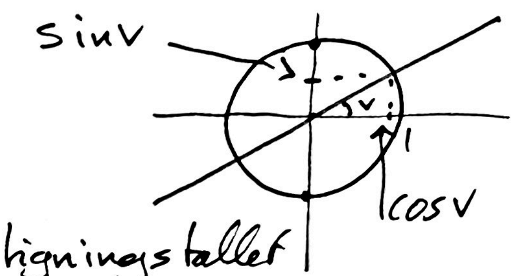
16.02
2021

11.6 Tangens

$\tan x = \dfrac{\sin x}{\cos x}$ def når $\cos x \neq 0$

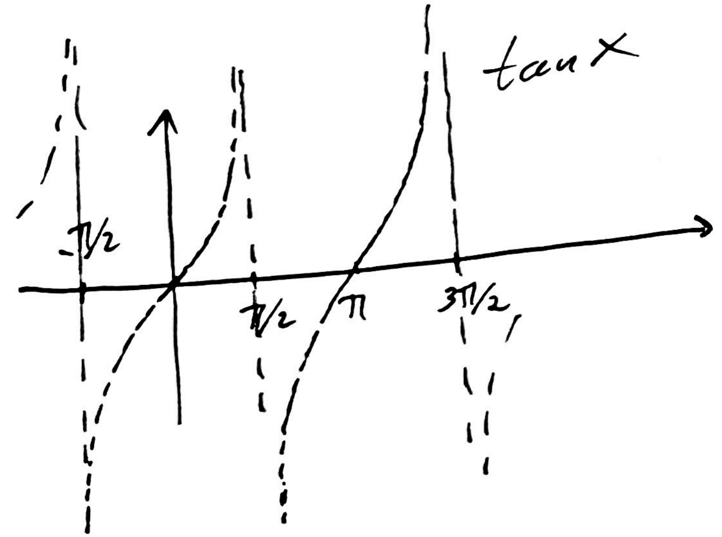
$x \notin \pi \cdot n + \frac{\pi}{2}$ n heltall.



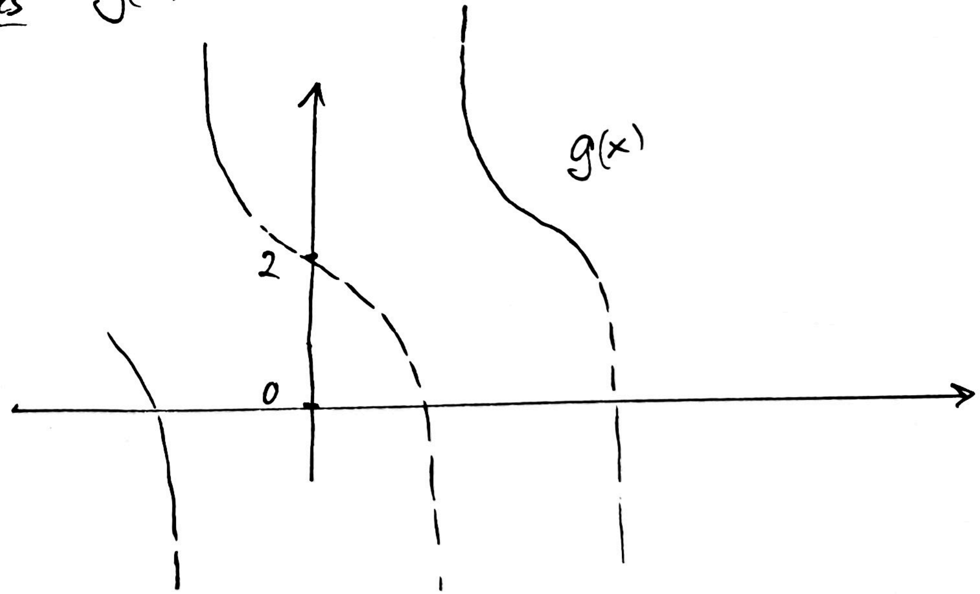
$\underline{\text{Stigningstallet}}$ til en linje med vinkel v er $\underline{\tan v}$.

