9 feb 2021

Funksjonsdrøfting 8.9.

$N(x) = e^{-x^2}$

$N(-x) = N(x)$ jevn funksjon

$\lim_{x\to\infty} e^{-x^2} = 0$ x-aksen er en horisontal asymptote.

$N'(x) = e^{-x^2}\cdot(-x^2)' = -2x\,e^{-x^2}$.

avtagende for $x > 0$

$f'(x) = 0$ for $x = 0$.

$N''(x) = (-2x)'\cdot e^{-x^2} + (-2x)(e^{-x^2})'$

$= -2e^{-x^2} + (-2x)(-2x e^{-x^2}) = (-2+4x^2)e^{-x^2}$

$= 4(x^2 - \frac{1}{2})e^{-x^2}$

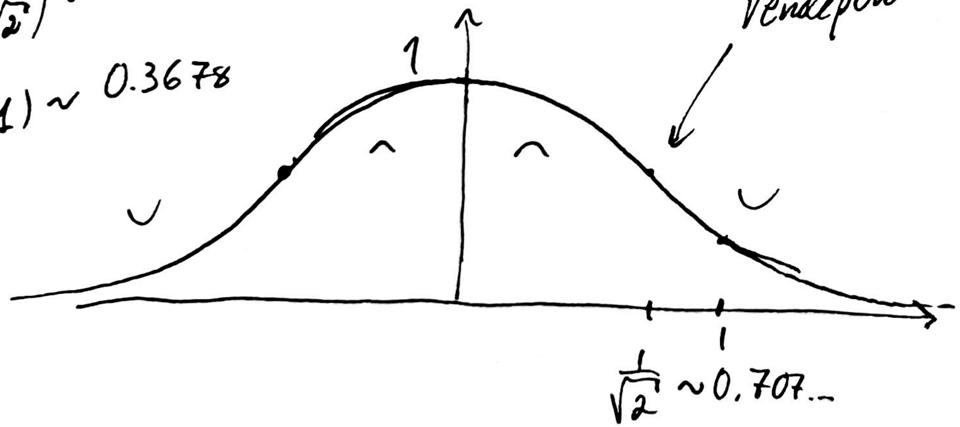
$f''(x) < 0$ når $|x| < \frac{1}{\sqrt{2}}$

$f''(x) = 0$ $x = \pm\frac{1}{\sqrt{2}}$

$f''(x) > 0$ $|x| > \frac{1}{\sqrt{2}}$

$f(\frac{1}{\sqrt{2}}) \sim 0.6065...$

$f(1) \sim 0.3678$



Normalfordelingskurven,

Viktig fordeling i sannsynlighetsregning p.g.a sentralgrenseteorem.

$P_0 = 1$

	$n =$ 1	12	365	10 000	$n \to \infty$
$r = 100\% = 1$	2	2.61	2.7145...	2.71815..	e
$r = 10\% = 0.1$	1.1	1.105	1.105		
$r = 1\% = 0.01$	1.01	1.01005	1.01005		

$$\left(1 + \frac{r}{n}\right)^n$$

$r = 100\%$ kont rente $\leftrightarrow$ 171.8 % årlig rente.

$\underline{r}$ —"— $\leftrightarrow$ $\underline{e^r - 1}$ —"—

kont. rente

$$P_t = e^{rt} P_0 = (e^r)^t \cdot P_0$$

(Banken bruker kont. renter)

oppg. $P_0 = 100$ kr årlig rente 10%

Tar ut pengene etter 4 mnd. ($1/3$ år) Hvor mye får dere?

