17.feb.2021

11.8. Vi skal syne at $(\sin x)' = \cos x$ (vinkel i radianer)

Fra definisjonen: $(\sin x)' = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

Addisjonsformelen for sin: $\sin(x+h) = \sin x \cos(h) + \sin(h) \cdot \cos x$

$$(\sin x)' = \lim_{h \to 0} \frac{\sin x(\cos(h) - 1) + \sin(h) \cdot \cos x}{h}$$

$$= \sin x \cdot \underbrace{\lim_{h \to 0} \frac{\cos(h) - 1}{h}}_{0} + \cos x \underbrace{\lim_{h \to 0} \frac{\sin h}{h}}_{1}$$

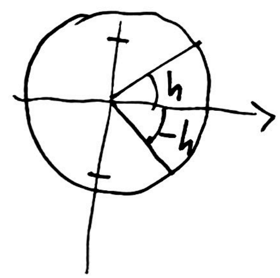
$$= 0 \cdot \sin x + 1 \cdot \cos x$$

$$\underline{(\sin x)' = \cos x}$$

"Vi viser nå grensene" →

$$\lim_{h\to 0^-} \frac{\sin h}{h} = \lim_{h\to 0^+} \frac{\sin(-h)}{-h} = \lim_{h\to 0^+} \frac{-\sin(h)}{-h}$$

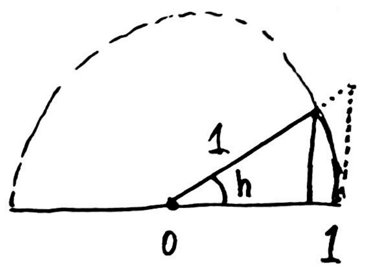
($\sin x$ er en odde funksjon så $\sin(-x) = -\sin x$)



$$\lim_{h\to 0^-} \frac{\sin h}{h} = \lim_{h\to 0^+} \frac{\sin h}{h}$$

Tilstrekkelig å vise at $\lim_{h\to 0^+} \frac{\sin(h)}{h} = 1.$

$\frac{\pi}{4} > h > 0$



Areal(liten trekant) < Areal(sirkelsegment) < Areal(stor trekant)

$$\underbrace{\frac{1}{2}\sin(h)\cos(h) < \frac{1}{2}h}_{\frac{\sin(h)}{h} < \frac{1}{\cos(h)}} \qquad \underbrace{< \frac{1}{2}\cdot 1\cdot \tan(h) = \frac{1}{2}\frac{\sin(h)}{\cos(h)}}_{\cos(h) < \frac{\sin(h)}{h}}$$

$$\cos(h) < \frac{\sin(h)}{h} < \frac{1}{\cos(h)}$$

$$\lim_{h\to 0} \cos(h) = 1 \quad \text{og} \quad \lim_{h\to 0} \frac{1}{\cos(h)} = \frac{1}{\lim_{h\to 0} \cos(h)} = 1.$$

Derfor må $\displaystyle\lim_{h\to 0} \frac{\sin(h)}{h} = 1.$ $\square$

$$\lim_{h\to 0} \frac{1-\cos(h)}{h} = \lim_{h\to 0} \frac{(1-\cos(h))(1+\cos(h))}{h \cdot (1+\cos(h))}$$

$$= \lim_{h\to 0} \frac{1-\cos^2(h)}{h(1+\cos(h))} = \lim_{h\to 0} \frac{\sin^2 h}{h(1+\cos h)}$$

$$= \lim_{h\to 0} h \cdot \left(\frac{\sin(h)}{h}\right)^2 \left(\frac{1}{1+\cos(h)}\right) = 0 \cdot \frac{1}{2} = 0. \quad \square$$

vi ser også:

$$\lim_{h\to 0} \frac{1-\cos h}{h^2} = \frac{1}{2}$$

h liten: $\cos(h) \sim 1 - \frac{h^2}{2}$

$\sin(h) \sim h$

V vinkel i grader:

$$\sin\Big(\overbrace{\frac{\pi}{180°}\cdot V}^{\text{radianer}}\Big)$$

$$\frac{d}{dV}\sin\left(\frac{\pi}{180°}\cdot V\right) = \underbrace{\cos\left(\frac{\pi}{180°}V\right)\cdot\frac{\pi}{180°}}_{\text{ved bruk av grader}}$$

$$(\cos x)' = \left(\sin\left(\frac{\pi}{2}-x\right)\right)' = \cos\left(\frac{\pi}{2}-x\right)\cdot\left(\frac{\pi}{2}-x\right)'$$

$$= -\cos\left(\frac{\pi}{2}-x\right)$$

$$= -\sin x.$$

$$\sin^3(5x+4) = \left(\sin(5x+4)\right)^3$$

$$\left(\sin^3(5x+4)\right)' = 3\left(\sin(5x+4)\right)^2 \cdot \left(\sin(5x+4)\right)'$$

$$\cos(5x+4) \cdot \underbrace{(5x+4)'}_{5}$$

$$= \underline{15 \sin^2(5x+4) \cos(5x+4)}$$

Deriver: $e^{-2x}\cos(3x)$

$$\left(e^{-2x}\cos(3x)\right)' = \left(e^{-2x}\right)'\cos(3x) + \left(e^{-2x}\right) \cdot \left(\cos(3x)\right)'$$

