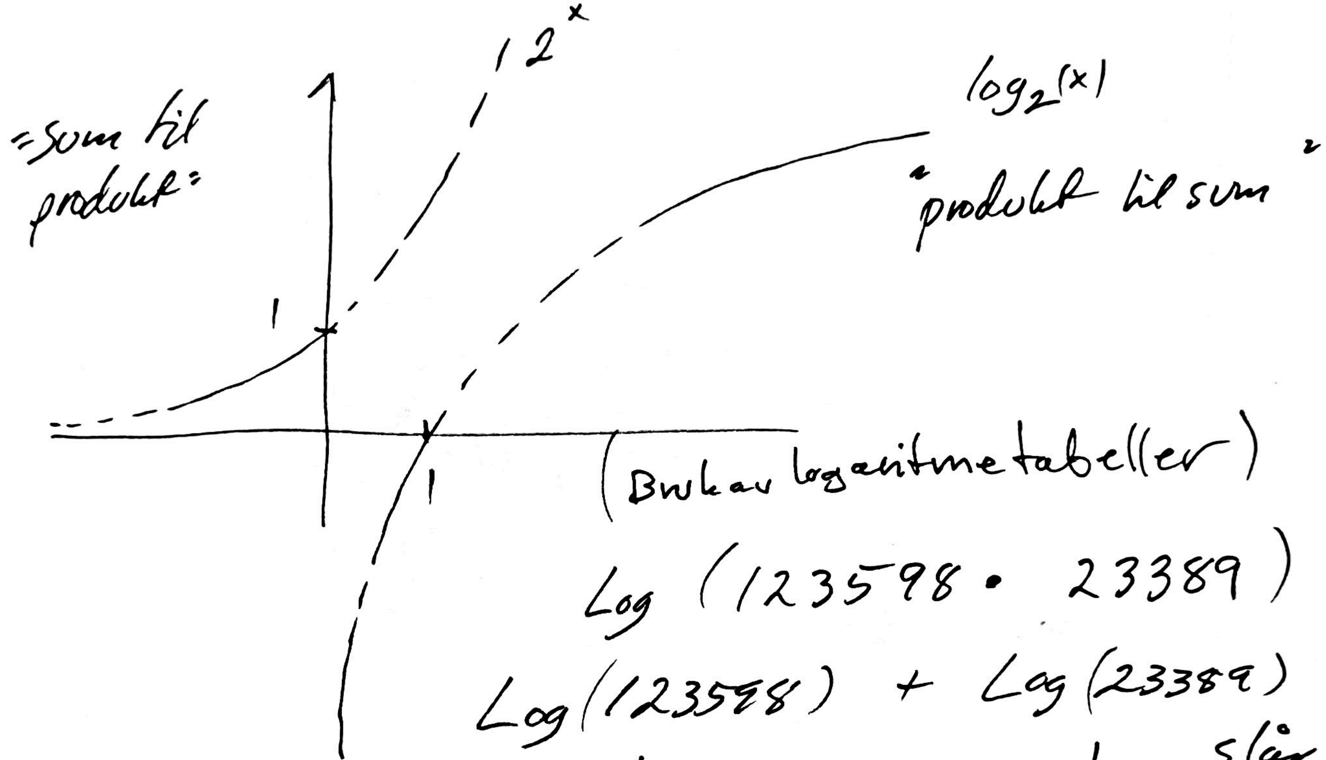
To løsninger

$$U = \left(\frac{1}{2}\right)^x = 3$$

$$U = \left(\frac{1}{2}\right)^x = -1$$ ingen løsning

$$x = \frac{\log 3}{\log(1/2)} = \frac{\log(3)}{-\log(2)}$$

$$\underline{x = -1{,}584\ldots}$$




(Bruk av logaritmetabeller)

$\text{Log}(123598 \cdot 23389)$

$\text{Log}(123598) + \text{Log}(23389)$ slår opp verdiene

$\downarrow \qquad\qquad \downarrow$

$\ldots + \ldots$

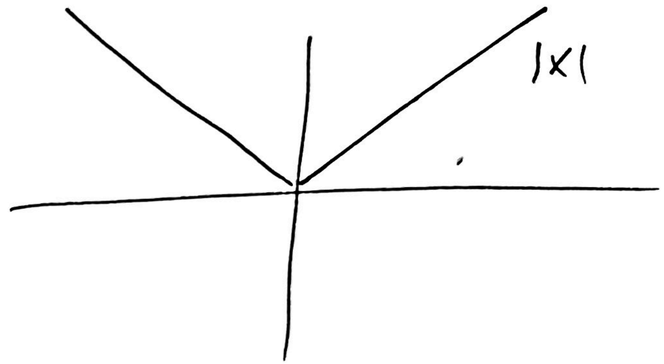
Finner summen. $S$

Finner $x$ slik at $\text{Log}(x)$ blir lik $S$.

Da er $x$ en tilnærming til produktet

$\underline{123598 \cdot 23389}$

$$|x| \overset{\text{def}}{=} \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$




$$|x|' = \begin{cases} +1 & x > 0 \\ -1 & x < 0 \end{cases}$$

ikke def. for $x = 0$.

$$= \frac{|x|}{x}$$

$$|2x+5| = \begin{cases} 2x+5 & x \geq -5/2 \\ -(2x+5) & x < -5/2 \end{cases}.$$