

3 feb
2021

Logaritmer 8.6 - 7.

a^x $a > 0$ $a > 1$ vokende
 $a \neq 1$ $0 < a < 1$ avtagende

$$a^x = (a^{-1})^{-x} = \left(\frac{1}{a}\right)^{-x}$$

$$(a^x)' = (\text{konstant}) \cdot a^x$$

$$(e^x)' = e^x$$

$a = e$ Eulers tall

$$e = 2.718281828 \dots$$

$$a = e^{\ln a}, \quad a^x = (e^{\ln a})^x = \frac{e^{(\ln a) \cdot x}}{}$$

$\ln x$ naturlig logaritme til x

$$= \log_e x$$

e^x , $\ln x$ inversfunksjoner

$$e^{\ln x} = x \quad \ln(e^x) = x$$

$\log\left(\frac{1}{a}\right) x$ uttrykt ved hjelp av $\log_a x$

$$\left(\frac{1}{a}\right)^{\log_a(x)} = x$$

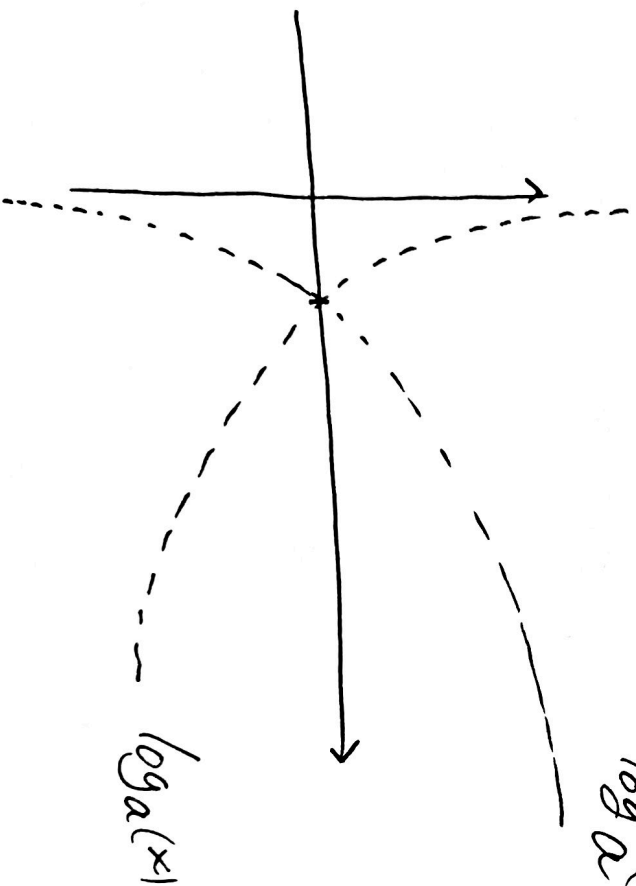
$$(a^{-1})^{\log_a(x)} = x = a^{\log_a x}$$

$$\text{Se } \frac{\log_a x}{\log_a x} = -\log_a x$$

$$\log_a(x) \quad a > 1$$

$$\lim_{x \rightarrow 0^+} \log_a(x) = -\infty$$

$$\lim_{x \rightarrow \infty} \log_a(x) = \infty$$



$$\log_a(x) \quad 0 < a < 1$$

$$\log_a(x) = \frac{\ln x}{\ln a}$$

$$(e^x)' = e^x$$

Eksempel

Derivation med e^x

$$(4e^{5x})' = 4 \cdot (e^{5x})' \cdot (5x)'$$

$$= 20e^{5x}$$

Deriver: $(e^x)^3$

ketjenergel

$$3(e^x)^2 \cdot (e^x)' = 3(e^x)^3$$

$$e^{3x}$$

$$e^{3x} (3x)' = 3e^{3x}$$

$$\left(\left(\frac{1}{2}\right)^x\right)'$$

$$= \left(e^{\ln(1/2)x}\right)' = \left(e^{\ln(1/2)x}\right)'$$

$$= e^{\ln(1/2)x} \cdot \left(\ln\left(\frac{1}{2}\right)x\right)'$$

$$= \ln\left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^x$$

$$= -\ln 2 \left(\frac{1}{2}\right)^x$$

$$= -\ln 2 \cdot 2^{-x}$$

$$(\ln x)' = \frac{1}{x}$$

(Derivate av
inversfunktioner)

$$e^{\ln x} = x$$

$x > 0$

$$(\ln x)' = (x)' = 1$$

$$x \cdot e^{\ln x}$$

Deriverer:

Se

$$(\ln x)' = \frac{1}{x}$$

$$\begin{aligned} (x^r)' &= r x^{r-1} \\ \left(\frac{x^r}{r}\right)' &= x^{r-1} \end{aligned}$$

alle $r \neq 0$.

$$\begin{aligned} r-1 &= -1 \quad \text{for} \quad r=0 \\ (x^0)' &= (1)' = 0 \cdot x^{-1} = 0 \end{aligned}$$

$$(\log_a x)' = \left(\frac{\ln x}{\ln a} \right)' = \frac{1}{(\ln a) \cdot x}$$

$$* \quad \ln(2x-3) \quad x > 3/2$$

$$(\ln(2x-3))' = \frac{1}{2x-3} (2x-3)' = \frac{2}{2x-3}$$

$$= \frac{3x^2}{x^3} = \frac{3}{x}$$

$$* \quad (\ln(x^3))' = \frac{1}{x^3} (\cancel{x^3})' = \frac{3x^2}{x^3} = \frac{3}{x}$$

$$= (3 \ln x)' = 3 \frac{1}{x}$$

bedre:

$$* \quad (\ln(2x-3) \cdot x^3)' = (\ln(2x-3) + \ln x^3)'$$

$$= \frac{2}{2x-3} + \frac{3}{x}$$

$$\ln(-x) \quad \text{defined for } x < 0$$

$$(\ln(-x))' = \frac{1}{-x} \cdot (-x)' = \frac{-1}{-x} = \frac{1}{x}$$

$$\ln|x| = \begin{cases} \ln x & x > 0 \\ \ln(-x) & x < 0 \end{cases}$$

$$\boxed{(\ln|x|)' = \frac{1}{x} \quad x \neq 0}$$

Deriver

oppo.

