5 feb. 2021

Deriver 1) e^{2x-1} 2) $\sqrt[3]{e^x}$ 3) $\frac{4}{e^x}$

Husk at $(e^x)' = e^x$ $(a^x)' = (\ln a)\, a^x$

$a^x = (e^{\ln a})^x = e^{\ln a \cdot x}$ euler tallet $e = 2.718281828459045\ldots$

1) $(e^{2x-1})' = e^{2x-1} \cdot (2x-1)' = \underline{2e^{2x-1}}$ $\left(= \frac{2}{e} e^{2x}\right)$

2) $\sqrt[3]{e^x} = (e^x)^{1/3} = e^{x/3}$

$(e^{x/3})' = e^{x/3} \cdot (x/3)' = \frac{1}{3} e^{x/3} = \underline{\frac{1}{3}\sqrt[3]{e^x}}$

(Alternativt: $((e^x)^{1/3})' = \frac{1}{3}(e^x)^{\frac{1}{3}-1}(e^x)'$

$= \frac{1}{3}(e^x)^{-2/3} \cdot e^x = \frac{1}{3}(e^x)^{-\frac{2}{3}+1} = \frac{1}{3}(e^x)^{1/3}$

$= \frac{1}{3}\sqrt[3]{e^x}$)

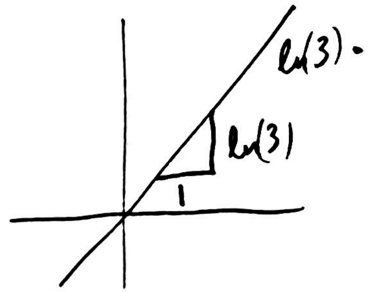
3) $\left(\frac{4}{e^x}\right) = 4 \cdot (e^x)^{-1} = 4 \cdot e^{-x}$

$\left(\frac{4}{e^x}\right)' = (4 \cdot e^{-x})' = 4(e^{-x})' = 4(e^{-x}) \cdot (-x)'$

$= -4e^{-x} = \underline{\underline{\frac{-4}{e^x}}}$

Deriver

1) 3^x 2) $\sqrt[5]{2^x}$ 3) $\dfrac{e^x}{\sqrt{3^x}}$



1) $3^x = \left(e^{\ln 3}\right)^x = e^{\ln 3 \cdot x}$

$$(3^x)' = \left(e^{\ln 3 \cdot x}\right)' = e^{(\ln 3)\cdot x} \cdot (\ln(3)\cdot x)'$$

$$= \ln 3 \cdot e^{\ln 3 \cdot x} = \underline{\ln(3)\cdot 3^x}$$

2) $\sqrt[5]{2^x} = \left(e^{\ln 2 \cdot x}\right)^{1/5} = e^{\left(\frac{\ln 2}{5}\right)x}$

$$\left(\sqrt[5]{2^x}\right)' = \left(e^{\left(\frac{\ln 2}{5}\right)x}\right)' = \left(e^{\frac{\ln 2}{5}x}\right)\left(\frac{\ln 2}{5}\cdot x\right)'$$

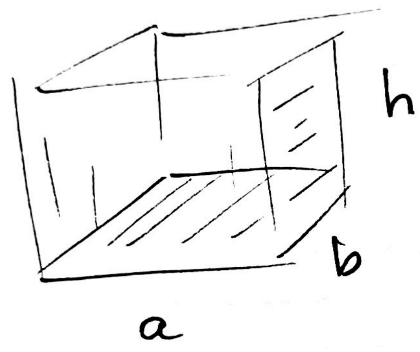
$$= \underline{\frac{\ln 2}{5}\sqrt[5]{2^x}}$$

3) $\dfrac{e^x}{\sqrt{3^x}} = e^x\left(3^x\right)^{-1/2} = e^x\left(e^{\ln 3 \cdot x}\right)^{-1/2} = e^x \cdot e^{-\frac{\ln 3}{2}x}$

$$= e^{\left(1-\frac{\ln 3}{2}\right)x}$$

$$\left(\frac{e^x}{\sqrt{3^x}}\right)' = e^{\left(1-\frac{\ln 3}{2}\right)x}\left(\left(1-\frac{\ln 3}{2}\right)x\right)' = \underline{\underline{\left(1-\frac{\ln 3}{2}\right)\frac{e^x}{\sqrt{3^x}}}}$$

ingen topp.