$e^r - 1 = 10\%$ p.a $\qquad e^r = 1 + r_{p.a}$, $\underline{r = \ln(1 + r_{p.a})}$

$r_{p.a} = 10\%$ kont. rente $r = \ln(1 + 10\%) = \ln(1.1)$

$\sim 0.09531.. \sim \underline{\underline{9.53\%}}$

$$P_{1/3} = e^{r \cdot 1/3} P_0 = (e^r)^{1/3} \cdot P_0$$

$$= \cancel{A+P_1} \; (1 + r_{pa})^{1/3} P_0$$

i vårt tilfelle: $P_{1/3} = (1 + 10\%)^{1/3} \cdot 100kr$

$$= \sqrt[3]{1{,}1} \cdot 100 \sim \underline{\underline{103.228}}$$

$$\underline{P_t = e^{rt} P_0 = (1 + r_{pa})^t P_0} \, .$$

Eks.

$$f(x) = \frac{1}{1+e^{-x}} \qquad x \in \mathbb{R}$$

Asymptoter?
Monotoniegenskape
Konkavitet.

$e^0 = 1$

$$f(0) = \frac{1}{2}$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1}{1+e^{-x}} = 1$$

horisontal asymptote $\underline{y = 1}$ og $y = 0$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1}{1+e^{-x}} = \lim_{z\to\infty} \frac{1}{1+e^{z}} = 0$$

$$f'(x) = \left(\left(1+e^{-x}\right)^{-1}\right)' = (-1)\left(1+e^{-x}\right)^{-2} \cdot \underbrace{\left(1+e^{-x}\right)'}_{(-e^{-x})} = \frac{e^{-x}}{\left(1+e^{-x}\right)^2} > 0 \text{ for alle } x$$

$f(x)$ er voksende på hele $\mathbb{R}$.

Forenkler: $$f'(x) = \frac{e^{-x}}{(1+e^{-x})^2} = \frac{(1+e^{-x})-1}{(1+e^{-x})^2} = \underbrace{\frac{1}{1+e^{-x}} - \frac{1}{(1+e^{-x})^2}}_{\left(f(x) - f(x)^2\right)}$$

$$f'(x) = f(x)\left(1 - f(x)\right)$$

Logistiske diff. likninging.

Differensiallikning.

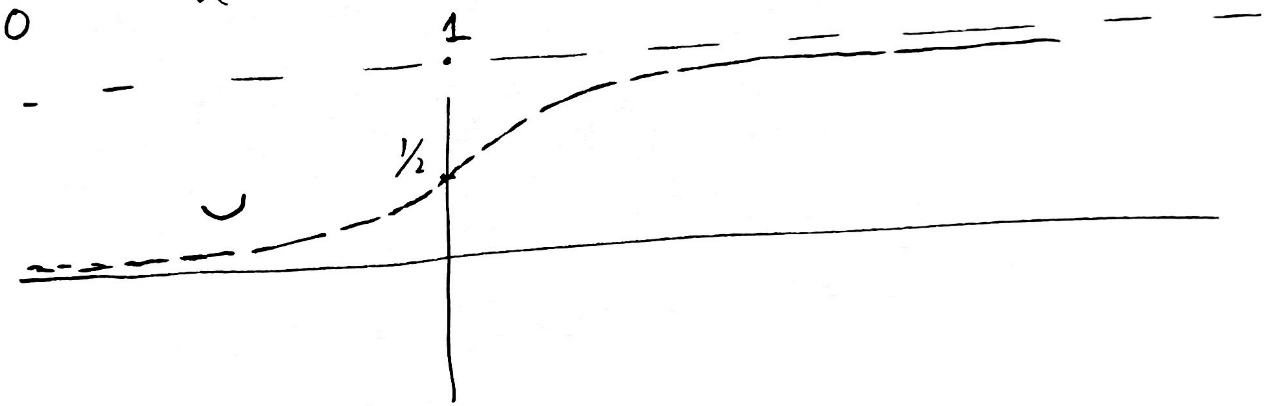
$$f''(x) = f'(x) - 2f(x)\cdot f'(x)$$

$$= \frac{e^{-x}}{(1+e^{-x})^2}\left(1 - \frac{2}{1+e^{-x}}\right) = \frac{e^{-x}}{(1+e^{-x})^2}\left(\frac{e^{-x}-1}{e^{-x}+1}\right)$$

$f''(x) > 0 \quad x < 0$ $\qquad f''(x) = 0$ for $x = 0$.