$$1) (\ln|4-8x|)' = \frac{1}{4-8x} \underbrace{(4-8x)'}_{-8} = \frac{-2}{1-2x}$$

$$2) (\ln(x^2+3))' = \frac{1}{x^2+3} (x^2+3)' = \frac{2x}{x^2+3}$$

Derive X^x $x > 0$

Hint: $X = e^{\ln x}$ so $X^x = (e^{\ln x})^x = e^{x \cdot \ln x} \dots$

$$(X \cdot \ln X)' = \text{prod. rule} \quad (X)' \cdot \ln X + X \cdot (\ln X)' = \underline{\ln X + 1}$$

$$\begin{aligned} \text{Note: } (X \ln X - X)' &= \ln X \\ (X^x)' &= (e^{x \ln x})' = e^{x \ln x} \cdot (x \cdot \ln x)' \\ &= X^x \cdot (\ln x + 1) \\ &= \underline{X^x + \ln X \cdot X^x} \end{aligned}$$

$$\ln(\ln x) \quad x > 1$$

$$(\ln(\ln x))' = \frac{1}{\ln x} \cdot (\ln x)' = \frac{1}{x \ln x}$$

Finne ekstremalpunkt i

$$f(x) = \begin{cases} x^2 \ln x & x > 0 \\ 0 & x = 0 \end{cases}$$

Lokalt ekstremalpunkt i origo.

$$f'(x) = (x^2 \ln x)'$$

$$= (x^2)' \ln x + x^2 (\ln x)'$$

$$= 2x \ln x + x^2 \cdot \frac{1}{x}$$

$$= x(2 \ln x + 1)$$

$$2 \ln x + 1 = 0$$

$$f'(x) = 0 \quad (\text{for } x > 0)$$

$$e^{\ln x} = e^{-1/2}$$

$$x = e^{-1/2} = \frac{1}{\sqrt{e}}$$

$$\sim 0.6065 \dots$$

$$\frac{x}{\ln x}$$

Resultat :

n	$\frac{n}{\ln(n)}$
1000	144
10000	1085
10^6	$\sim 73\ 000$
10^9	$\sim 48 \cdot 10^6$

$$(\# \text{ primtall} \leq n) \cdot \frac{\ln n}{n} \rightarrow 1$$

n stor

$$\begin{aligned}
 & \left(\frac{\ln x}{x} \right)' = \left((\ln x)' \cdot \frac{1}{x} \right)' \\
 & = (\ln x)' \cdot \frac{1}{x} + \ln x \cdot \left(\frac{1}{x} \right)' \\
 & = \frac{1}{x} \cdot \frac{1}{x} + \ln x \cdot \left(-\frac{1}{x^2} \right) \\
 & = \frac{1 - \ln x}{x^2}
 \end{aligned}$$

$$x=e$$

Mer
likninger

$$e^{3x} + 1 - 12e^{-3x} = 0$$

$$U + 1 - \frac{12}{U} = 0 \quad | \cdot U$$

$$U = e^{3x}$$
$$\ln U = 3x$$
$$x = \frac{\ln U}{3}$$

$$U^2 + U - 12 = 0$$

$$(U+4)(U-3) = 0$$

2. grads likning

$$U = 3 \text{ og } U = -4.$$

$$\frac{\ln 3}{3} \approx 0.3662$$

Så løsningene er

$$(\ln x)^3 - 4 \ln x = 0.$$

$$e^U = \ln x$$

$$x = e^U$$

$$U(U^2 - 4) = 0$$

$$U(U-2)(U+2) = 0$$

$$U = 0, -2, 2.$$

Løsningene er $e^{-2}, 1, e^2$

Ekspont funksjonar voksa raskare enn polynome

$$\lim_{x \rightarrow \infty} \frac{x^n}{e^x} = 0$$

$$\begin{aligned} (x^n e^{-x})' &= (x^n)' \cdot e^{-x} + (x^n) (e^{-x})' \\ &= n x^{n-1} e^{-x} + x^n (-e^{-x}) \\ &= x^{n-1} \cdot e^{-x} (n - x) \end{aligned}$$

$x > 0$

sjåer synker

$x = n$

begrense $\leq \left(\frac{n+1}{e}\right)^{n+1}$

toppunkt $\left(n, \left(\frac{n}{e}\right)^n\right)$

$$\lim_{x \rightarrow \infty} \frac{x^n}{e^x}$$

$$\frac{x^n}{e^x}$$

$$\lim_{x \rightarrow \infty} \frac{1}{x}$$

$$\cdot$$

$$\underbrace{(x^{n+1} e^{-x})}_{\text{begrense}} = 0$$

$$\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt[n]{x}} = 0$$

følger siden:

$$\frac{\ln x}{\sqrt[n]{x}}$$

=

$$\frac{\ln e^x}{(e^x)^{1/n}}$$

=

$$\frac{x}{e^{x/n}}$$

$$x = e^x$$