Volum

$V = a \cdot b \cdot h = 1 m^3.$

Minimere overflaten.

Overflaten:

$$O = \underbrace{a \cdot b}_{\text{bunn}} + \underbrace{2(a+b) \cdot h}_{\text{sidevegger}}$$

skal minimeres.

side 296-7

P_{2016}

$$P_{2017} = P_{2016} + 0.95\% \cdot P_{2016}$$
$$= P_{2016}(1 + 0.95\%)$$
$$P_{2016}(1 + 0.0095)$$
$$(1,0095)$$

8.15 e) $\frac{\log 81}{\log 9} = \frac{\log 9^2}{\log 9} = \frac{2\log 9}{\log 9} = \underline{2}$

f) $\frac{\log x^3 + \log x^5}{\log x^2}$ $\quad x > 0, \neq 1$

$= \frac{3\log x + 5\log x}{2\log x} = \frac{(3+5)\log x}{2\log x} = \frac{8}{2} = \underline{\underline{4}}$

8.33 c) Løs likningen. $(\log x)^3 - (\log x)^2 - 2(\log x) = 0$

$U = \log_{10} x$

10^U

$\underline{x = 10^U}$

$U^3 - U^2 - 2U = 0$

$U(U^2 - U - 2) = 0$

$U(U-2)(U+1) = 0$

$U = 0, -1, 2$

Løsningene e $x = 10^0 = \underline{1}$, $x = 10^{-1} = \underline{\frac{1}{10}}$ og $x = 10^2 = \underline{100}$

Deriver :

$(\ln x)' = \frac{1}{x}$

$\log_a(x) = \frac{\ln x}{\ln(a)}$

$\ln x = \log_e x$

1) $\ln 3x$ 2) $\log(3x+1)$

3) $\ln((x-3)^3)$ 4) $\ln\left(\frac{x}{x^2+1}\right)$.

1) $(\ln(3x))' = (\ln(3) + \ln(x))' = \frac{1}{x}$.

(Alternativ : $(\ln(3x))' = \frac{1}{3x} \cdot (3x)' = \frac{3}{3x} = \frac{1}{x}$ ✓

2) $(\log(3x+1))' = \left(\frac{\ln(3x+1)}{\ln(10)}\right)' = \frac{1}{\ln 10}(\ln(3x+1))'$

$= \frac{1}{\ln 10} \cdot \frac{1}{3x+1} \cdot (3x+1)' = \frac{1}{\ln(10)} \cdot \frac{3}{3x+1}$

$= \frac{1}{(\ln 10)(x+1/3)}$

($\ln 10 \sim 2.302\ldots$

$(\log_a x)' = \left(\frac{\ln x}{\ln a}\right)' = \frac{1}{(\ln a) \cdot x}$.

$(\ln(u(x)))' = \frac{1}{u(x)} \cdot u'(x)$

$= \frac{u'(x)}{u(x)}$

3) $\left(\ln\left((x-3)^3\right)\right)' = \left(3\ln(x-3)\right)'$

$= 3\frac{1}{x-3}\cdot(x-3)' = \underline{\underline{\frac{3}{x-3}}}$

4) $\ln\left(\frac{x}{x^2+1}\right) = \ln\left(x\cdot(x^2+1)^{-1}\right)$

$= \ln x + (-1)\ln(x^2+1)$

$\left(\ln\frac{x}{x^2+1}\right)' = \left(\ln x - \ln(x^2+1)\right)'$

$= \frac{1}{x} - \frac{1}{x^2+1}(x^2+1)' = \underline{\frac{1}{x} - \frac{2x}{x^2+1}}$

$= \frac{(x^2+1) - 2x(x)}{x(x^2+1)} = \underline{\underline{\frac{1-x^2}{x(x^2+1)}}}$

Deriver $x^{\sqrt{2}}$ og $\sqrt{2}^x$.