$\tan(v+\pi) = \tan v$

tan. er periodisk med periode π

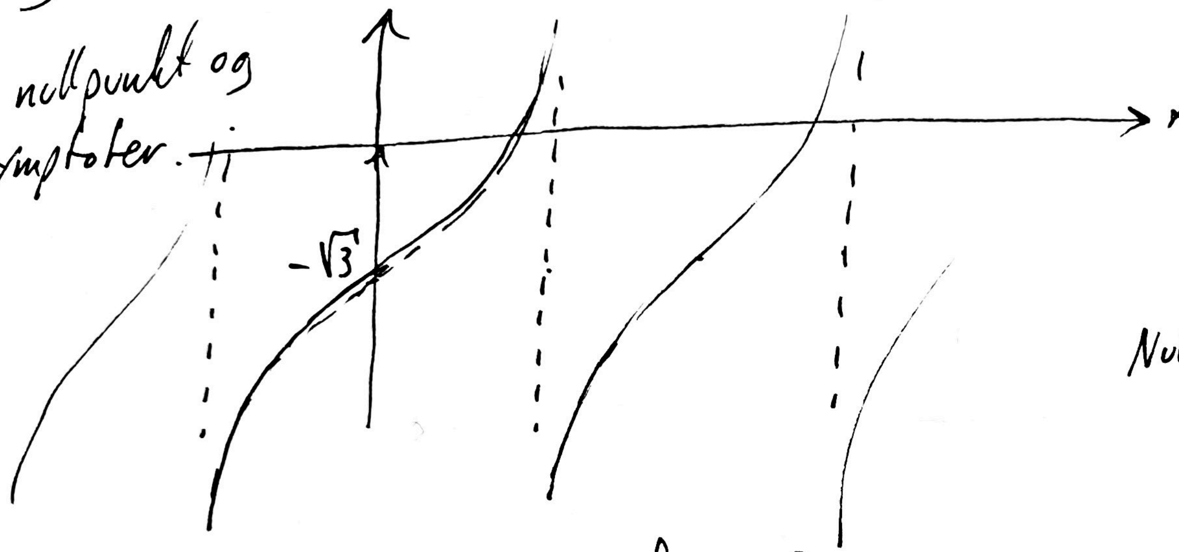


<u>Eks</u> $g(x) = -\tan(x) + 2$



$f(x) = 3\tan(\pi x) - \sqrt{3}$ $\quad x \neq \frac{1}{2} + n$, n heltall

Finn nullpunkt og asymptoter.



Asymptoter i $\underline{x = \frac{1}{2} + n}$ n heltall

Nullpunkt: $\underline{x = \frac{1}{6} + n}$

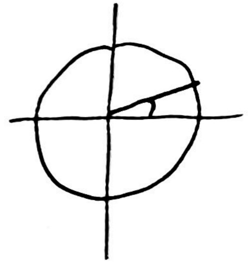
f(x) krysser x-aksen:

$$f(x) = 0$$

$$3\tan(\pi x) - \sqrt{3} = 0$$

$$\tan\underbrace{(\pi x)}_{U} = \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

$$\tan(U) = \frac{1}{\sqrt{3}}$$



$\sin(\pi/6) = 1/2$

$\cos(\pi/6) = \sqrt{3}/2$

så $\frac{\sin(\pi/6)}{\cos(\pi/6)} = \frac{1}{\sqrt{3}}$.