$$= e^{-2x}(-2x)'\cos(3x) + e^{-2x}\left(-\sin(3x)\right) \cdot (3x)'$$

$$= -2e^{-2x}\cos(3x) - 3e^{-2x}\sin(3x)$$

$$= e^{-2x}\left(-2\cos(3x) - 3\sin(3x)\right)$$

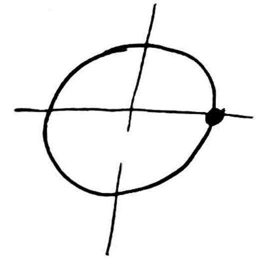
$f(x) = \sin x - x$

$f'(x) = \cos x - 1 \quad \leq 0$ for alle x

$f(x)$ er avtagende.

$f'(x) = 0$: $\quad \cos x = 1$

$x = 0 + 2\pi \cdot n$



$2\pi \sim 6.28$

$f''(x) = (\cos x - 1)' = -\sin x$

konkav ned $\quad f''(x) < 0 \qquad x \in \langle 0, \pi \rangle$ opp til et helt omløp

— opp $\quad f''(x) > 0 \qquad x \in \langle \pi, 2\pi \rangle$ opp til et helt omløp

$f''(x) = 0 \qquad \sin x = 0$:

$x = \pi \cdot n \qquad n$ heltall

vendepunkt i $(\pi \cdot n, -\pi \cdot n)$

Oppg $\quad f(x) = e^{-x}\cos x$

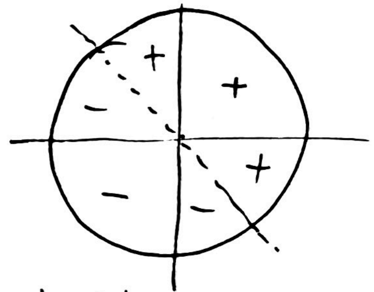
Monotoniegenskaper

Konkavitet.

$$f'(x) = \underbrace{(e^{-x})'}_{(-e^{-x})}\cos x + e^{-x}\cdot \underbrace{(\cos x)'}_{-\sin x}$$

$$= -e^{-x}\cos x - e^{-x}\sin x$$

$$= \underline{-e^{-x}(\cos x + \sin x)}$$



$f'(x) = 0 \; : \; \cos x = -\sin x \Leftrightarrow \tan x = -1,$

$x = -\frac{\pi}{4} + \pi\cdot n$

$f'(x) > 0$ når $\cos x + \sin x < 0 \; : \; \left\langle \frac{3\pi}{4}, \frac{7\pi}{4} \right\rangle$ opp til helt omløp.

$\cos x + \sin x > 0 \quad \left\langle -\frac{\pi}{4}, \frac{3\pi}{4} \right\rangle$ —

$f'(x) < 0$

$$f''(x) = \left(-e^{-x}\left(\cos x + \sin x\right)\right)'$$

$$= \left(-e^{-x}\right)'\left(\cos x + \sin x\right) + \left(-e^{-x}\right)\left(\cos x + \sin x\right)'$$

$$= \left(-e^{-x}(-x)'\right)\left(\cos x + \sin x\right) + \left(-e^{-x}\right)\left(-\sin x + \cos x\right)$$

$$= e^{-x}\left(\cos x + \sin x\right) - e^{-x}\left(-\sin x + \cos x\right)$$

$$= e^{-x}\left[\cos x + \sin x + \sin x - \cos x\right]$$

sin(v) 

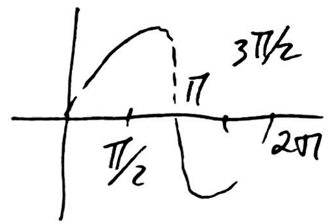
$$f''(x) = \underline{2\sin x \cdot e^{-x}}$$

$f''(x) > 0$ $x \in [0, \pi)$ opp til helt omløp.

$f''(x) < 0$ $x \in (\pi, 2\pi)$

vendepunkt: $x = \pi \cdot n$

$$g(x) = \sin(\ln x)$$



Ekstremalpunkt:

$g(x)$ størst når $\sin(\ln x) = 1$

da må $\ln(x) = \frac{\pi}{2} + 2\pi \cdot n$

$$x = e^{\ln x} = e^{\frac{\pi}{2} + 2\pi \cdot n}$$

toppunkter $\left(e^{\frac{\pi}{2} + 2\pi \cdot n}, 1\right)$

tilsvarende bunnpunkt $\left(e^{\frac{3\pi}{2} + 2\pi \cdot n}, -1\right)$

heltall n

Nullpunkt: $\sin(\ln x) = 0$

$$\ln x = \pi \cdot n$$

$$x = e^{\pi \cdot n}$$

$$f'(x) = (\sin(\ln x))' = \cos(\ln x) \cdot (\ln x)' = \frac{\cos(\ln x)}{x}$$

$$f''(x) = \left(\cos(\ln x) \cdot \frac{1}{x}\right)' = \frac{-\sin(\ln x)}{x} \cdot \frac{1}{x} + \cos(\ln x) \frac{-1}{x^2}$$

$$= \frac{-(\sin(\ln x) + \cos(\ln x))}{x^2}$$