2 feb. 2021

8.5

$$e^x = 3$$
$$\text{Log } e^x = \text{Log } 3$$
$$x \text{ Log } e = \text{Log } 3$$
$$x = \frac{\text{Log } 3}{\text{Log } e}$$

$$\ln e^x = \ln 3$$
$$x = \ln 3$$
$$\sim 1.09861\ldots$$

Vi sàg sist gang ~~at~~

$$\text{Log}_a\, x = \frac{\text{Log } x}{\text{Log}(a)}$$

så $\text{Log}_e(x) = \dfrac{\text{Log } x}{\text{Log}(e)}$

$$\text{Log}_e(x) = \ln(x)$$

oppg.

$$2e^{3x} = 10$$
$$e^{3x} = 10/2 = 5$$
$$\ln e^{3x} = \ln 5$$
$$3x = \ln 5$$
$$x = (\ln 5)/3 = 0.536479\ldots$$

$$\ln y = -3$$
hva er y?
$$e^{\ln y} = e^{-3}$$
$$y = e^{-3}$$
$$\sim 0.04979$$

$$10^x = \left(e^{\ln 10}\right)^x = e^{\ln(10)\cdot x}$$

$$10 = e^{\ln 10}$$

Kjerneregelen gir

$$(10^x)' = \Big(e^{\overbrace{(\ln(10)\cdot x)}^{u \text{ kjerne}}}\Big)' = \frac{de^u}{du}\cdot\frac{du}{dx} = e^u\cdot(\ln(10))$$

$$(10^x)' = \ln(10)\cdot 10^x \sim (2.305\ldots)\cdot 10^x$$

$$(a^x)' = \left(e^{\ln a\cdot x}\right)' = \underline{\ln a\cdot a^x}$$

$a > 0$

$$(2^x)' = (\ln 2)\cdot 2^x \sim (0.69315\ldots)\cdot 2^x$$

$(a^x)' = a^x \cdot \text{konstant}$

Hvis $a = e$ er konstanten lik 1.

$$\underline{(e^x)' = e^x}$$

Naturlig logaritme er Logaritme med basis e.

$$\ln x = \log_e x$$

$e^{\ln x} = x \qquad x > 0$

$\ln(e^x) = x \qquad \text{alle } x$

$\ln(e) = 1 \qquad \ln(e^{-2}) = -2$

$e^{\ln(10)} = 10, \qquad \ln(10) = 2.3059\ldots$

8.4 EULER TALLET e

$e = 2.7\underbrace{1828}\,\underbrace{1828}459\ldots$ irrasjonalt tall.

Deriverte til eksponentfunksjoner.

$$(a^x)' = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} \quad , \quad a^{x+h} = a^x \cdot a^h$$

$$= \lim_{h\to 0} \frac{a^x(a^h - 1)}{h}$$

$$= a^x \cdot \left(\lim_{h\to 0} \frac{a^h - 1}{h} \right)$$

$$\left(h = 1/n \quad \frac{a^h - 1}{\frac{1}{n}} = n(a^{1/n} - 1) \right)$$

Grensen $\lim_{h\to 0} \frac{a^h - 1}{h} = 1$ når $a = e$

$$a = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$$

Løs likningen

$$2\log(x) = \log(2x+3)$$

$x > 0$

$(x > -3/2)$

$$\Rightarrow 10^{\log(x^2)} = 10^{\log(2x+3)}$$

$\log a = \log b$

$\Updownarrow$

$a = b \ (>0)$

$$\Rightarrow x^2 = 2x+3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$\underline{\underline{x = 3}}$$

(og $x = -1$ utgår fordi: $x > 0$)

$$\left(\begin{aligned} &\log x^r = r\log x \\ &\log x^2 = 2\log x \\ &\qquad\quad x > 0 \end{aligned}\right)$$