$\left(\frac{1/2}{\sqrt{3}/2} \cdot \frac{2}{2} = \frac{1}{\sqrt{3}}\right)$

Løsningen er

$$U = \arctan\left(\frac{1}{\sqrt{3}}\right) + \pi \cdot n$$

$$= \frac{\pi}{6} + \pi \cdot n.$$

$U = \pi x$ så

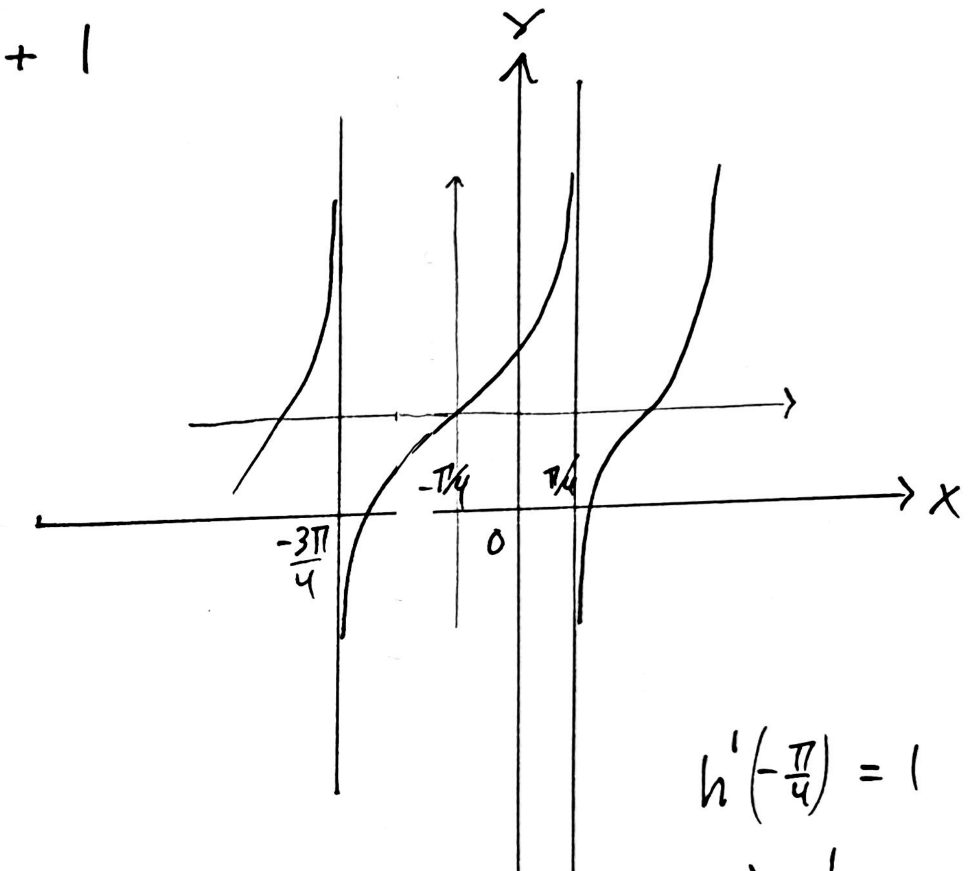
$$x = \frac{U}{\pi} = \underline{\frac{1}{6} + n}$$

n heltall

999. $h(x) = \tan\left(x + \frac{\pi}{4}\right) + 1$

Lag skisse av grafen til $h(x)$

- Finn nullpunkt og asymptoter.



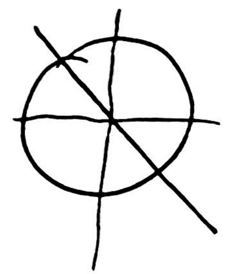
Asymptoter

$x + \frac{\pi}{4} = \frac{\pi}{2} + \pi \cdot n$

$x = \frac{\pi}{2} - \frac{\pi}{4} + \pi \cdot n$

$\underline{x = \frac{\pi}{4} + \pi \cdot n}$ n heltall

$h'\left(-\frac{\pi}{4}\right) = 1$



Nullpunkt:

$h(x) = 0$

$\tan\left(x + \frac{\pi}{4}\right) + 1 = 0$

$\tan\underbrace{\left(x + \frac{\pi}{4}\right)}_{U} = -1$

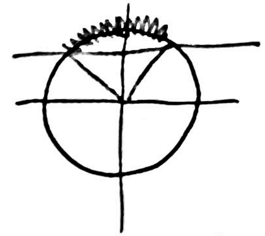
$x + \frac{\pi}{4} = U = \frac{-\pi}{4} + \pi \cdot n$

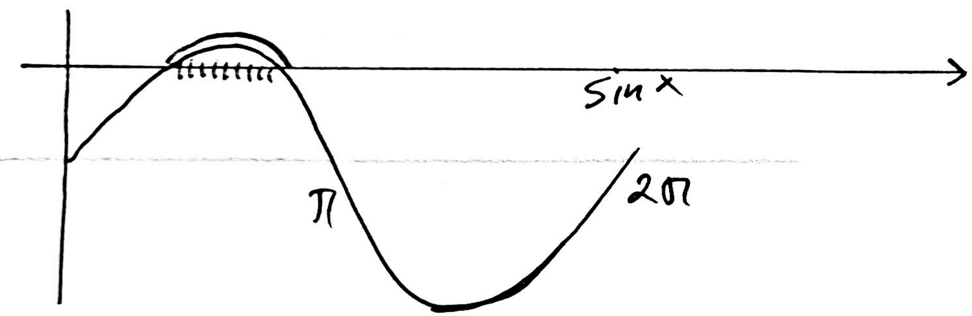
Så $x = \frac{-\pi}{4} - \frac{\pi}{4} + \pi \cdot n = \underline{\frac{-\pi}{2} + \pi \cdot n}$ ~ $-1.57 + 3.14 \cdot n$

11.7 Trigonometriske ulikheter.

I $\sin x > \frac{\sqrt{3}}{2}$ $\qquad x \in [0, 2\pi]$

II $\sin x - \frac{\sqrt{3}}{2} > 0$





$\sin x = \sqrt{3}/2$

$x = \frac{\pi}{3}$ og $\frac{2\pi}{3}$ $(+2\pi \cdot n)$

ser at løsningen i

$[0, 2\pi]$ er

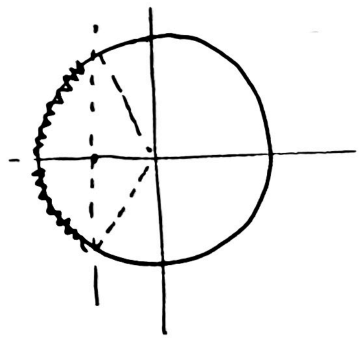
$\underline{x \in \langle \frac{\pi}{3}, \frac{2\pi}{3} \rangle}$

oppg. Løs ulikheten

$\cos x < \frac{-1}{2}$ $\qquad x \in [-\pi, \pi]$.

$\cos x = -1/2$ $\qquad x = \frac{2\pi}{3} + 2\pi \cdot n$

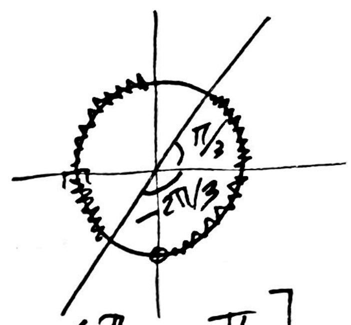
$\qquad\qquad\qquad\quad -\frac{2\pi}{3} + 2\pi \cdot n$



Løsningen til ulikheten er $\underline{[-\pi, -\frac{2\pi}{3} \rangle \cup \langle \frac{2\pi}{3}, \pi]}$