$f''(x) < 0 \quad x > 0$

Skisse av $f(x)$



Variant av ob/5 #11

$$g(x) = (x-1)\,e^{-x} \qquad x \geq 0$$

$$g'(x) = (x-1)'\,e^{-x} + (x-1)(e^{-x})'$$

$$= 1\cdot e^{-x} + (x-1)(e^{-x})\cdot\underbrace{(-x)'}_{-1}$$

$$g'(x) = e^{-x} - (x-1)\,e^{-x} = e^{-x}(1 + 1 - x) = \underline{(2-x)\,e^{-x}}$$

$$g''(x) = (g'(x))' = ((2-x)\cdot e^{-x})'$$

$$= (2-x)'e^{-x} + (2-x)(e^{-x})'$$

$$= -1\cdot e^{-x} + (2-x)(-e^{-x}) = e^{-x}(-1 + x - 2)$$

$$g''(x) = (x-3)\,e^{-x}$$

$$g'(x) = (2-x)e^{-x} \text{ og } g''(x) = (x-3)\,e^{-x}.$$

monotoni-egenskaper
$g'(x) > 0$ $\quad 0 \leq x < 2$ $\quad$ voksende
$g'(x) < 0$ $\quad x > 2$ $\quad$ avtagende

$g'(2) = 0$ $\quad$ maksimumspunkt $x=2$, $\quad$ toppunkt $(2, \frac{1}{e^2})$

Endepunkt i $x=0$. bunnpunkt $\cancel{(0, \tfrac{1}{e})}$

$(0, -1)$.

Ekstremalpunktene : $(0,-1)$ bunnpunkt

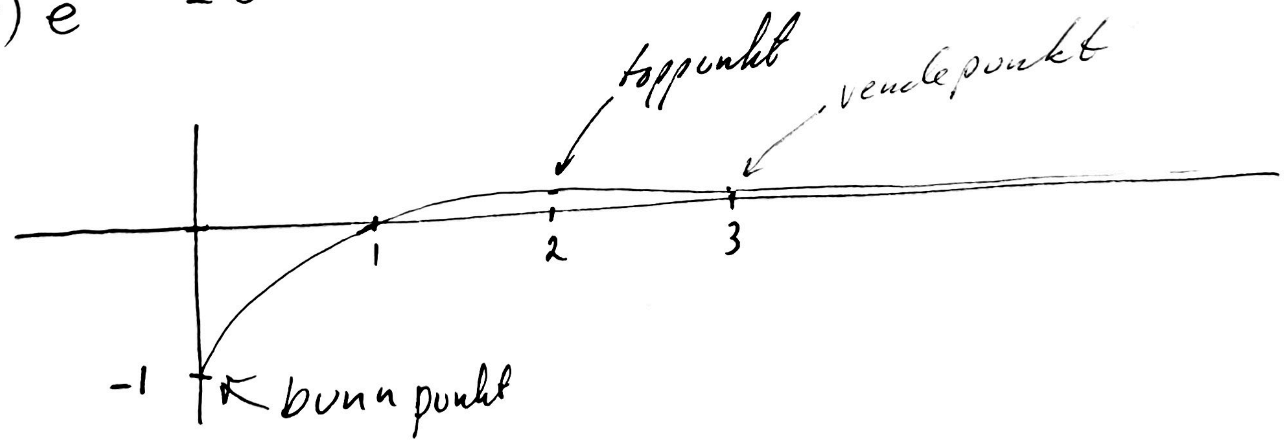
$(2, \frac{1}{e^2})$ toppunkt. $\sim (2, 0.135)$

$g''(x) > 0$ for $x > 3$ konkav opp

$g''(x) < 0$ $\quad 0 \leq x < 3$ konkav ned.

$g''(3) = 0$ vendepunkt $\left(3, 2\frac{1}{e^3}\right) \sim (3, 0.0996)$

$\lim_{x \to \infty} (x-1) e^{-x} = 0$ x-aksen er en horisontal asymptote.