$*$

$$\log x = 10$$

$$x = 10^{\log x} = 10^{10} = 10^{9+1}$$

$$= 10 \cdot 10^9 = 10 \text{ milliarder.}$$

$$\log(x+1) = \log(x-1) + 1$$

$$\log(x+1) - \log(x-1) = 1$$

$$\log(x+1) + \log((x-1)^{-1}) = 1$$

$$\log((x+1)(x-1)^{-1}) = 1$$

$$\log\left(\frac{x+1}{x-1}\right) = 1$$

$$\frac{x+1}{x-1} = 10^{\log \frac{x+1}{x-1}} = 10^1 = 10 \quad | \, x-1$$

$$x+1 = 10(x-1) = 10 \cdot x - 10$$

$$1+10 = 10x - x = (10-1)x = 9x$$

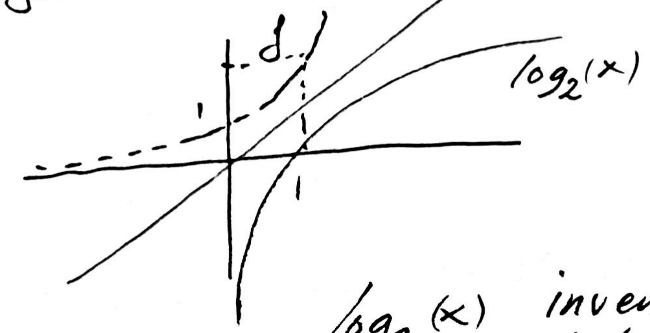
Så $\underline{\underline{x = \frac{11}{9}}}$

tegnet i geogebra
ser ut som løsningen
er 1.222...

$$1 + \frac{2}{9} = \frac{11}{9}.$$

8.3 Logaritme likninger

$f(x) = 2^x$



$2^{\log_2 x} = x$

$\log_2 2^x = x$

$\log_2(x)$ inversfunksjonen til 2^x

eksponentfunksjoner $\quad a^x \quad a > 0, \ a \neq 1$

$\text{Log}_a(x)$ ("a-logaritmen til x" "Logaritmen med basis a av x")

Logaritmereglene

$$\boxed{\begin{aligned} \text{Log}_a(x \cdot y) &= \text{Log}_a(x) + \text{Log}_a(y) \\ \text{Log}_a(x^r) &= r \cdot \text{Log}_a x \end{aligned}}$$

$\text{Log}_{10} = \text{Log} = \lg$

Briggske logaritme

$\mathrm{Log}\, x = 2$ gir $x = 10^2 = 100$

$10^{\mathrm{Log}\, x} = 10^2$

$x = 10^2 = 100.$

* $\mathrm{Log}\, x = 3/2$, $10^{\mathrm{Log}\, x} = 10^{3/2}$

$x = 10^{3/2}$

$= 10^{1+1/2} = 10\sqrt{10} \sim 31.6228\ldots$

oppg. Løs likningen $\mathrm{Log}_2\, x = 8$

$\left(\mathrm{Log}_2 (8) = \mathrm{Log}_2 (2^3) = 3. \right)$

$x = 2^{\mathrm{Log}_2\, x} = 2^8 = 256$

* $3\,\mathrm{Log}(x) + 1 = 10$ løser den lineære likningen i $\mathrm{Log}(x)$

$\mathrm{Log}(x) = \frac{10-1}{3} = 3$

$x = 10^{\mathrm{Log}(x)} = 10^3 = 10 \cdot 10 \cdot 10 = \underline{\underline{1000}}$

Øving 8.33 c) $(\log x)^3 - (\log x)^2 - 2(\log x) = 0 \quad (*)$

$U = \log x$ gir $x = 10^U$.

3.grads likning.

$U^3 - U^2 - 2U = 0$

$U(U^2 - U - 2) = 0$

$U = 0$ eller $U^2 - U - 2 = 0$

$(U-2)(U+1) = 0$

$U = 2$ og $U = -1$

Mulige løsninger

$0 = \log x \quad , \quad x = 10^0 = 1$

$-1 = \log x \quad , \quad x = 10^{-1} = 1/10$

$2 = \log x \quad , \quad x = 10^2 = 100$

Løsningene til $(*)$ er $\underline{x = \frac{1}{10}, 1 \text{ og } 100}$

$$x = 2^{\log_2 x} = 2^8 \qquad x = 2^8 = 256$$

8.51 c)

