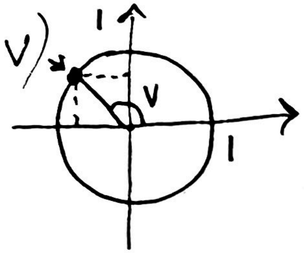
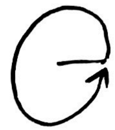
15.02.2021

11 Trigonometriske funksjoner.

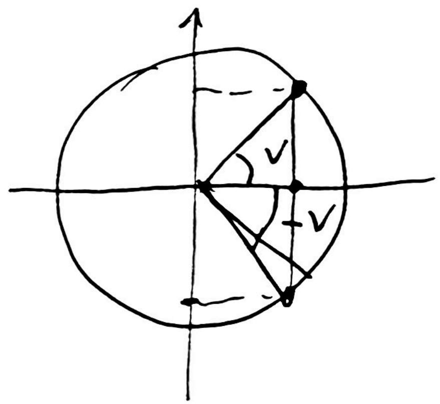


Ett omløp  2π radian

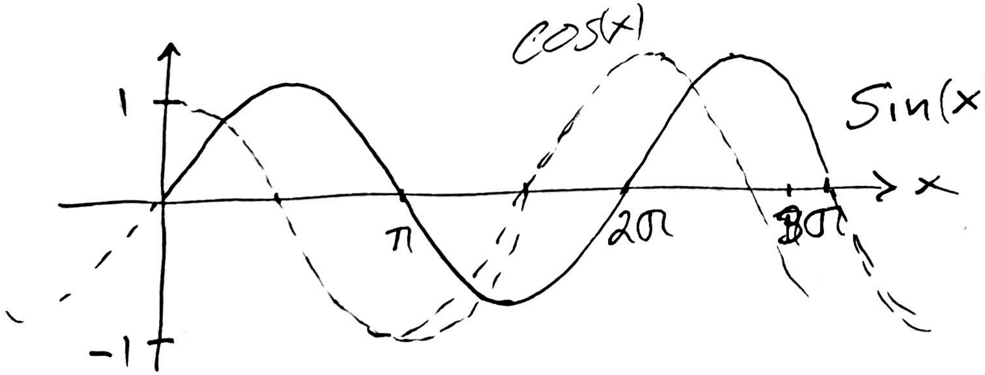
 $\frac{\pi}{2}$ rad.

Pytagoras
$\cos^2 v + \sin^2 v = 1$
for alle v.

$\cos(v+2\pi) = \cos v$ periodisk med periode 2π.
$\sin(v+2\pi) = \sin v$

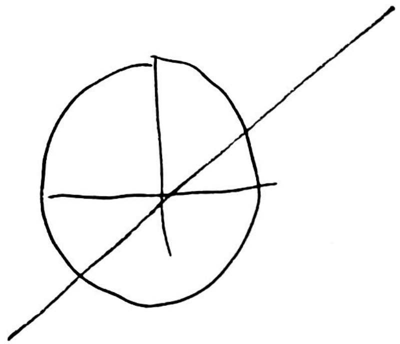


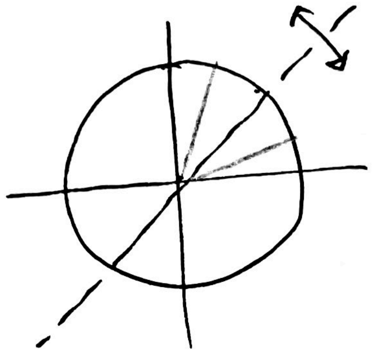
$\cos(-v) = \cos(v)$ jevn funksjon
$\sin(-v) = -\sin(v)$ ~~jev~~ odde funksjon