$(\sqrt{2} \sim 1.4142\ldots)$

$\left(x^{\sqrt{2}}\right)' = \underline{\sqrt{2}\,x^{\sqrt{2}-1}}$

$\left(\sqrt{2}^x\right)' = \left(\left(e^{\ln\sqrt{2}}\right)^x\right)'$

$= e^{(\ln\sqrt{2})x}\cdot(\ln\sqrt{2}\cdot x)'$

$= \underline{\ln\sqrt{2}\cdot\sqrt{2}^x} = \ln 2^{1/2}(\sqrt{2})^x$

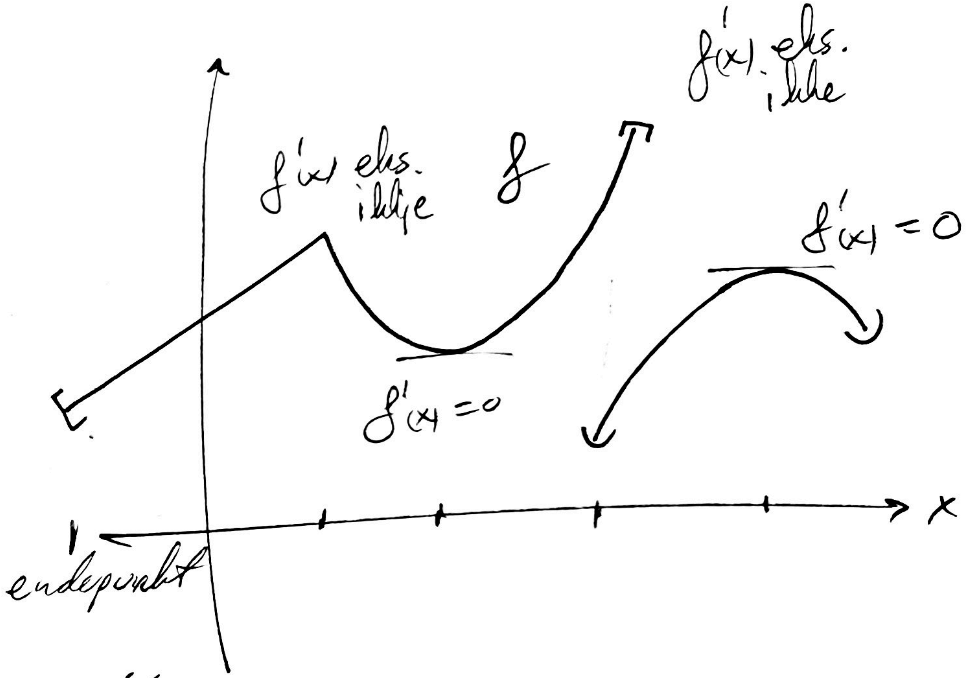
$= \underline{\underline{\left(\frac{1}{2}\ln 2\right)\left(\sqrt{2}\right)^x}}$

Kritiske punkt til f, D_f

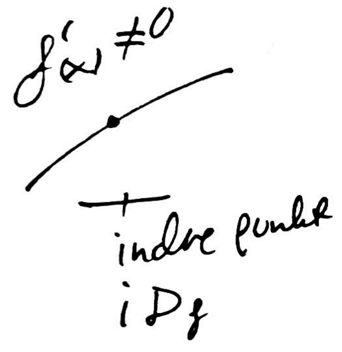
1 x s.a. $f'(x) = 0$

2 x s.a. $f'(x)$ ikkje eksisterer

3 x s.a. x er et endepunkt i D_f



(Har ikkje ekstremalpunkt:



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Alle ekstremalpunkt er kritiske punkter.

Tilstrekkelig å undersøke de kritiske punktene for å finne (lokale) ekstremalpunkt.

(spørsmål om kritiske punkt som henvises til i oppg 9 ob15)

Se også 7.1 og 7.2 i boka.