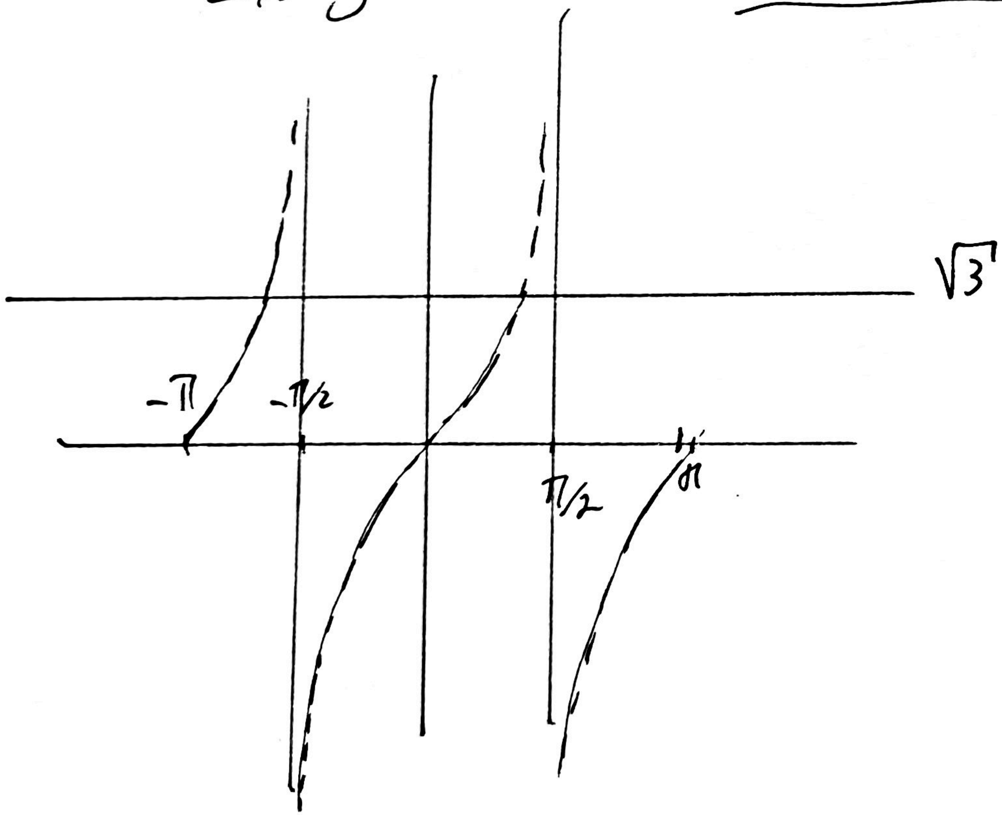
$\tan x < \sqrt{3}$

$x \in [-\pi, \pi]$
i def.mengden



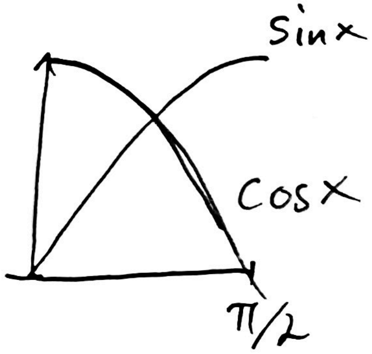
$\left(\begin{array}{l} \tan x = \sqrt{3} \\ x = \pi/3 + \pi \cdot n \end{array} \right)$

Løsningene blir $[-\pi, -\frac{2\pi}{3}\rangle \cup \langle -\frac{\pi}{2}, \frac{\pi}{3}\rangle \cup \langle \frac{\pi}{2}, \pi]$