viser senere
$(\sin(x))' = \cos(x)$
$(\cos(x))' = -\sin(x)$

Reflekterer om $x=y$

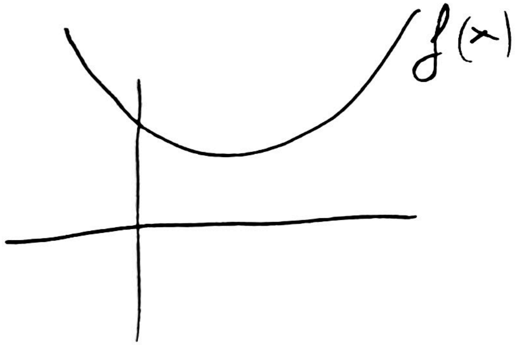


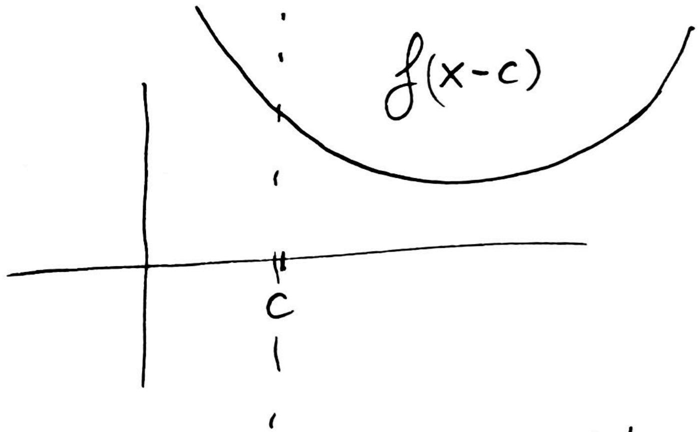


$v \leftrightarrow \frac{\pi}{2} - v$

x akse $\leftrightarrow$ y-akse

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$
$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$





Grafen til $f(x-c)$ er lik grafen til $f(x)$ forskyvd med c mot høyre.

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$
$$= \cos\left(-\left(\frac{\pi}{2} - x\right)\right)$$
$$= \cos\left(x - \frac{\pi}{2}\right)$$

så grafen til $\sin x$ er lik grafen til $\cos x$ forskyvd med $\frac{\pi}{2}$ mot høyre.

Addisjonsformlene $\sin(U+V) = \sin(U)\cdot\cos(V) + \sin(V)\cdot\cos(U)$

$U=V$ dobling av vinkel: $\sin(2U) = 2\sin(U)\cos(U)$

$\cos(U+V) = \cos(U)\cdot\cos(V) - \sin(U)\cdot\sin(V)$

$U=V$: $\cos(2U) = \cos^2 U - \sin^2 U$

Kombinert med Pythagoras $\cos^2 U + \sin^2 U = 1$

$\cos(2U) = \cos^2 U - (1-\cos^2 U)$

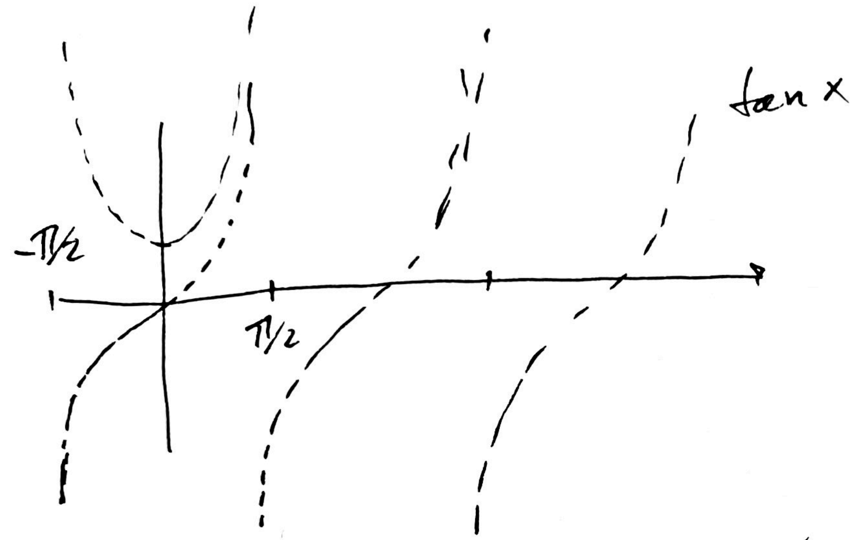
$= 2\cos^2 U - 1$

Så $\underline{\cos^2(U) = \frac{1}{2}(\cos(2U)+1)}$

Eksempel $\cos(30°) = \frac{\sqrt{3}}{2}$

$\cos^2(15°) = \frac{1}{2}(\cos(2\cdot 15°)+1) = \frac{1}{2}\left(\frac{\sqrt{3}}{2}+1\right)$