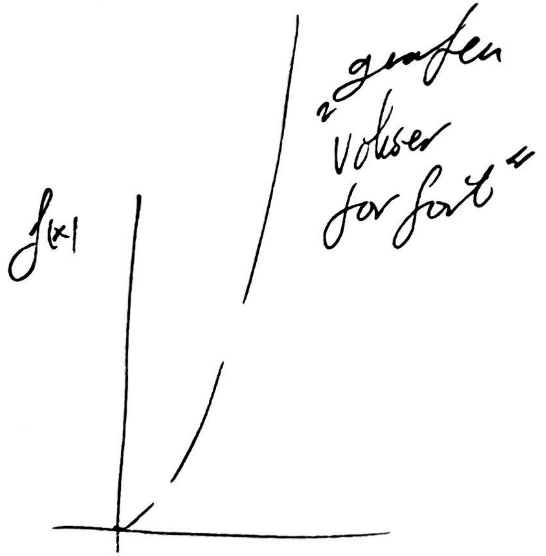
$$10 \cdot \text{Log}\left(\frac{E_2}{E_0}\right) \quad dB.$$

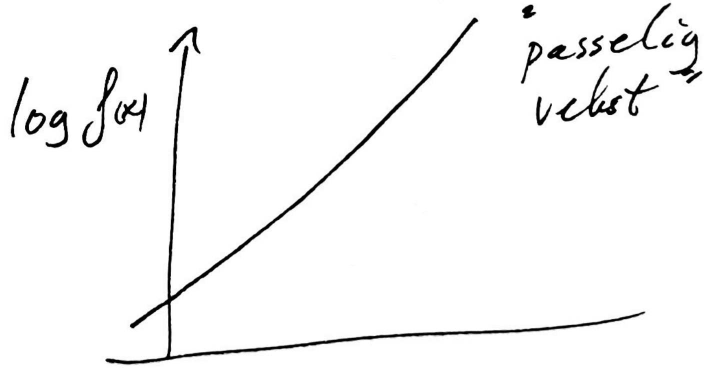
E_0 <u>referansen</u> $\qquad 0\,dB$

$$10 \cdot \text{Log}\,\frac{E_0}{E_0} = 0.$$

$$X = 10\,\text{Log}\,\frac{E}{E_0} \quad dB.$$

$$\underline{E = E_0 \cdot 10^{X/10}}$$





Renter. Årlig rente p.o. r_{po}.

Etter 1 år økes pengemengden med r_{po}

$P_1 = (1 + r_{po}) P_0$

% prosent $\frac{1}{100}$ $10\% = 10 \cdot \frac{1}{100} = 0.1$

‰ promille $\frac{1}{1000}$

prosenpoeng.

$4\% \to 6\%$ steget med $(6-4)\% = 2\%$ prosentpoeng.

$4\% \underbrace{(1 + 0{,}02)}_{2\%} = 4{,}08\%$

basispunkt 0.01% (bukt additivt)

13.3.20 $1{,}5\% \to 1\%$

Renten har gått ned med 0.5 prosentpoeng

50 basispunkt

20.3 $1\% \to 0{,}25\%$

8.5 $0.25\% \to 0\%$

<u>Kontinuerlig rente</u>:

årlig rente r

$P_1 = (1+r) P_0$

Etter 1 år

månedlig rente $\left(1 + \frac{r}{12}\right)^{12}$

daglig rente $\left(1 + \frac{r}{365}\right)^{365}$

kontinuerlig rente $\lim_{n \to \infty} \left(1 + \frac{r}{n}\right)^n$

oblig 5 variant

#10 f)

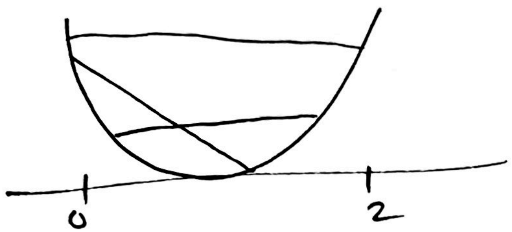
$$(\ln|x|)' = \frac{1}{x} \quad x \neq 0$$