11.8 Derivasjon av trig. funksjoner.

$(\sin x)' = \cos x$ $\qquad$ $(\cos x)' = -\sin x$



Viser hvordan $(\cos x)'$ kan regnes ut når vi vet $(\sin x)' = \cos x$

$$\cos x = \sin\left(\frac{\pi}{2} - x\right)$$

(refleksjon om linjen $y = x$)

$$(\cos x)' = \left(\sin\left(\frac{\pi}{2} - x\right)\right)'$$
$$= \cos\left(\frac{\pi}{2} - x\right) \cdot \underbrace{\left(\frac{\pi}{2} - x\right)'}_{-1}$$
$$= -\cos\left(\frac{\pi}{2} - x\right)$$
$$(\cos(x))' = -\sin(x)$$

oppg. Deriver $\sin(3x) + 3\sin(x)$

$$(\sin(3x) + 3\sin x)'$$

$$= \cos(3x) \cdot \underbrace{(3x)'}_{3} + 3\cos(x)$$

$$= \underline{3\cos(3x) + 3\cos(x)}$$

Deriver: $\cos(x^2-3)$

$$(\cos(x^2-3))'$$

$$= \underbrace{-\sin(x^2-3)} \cdot \underbrace{(x^2-3)'}_{2x}$$

$$= \underline{-2x\sin(x^2-3)}$$

$$a \cdot (\sin(1))$$

$$\neq \sin(a)$$

Deriver: $\dfrac{1}{\cos^2 x} = \dfrac{1}{(\cos x)^2} = (\cos x)^{-2}$ hvor $U(x) = \cos x$

$$\left(\frac{1}{\cos^2 x}\right)' = \underset{\text{kjerneregel}}{-2U^{-3} \cdot U'}$$

$$= -2(\cos x)^{-3} \underbrace{(\cos x)'}_{-\sin x} = \frac{+2\sin x}{(\cos x)^3} = \frac{+2\sin x}{\cos^3 x}$$

$\dfrac{1}{\cos^2 x} = \dfrac{1}{V}$ hvor $V = \cos^2 x$

$$\left(\frac{1}{\cos^2 x}\right)' = \frac{-1}{V^2} \cdot V' = \frac{-1 \cdot 2\cos x(-\sin x)}{(\cos^2 x)^2}$$

$$= \frac{+2\sin x \cdot \cos x}{\cos^4 x} = \underline{\underline{\frac{+2\sin x}{\cos^3 x}}}$$

$$\sqrt{\sin(x^2)} = \left(\sin(x^2)\right)^{1/2}$$

kjerneregel 2ganger

$$\left(\sqrt{\sin(x^2)}\right)' = \frac{1}{2}\left(\sin(x^2)\right)^{-1/2} \cdot \left(\sin(x^2)\right)'$$

$$\cos(x^2) \cdot (x^2)'$$

$$= \frac{1}{2}\frac{1}{\sqrt{\sin(x^2)}} \cdot \cos(x^2) \cdot 2x$$

$$= \underline{\frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}}$$

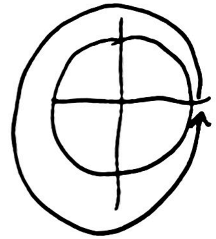
Vis at $\tan(x) \cdot \tan\left(\frac{\pi}{2} - x\right) = 1$

hvor definert

$x \neq \frac{\pi}{2} \cdot n$

$$\tan\left(\frac{\pi}{2} - x\right) = \frac{1}{\tan(x)}$$

Viser dette: $\underline{\tan\left(\frac{\pi}{2} - x\right)} = \frac{\sin\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right)} = \frac{\cos(x)}{\sin x} = \frac{1}{\sin x / \cos x} = \underline{\frac{1}{\tan x}}$



11.21 Dobling av vinkel

$$\cos(2v) = \cos^2 v - \underbrace{\sin^2 v}_{1-\cos^2 v} \quad \text{Pytagoras}$$

$$= 2\cos^2 v - 1$$

a) $$\cos^2 v = \frac{1}{2}(\cos(2v) + 1)$$

$$\cos v = \pm \frac{1}{\sqrt{2}} \sqrt{\cos(2v) + 1}.$$

b) $$\cos(22{,}5^\circ) = \underset{(>0)}{\cos\left(\frac{\pi}{8}\right)} = + \frac{1}{\sqrt{2}} \sqrt{\cos\left(\frac{\pi}{4}\right) + 1}$$

$$= \frac{\sqrt{1/\sqrt{2} + 1}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

11.21 $\cos\left(\frac{\pi}{8}\right) = \frac{\sqrt{2\left(\frac{1}{\sqrt{2}}+1\right)}}{2} = \underline{\frac{\sqrt{\sqrt{2}+2}}{2}} \sim \underline{0.92388..}$

c) $\cos^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{\pi}{8}\right) = 1$ Pyt.