$\cos(15°)>0$ så $\cos(15°) = \sqrt{\frac{\sqrt{3}+2}{2\cdot 2}} = \underline{\underline{\frac{\sqrt{\sqrt{3}+2}}{2}}}$



$$\tan(x) = \frac{\sin x}{\cos x}$$

Finner $(\tan x)'$

bruker kvotientregelen:

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)'$$

$$= \frac{(\sin x)' \cdot \cos x - \sin x (\cos x)'}{(\cos x)^2}$$

$$= \frac{(\cos x)^2 - \sin x(-\sin x)}{(\cos x)^2}$$

$$= \frac{(\cos x)^2 + (\sin x)^2}{(\cos x)^2}$$

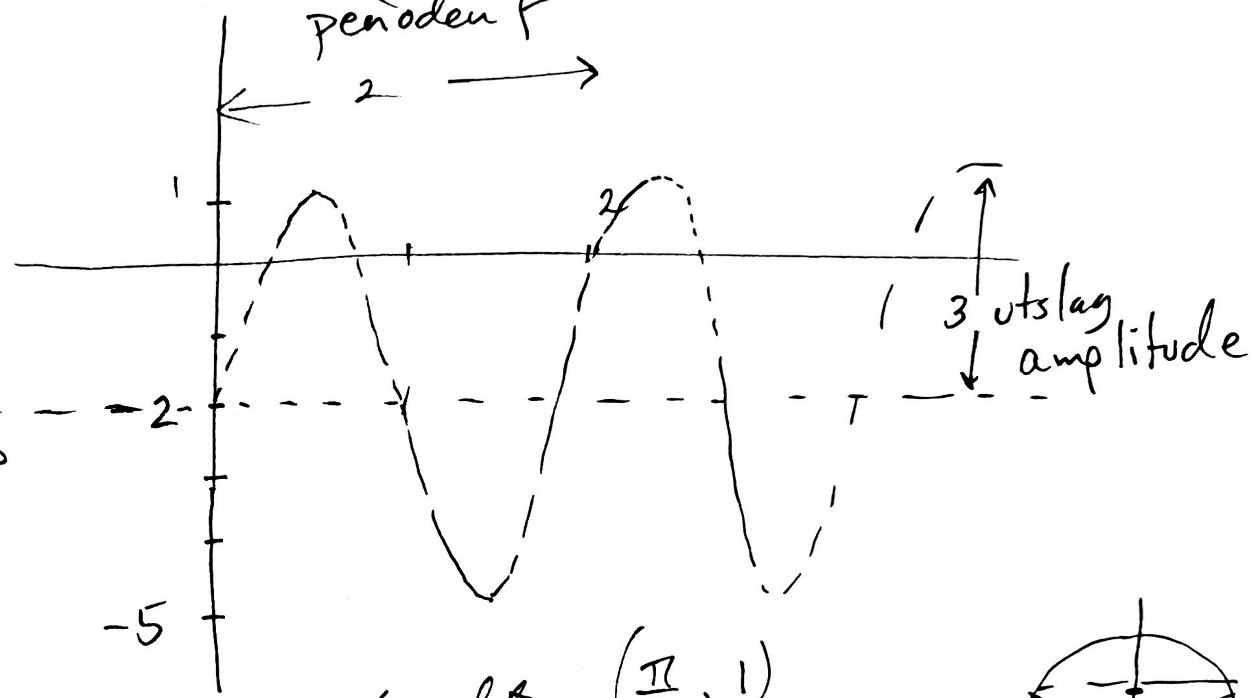
Pytagoras $= \frac{1}{\cos^2 x}$

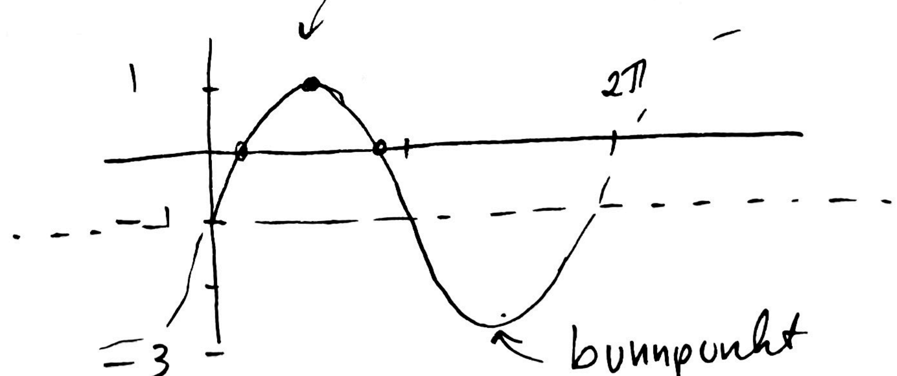
$$= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x$$

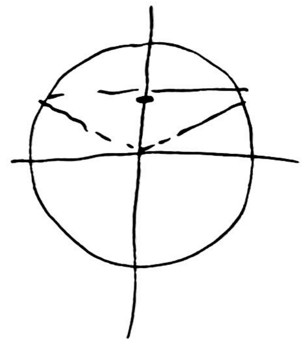