$$\frac{2e^x - 3}{e^x - 1} = \frac{e^x}{2} \quad (**) \qquad e^x = U, \quad x = \ln(U)$$

$$\frac{2 \cdot U - 3}{U - 1} = \frac{U}{2} \quad | (U-1)$$

$$2 \cdot U - 3 = \frac{U}{2}(U-1) \quad | \cdot 2, \text{ 2.grads uttrykk i } U$$

$$4U - 6 = U^2 - U$$

$$U^2 - 5U + 6 = 0$$

$$(U-2)(U-3) = 0, \quad U = 2 \text{ og } U = 3.$$

Løsningene til likning (**) er $\underset{\sim 0.693..}{\ln 2}$ og $\underset{\sim 1.0986}{\ln 3}$

Deriver: 1) e^{3x} 2) e^{x^2} 3) $x \cdot e^x$

(Minner om $(e^x)' = e^x$ $(a^x)' = (\ln a) \cdot a^x$)

$= ((e^{\ln a})^x)'$

$(3^x)'$

$= (e^{\ln 3 \cdot x})'$

$= \ln(3) \cdot 3^x$

1) $(e^{3x})' = \frac{de^u}{du} \cdot \frac{du}{dx}$ $u = 3x$

$= e^{3x} \cdot 3$

$(e^{3x})' = 3e^{3x}$

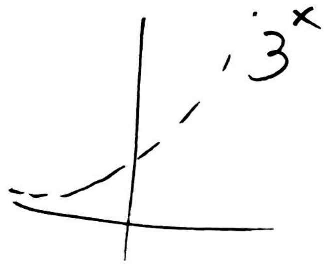
2) $(e^{x^2})' = e^{x^2} \cdot (x^2)' = 2xe^{x^2}$

3) $(x \cdot e^x)' = (x)' \cdot e^x + x(e^x)'$

$= 1 \cdot e^x + x \cdot e^x$

$= (1+x)e^x$

generelt $(f(x) \cdot e^x)' = (f'(x) + f(x))e^x$.



$3^a = 3^b \iff a = b$

$3^{x^2} = 3^x \iff x^2 = x \iff x^2 - x = 0$

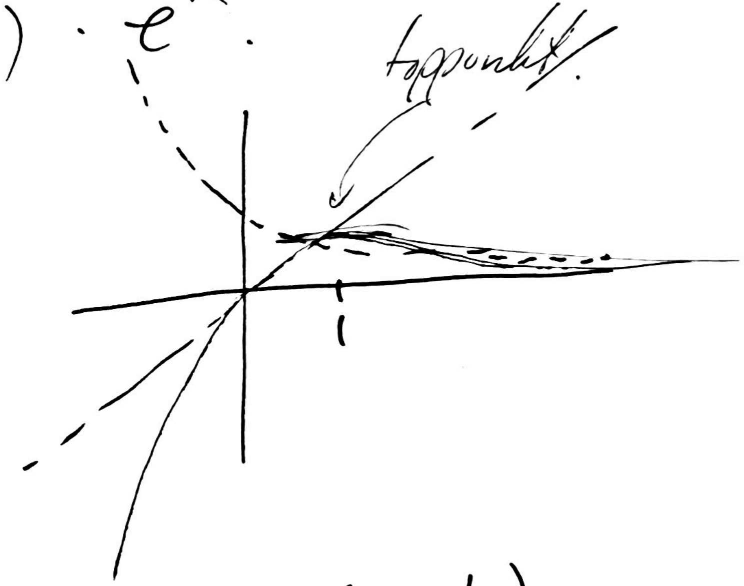
$\iff x(x-1) = 0$

Log på begge sid

$3^a = 10^b = a \log 3 = b \log_{10} 10$

$\underline{a \log 3 = b}$

$(x^2 \cdot e^x)' = (2x + x^2) \cdot e^x.$



$f(x) = x e^{-x}$

Finn toppunkt til $f(x)$.

$f'(x) = (x)' e^{-x} + x \cdot (e^{-x})'$

$= e^{-x} + x(e^{-x} \cdot (-x)')$

$= \underline{(1-x) e^{-x}}$

toppunkt $\underline{\underline{(1, \frac{1}{e})}}$