$\sin\left(\frac{\pi}{8}\right) = \sqrt{1-\cos^2\left(\frac{\pi}{8}\right)}$ siden $\sin\left(\frac{\pi}{8}\right) > 0$

$= \sqrt{1 - \frac{\sqrt{2}+2}{4}} = \sqrt{\frac{4-(\sqrt{2}+2)}{4}}$

$= \frac{\sqrt{2-\sqrt{2}}}{2} \sim \underline{0.38268..}$

$\tan\left(\frac{\pi}{8}\right) = \frac{\sin(\pi/8)}{\cos(\pi/8)} = \frac{\sqrt{\sqrt{2}+2}\,/2}{\sqrt{2-\sqrt{2}}\,/2} = \underline{\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}}$

$= \sqrt{\frac{(2+\sqrt{2})^2}{(2-\sqrt{2})(2+\sqrt{2})}} = \sqrt{\frac{4+4\sqrt{2}+2}{2}}$

$= \underline{\sqrt{3+2\sqrt{2}}}$

Addisjonsformel for tan.

oppg 11.14

$$\tan(u+v) = \frac{\sin(u+v)}{\cos(u+v)} = \frac{\sin u \cos v + \sin v \cos u}{\cos u \cdot \cos v - \sin u \sin v}$$

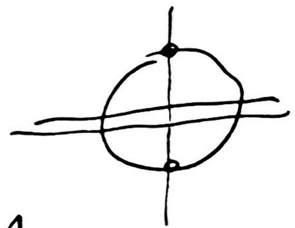
deler nevner og teller med $\cos u \cdot \cos v$