$$(\ln(-x))' = \frac{1}{-x}(-x)' = \frac{-1}{-x} = \frac{1}{x}$$

$$f(x) = \ln|-3x+5| - \ln|x^3+x|$$

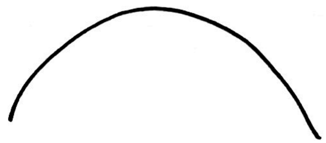
$$f'(x) = \frac{1}{-3x+5}\underbrace{(-3x+5)'}_{-3} - \frac{1}{x^3+x}\underbrace{(x^3+x)'}_{3x^2+1}$$

$$= \frac{-3}{-3x+5} - \frac{3x^2+1}{x^3+x}$$



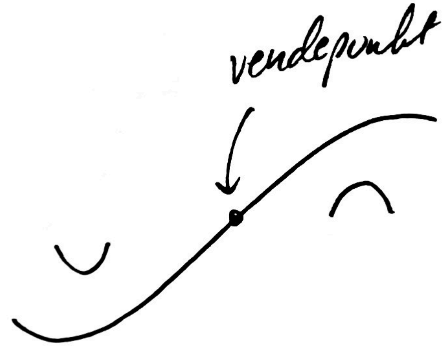
$f''(x) > 0$

$\dfrac{\text{konkav opp}}{\text{konveks}}$ i en intervall



$f''(x) < 0$

$\dfrac{\text{konkav ned}}{\text{konkav}}$



Etter et halvt år har pengemengden i banken økt med 10%. Hva er årlig rente i banken r_{pa}. (vendret gjennom året)

$$P_t = (1+r_{pa})^t P_0$$

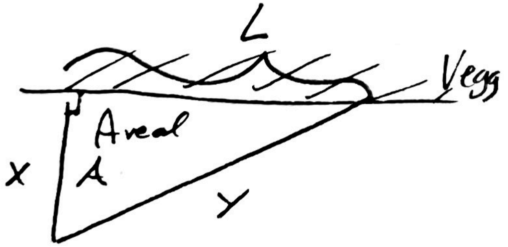
$$P_{1/2} = (1+r_{pa})^{1/2} P_0 = \sqrt{1+r_{pa}}\, P_0 = (1+10\%)\, P_0 = 1.1 \cdot P_0 .$$

$$\sqrt{1+r_{pa}} = 1.1$$

$$1+r_{pa} = (1.1)^2 = 1.21$$

$$r_{pa} = 0.21 \underline{= 21\%}$$

optimalisering



$x + y = 10$ (meter)

Hva må x (og da y) være for at arealet A skal bli størst mulig?

$L^2 + x^2 = y^2$

$L = \sqrt{y^2 - x^2}$

$(L > 0)$

Arealet $A = \frac{1}{2} x \cdot L$

$A = \frac{1}{2} x \sqrt{y^2 - x^2}$.

$x + y = 10$

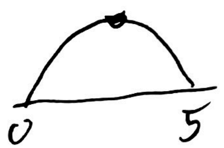
$y = 10 - x$

$y^2 = (10 - x)^2 = x^2 - 20x + 100$

$L = \sqrt{y^2 - x^2} = \sqrt{100 - 20x}$.

$A(x) = \frac{1}{2} x \sqrt{100 - 20x}$

ønsker å finne x (mellom 0 og 5m) slik at A(x) blir størst.



$A'(x) = \frac{1}{2}(x)' \sqrt{100-20x} + \frac{1}{2} x \left(\sqrt{100-20x}\right)'$

$= \frac{1}{2}\sqrt{\sim} + \frac{x}{2} \cdot \frac{1}{2} \frac{1}{\sqrt{\sim}} \cdot \underbrace{(100 - 20x)'}_{-20}$

$= \frac{1}{2\sqrt{\sim}}\left[(100 - 20x) + \frac{x}{2} \cdot (-20)\right]$

$= \frac{1}{2\sqrt{\sim}}\left[100 - 30x\right]$

så $A'(x) = 0$ for

$x = \frac{100}{30} = \frac{10}{3} \sim \underline{3.33m}$

2x = y gir størst areal.

$y = 10 - \frac{10}{3} = \frac{2}{3} \cdot 10 \sim \underline{6.66m}$