$$(\tan x)' = \frac{1}{\cos^2 x}$$

$$= 1 + \tan^2 x$$

Grafen til
$3\sin(\pi x) - 2$







$f(x) = 2\sin(x) - 1$

Nullpunkt $f(x) = 0$

$2\sin x = 0$
-1

$\Leftrightarrow \sin x = \frac{1}{2}$

Nullpunkt: $\frac{\pi}{6} + 2\pi n, \quad \frac{5\pi}{6} + 2\pi n$

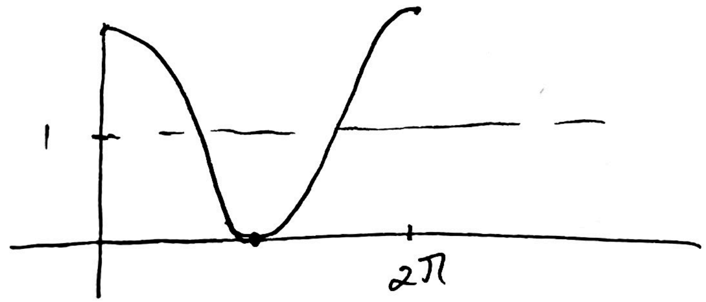
oppg

$f(x) = -\sin\left(x - \frac{\pi}{2}\right) + 1 \qquad 0 \leq x \leq 2\pi$

Finn
topp/bunn punkt
0-punkt.

Forskyve med $\frac{\pi}{2}$ →

↑ forskyve opp med 1



toppunkt $(0, 2)$
$(2\pi, 2)$

bunnpunkt $(\pi, 0)$
det er også nullpunktet.

11.4 Sinusfunksjoner er på formen

$$f(x) = a\sin(kx + c) + d$$
$$= a\sin\left(k\left(x + \frac{c}{k}\right)\right) + d$$