$$\tan(u+v) = \frac{\tan(u) + \tan(v)}{1 - \tan(u)\tan(v)}$$ når definert

Dobling av vinkel

$u = v$

$$\tan(2u) = \frac{2\tan(u)}{1 - \tan^2(u)}$$



ikke definert for $2x = \frac{\pi}{2} + \pi \cdot n$

$x = \frac{\pi}{4} + \frac{\pi}{2} \cdot n$

11.60 b) $f(x) = \tan(2x)$

Bruddpunkt (ikke definert) når

$\underline{x = \frac{\pi}{4} + \frac{\pi}{2} \cdot n}$ n heltall

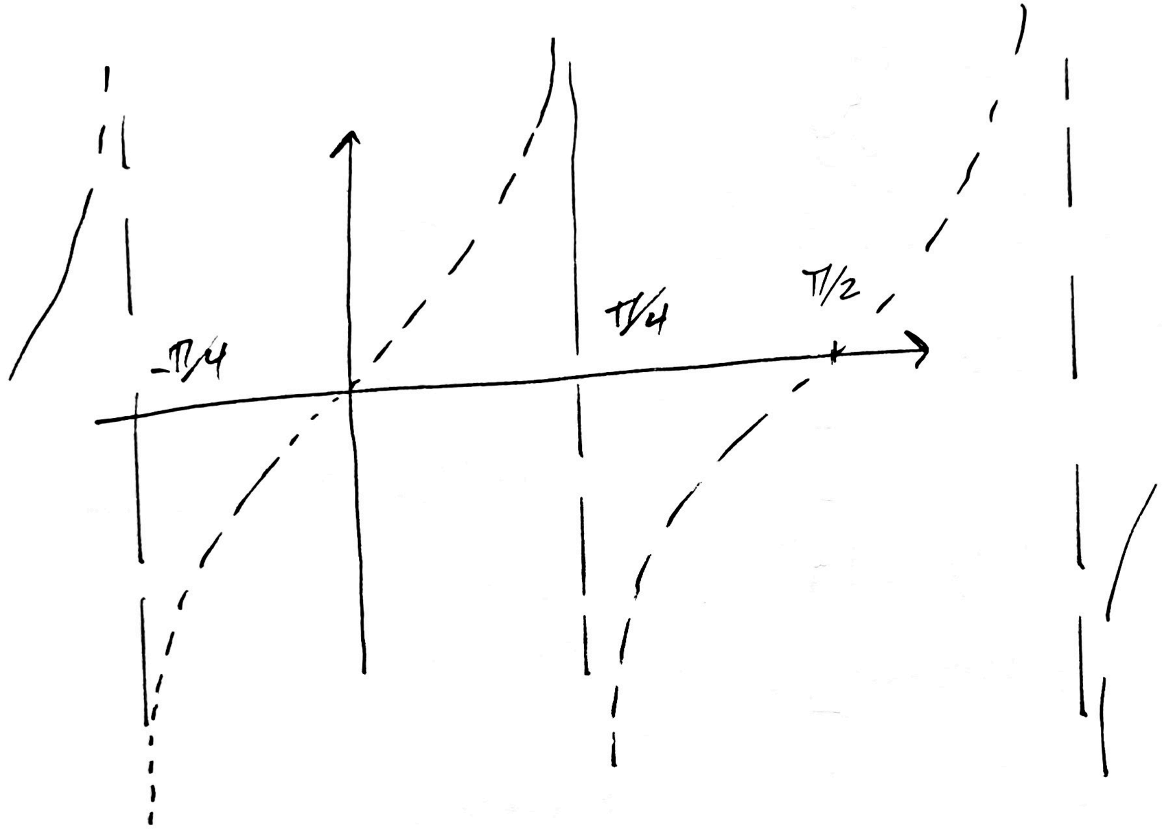
Nullpunkt $f(x) = 0$

$\tan(2x) = 0$

$2x = \pi \cdot n$ så

Nullpunkt: $\underline{x = \frac{\pi}{2} \cdot n}$ n heltall

1(.60b)



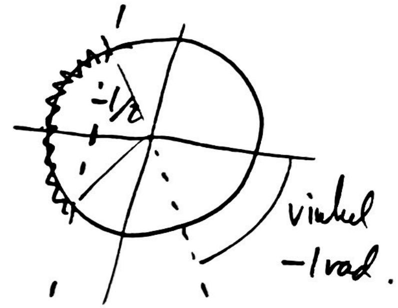
Løs ulikheten $\cos(2x-1) < \frac{-1}{2}$ $\quad x \in [0, \pi]$.

Numerisk: $x \in \langle 1{,}5472,\ 2{,}5944 \rangle$

$2x - 1 = u$

$-1 \leq u \leq 2\pi - 1$

$\cos(u) < \frac{-1}{2}$



$\frac{2\pi}{3} < u < \frac{4\pi}{3}$

er løsningen for $u \in [-1, 2\pi - 1]$.

$2x - 1 = u$ så $x = \frac{u+1}{2}$

$$\frac{\frac{2\pi}{3} + 1}{2} < \frac{u+1}{2} < \frac{\frac{4\pi}{3} + 1}{2}$$

$$\frac{\pi}{3} + \frac{1}{2} < x < \frac{2\pi}{3} + \frac{1}{2}$$

Løsningen er alle x: $\quad x \in \langle \frac{\pi}{3} + \frac{1}{2},\ \frac{2\pi}{3} + \frac{1}{2} \rangle$

$\left(\frac{\pi}{3} + \frac{1}{2} = 1{,}5472\ldots \quad \frac{2\pi}{3} + \frac{1}{2} = 2{,}5944\ldots \right)$