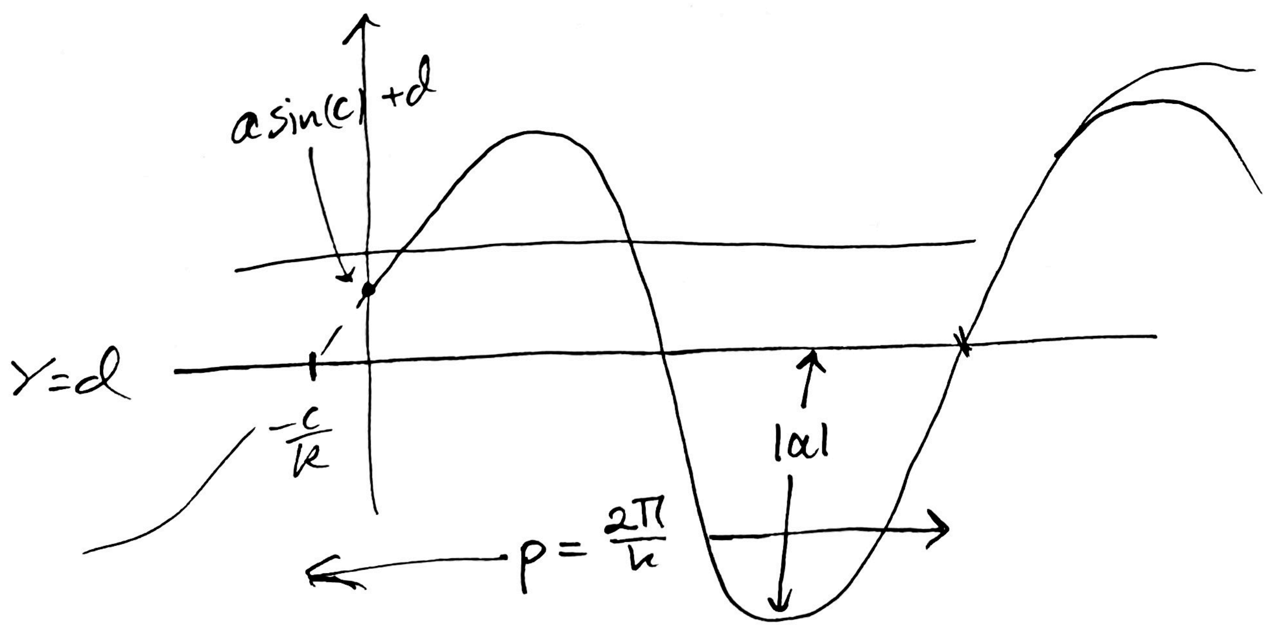
$y = d$ jamvektslinjen

$|a|$ amplitude

periode $p = \frac{2\pi}{|k|}$

$-c$ faseforskyvningen.

grafen forskyves $-\frac{c}{k}$ mot høyre.



$f(x) = 5\sin(\pi x - 1) + 3$

amplituden er 5

jamvektslinjen er $y = 3$

perioden $p = \frac{2\pi}{\pi} = 2$

fasen $-c = 1$

Grafen er forskyvd $\frac{1}{\pi}$ mot høyre.

$g(x) = -3\sin(-2x + 3) - 1$

amplituden $|a| = 3$

jamvektslinjen $y = -1$

periode $p = \frac{2\pi}{|k|} = \frac{2\pi}{2} = \underline{\underline{\pi}}$

Faseforskyvning $-c = \underline{-3}$

Grafen er forskyvd $\frac{-c}{k}$ mot høyre

$\underline{3/2}$

Beskriv $\sin^2 x$ som en sinuskurve.
på forma $a \sin(kx + c) + d$.

$$\left(\cos(2x) = \cos^2 x - \sin^2 x \overset{\text{Pyt}}{=} \overbrace{(1 - \sin^2 x)}^{\cos^2 x} - \sin^2 x = 1 - 2\sin^2 x \right)$$

$$\sin^2 x = \frac{1}{2}\left(1 - \cos(2x)\right) = \frac{1}{2}\left(1 - \left(-\sin\left(2x - \frac{\pi}{2}\right)\right)\right) = \underline{\underline{\frac{1}{2} + \frac{1}{2}\sin\left(2x - \frac{\pi}{2}\right)}}$$

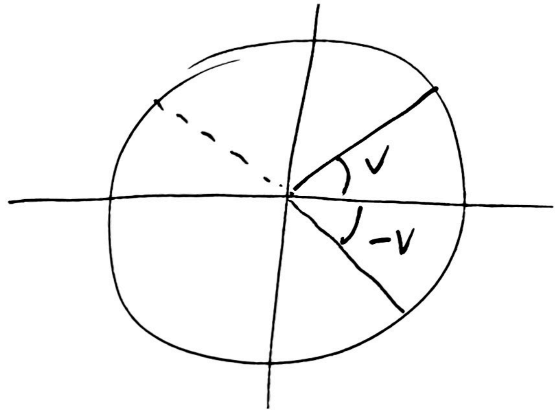
$$\cos(u) = \sin\left(\frac{\pi}{2} - u\right) = -\sin\left(u - \frac{\pi}{2}\right)$$
(sin odde funksjon)

Så $\cos(x)$ er en sinus-funksjon ...

11.10

iboka.

$v \in [0, \frac{\pi}{2}]$ $\sin v = \frac{1}{3}.$



a) $\sin(-v) = -\sin v = -\frac{1}{3}$

b) $\sin(\pi - v) = \sin(v) = \frac{1}{3}.$

c) $\cos(\frac{\pi}{2} - v) = \sin(v) = \frac{1}{3}$

d) $\cos v$: $\sin^2 v + \cos^2 v = 1$

$\cos^2 v = 1 - \frac{1}{9} = \frac{8}{9}$

$\cos v = \pm \frac{\sqrt{8}}{3} = \pm \frac{2\sqrt{2}}{3}.$

$0 \leq v \leq \frac{\pi}{2}$

så $\cos v \geq 0$

$\cos v = \frac{2\sqrt{2}}{3}$

e) $\cos(-v) = \cos(v) = \frac{2\sqrt{2}}{3}.$

f) $\cos(\pi - v) = -\cos(v) = -\frac{2\sqrt{2}}{3}.$

g) $\sin(\frac{\pi}{2} - v) = \cos(v) = \frac{2\sqrt{2}}{3}.$

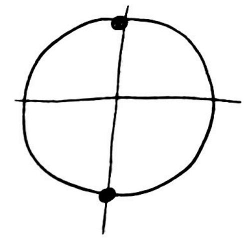
h) $\tan v = \frac{\sin v}{\cos v}$

$= \frac{1/3}{2\sqrt{2}/3}$

$\tan v = \frac{1}{2\sqrt{2}}$

$f(x) = 2\sin\left(3x - \frac{\pi}{2}\right) + \sqrt{3}. \quad = 2\sin(u) + \sqrt{3}.$

Finn topp & bunnpunkt og nullpunkt



$f(x)$ størst når $\sin(u) = 1$

$$u = \frac{\pi}{2} + 2\pi \cdot n$$

$$3x - \frac{\pi}{2} = \frac{\pi}{2} + 2\pi \cdot n$$

$$3x = \pi + 2\pi \cdot n.$$

$$x = \underline{\frac{\pi}{3} + \frac{2\pi}{3} \cdot n}$$

maksimumspunkt

maksimalverdien er alle $2 + \sqrt{3}$.

$f(x)$ minst når $\sin(u) = -1$ $\quad u = \frac{-\pi}{2} + 2\pi \cdot n$

$$3x - \frac{\pi}{2} = \frac{-\pi}{2} + 2\pi \cdot n$$

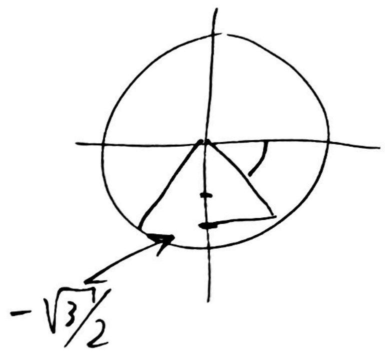
minimumspunkter $\quad x = \underline{\frac{2\pi}{3} \cdot n}$

minimalverdiene er alle $-2 + \sqrt{3}$.

Nullpunkt til $2\sin\left(3x - \frac{\pi}{2}\right) + \sqrt{3}$.

$\sin\left(3x - \frac{\pi}{2}\right) = \frac{-\sqrt{3}}{2}$.

$\sin(U) = -\frac{\sqrt{3}}{2}$



$U = 3x - \frac{\pi}{2}$.

$x = \frac{U + \pi/2}{3}$.

$U = \frac{-\pi}{3} + 2\pi \cdot n$

$U = \frac{-2\pi}{3} + 2\pi \cdot n$

Så nulpunkt:

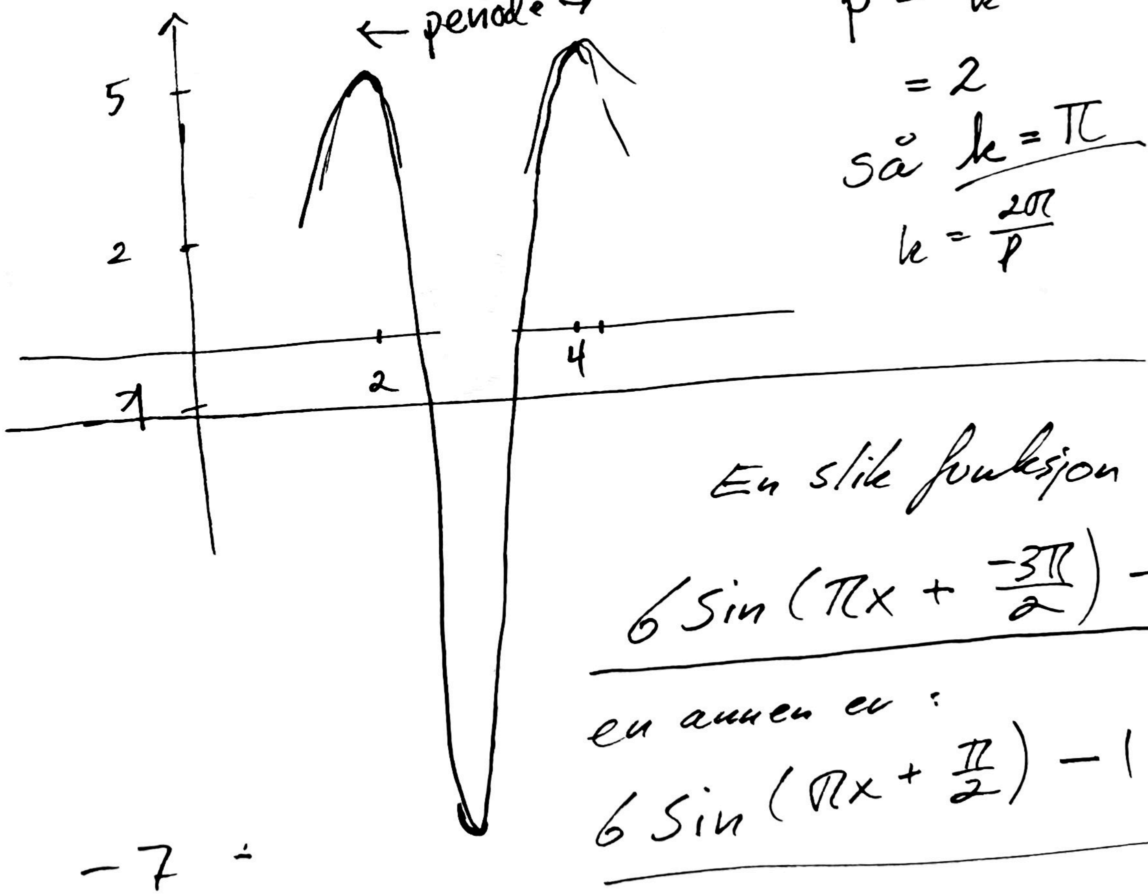
$x = \frac{\left(-\frac{\pi}{3} + 2\pi \cdot n\right) + \frac{\pi}{2}}{3} = \underline{-\frac{\pi}{18} + \frac{2\pi}{3} \cdot n}$

$x = \frac{\left(-\frac{2\pi}{3} + 2\pi \cdot n\right) + \frac{\pi}{2}}{3} = \underline{\frac{\pi}{18} + \frac{2\pi}{3} \cdot n}$

Sinusbølge slik at (2,5) og (4,5) er etterfølgende toppunkt.

Amplituden er 6.

Finn en sinuskurve med disse egenskapene.



$p = \frac{2\pi}{k}$

$= 2$

så $k = \pi$

$k = \frac{2\pi}{p}$

$6\sin(\pi x + c) - 1$

ønsker c slik at

$x = 2$:

$\pi \cdot 2 + c = \frac{\pi}{2}$

$c = \frac{\pi}{2} - 2\pi$

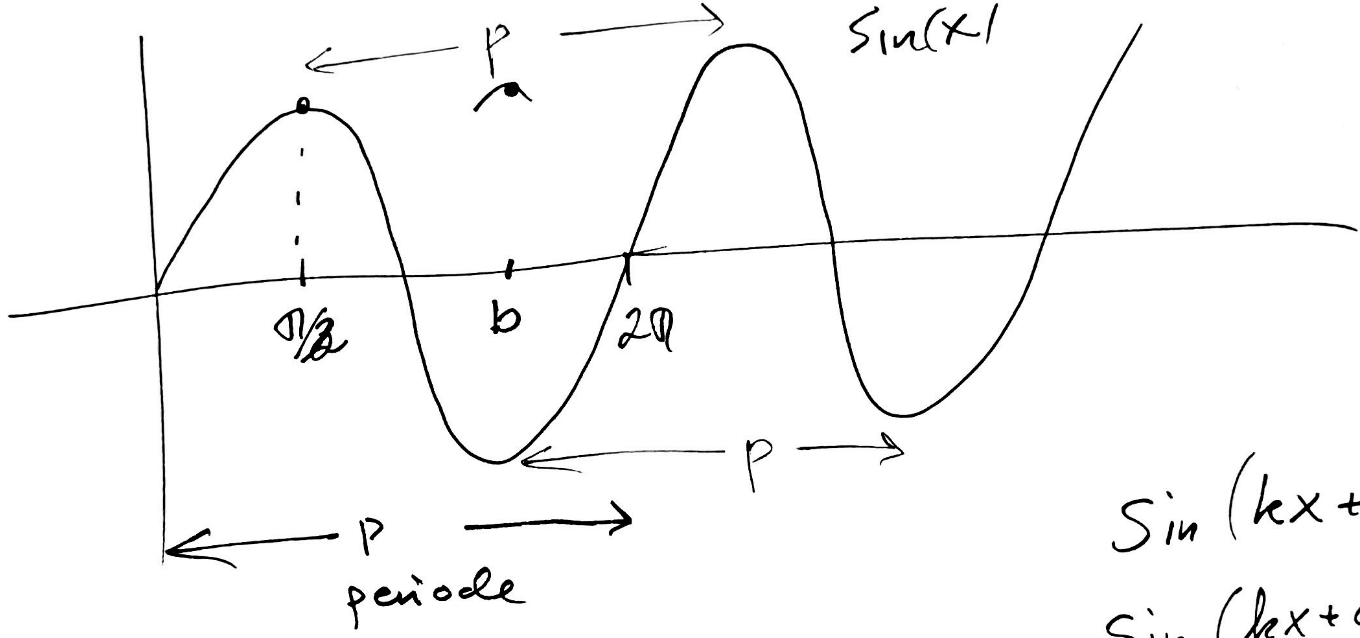
$= \frac{-3\pi}{2}$

En slik funksjon

$6\sin\left(\pi x + \frac{-3\pi}{2}\right) - 1$

en annen er :

$6\sin\left(\pi x + \frac{\pi}{2}\right) - 1$



$$\sin(kx+c)$$

$$\sin\Big(kx+c+\underbrace{k\Delta x}_{2\pi}\Big)$$

$k \cdot p = 2\pi$

så $p = \frac{2\pi}{k}$.

$\sin(x+c)$ slik at vi får en topp i $x=b$.

$\sin(x-(b-\pi/2)) = \underline{\sin(x+\frac{\pi}{2}-b)}$.

setter $x=b$: $\quad x+c = \frac{\pi}{2}$

$b+c = \frac{\pi}{2} \Rightarrow \underline{c = \frac{\pi}{2}-b}$