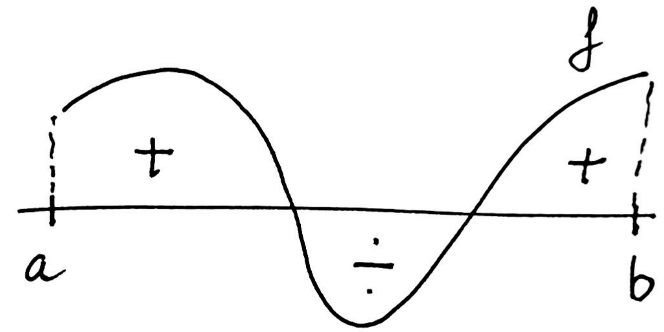
3mars 2021

15.5 Bestemte integral

$\int_a^b f(x)\,dx$ = areal med fortegn mellom grafen til f og x-aksen



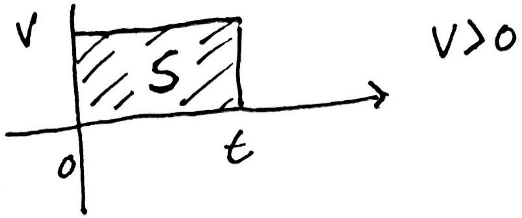
"Det bestemte integralet av $f(x)$ med hensyn til x fra a til b"

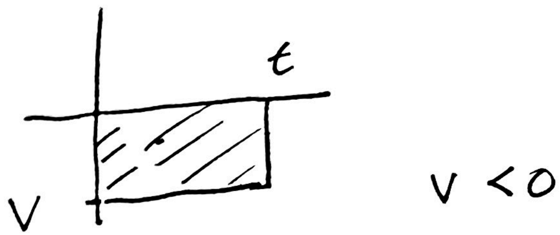
$\int_a^b$

b ← øvre (integral)grense

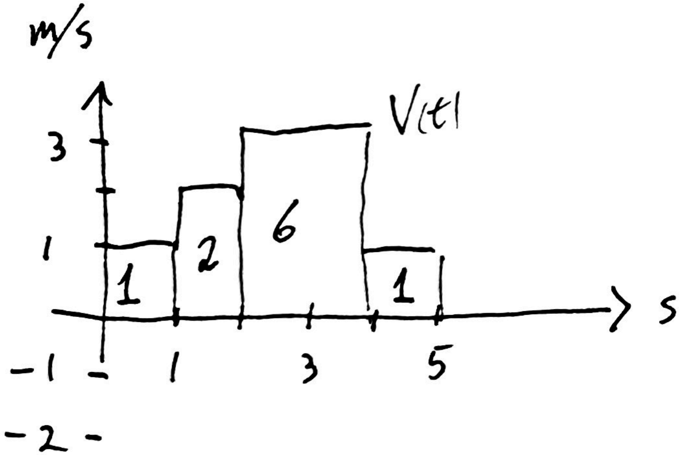
a ← nedre (integral) grense

Vei = fart · tid





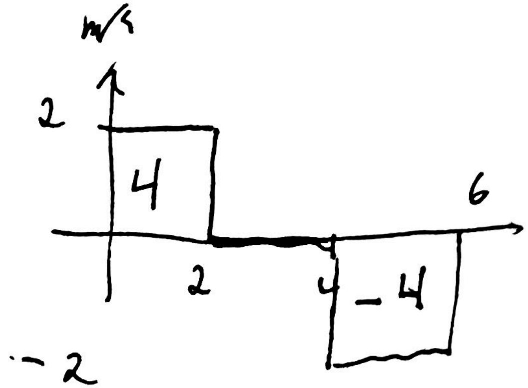
Forflytningen
$S = v \cdot t$ " minus arealet til rektangelet"



Distansen forflyttet:

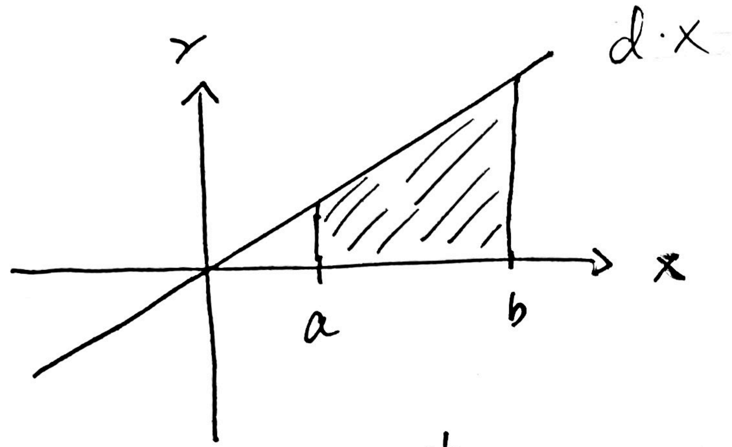
$S = 1 + 2 + 6 + 1 = 10$ (meter)

= arealet m. fortegn mellom $v(t)$ og t-aksen.



Forflyttet distanse:

$4m + 0m + (-4m) = 0$ (meter)



$$\int_a^b d \cdot x \, dx = \frac{b \cdot (d \cdot b)}{2} - \frac{a(d \cdot a)}{2}$$

$$= \frac{d}{2}(b^2 - a^2)$$

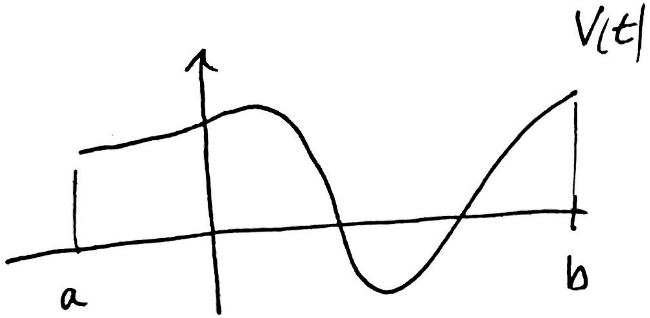
alle a og b.

Merk $\quad \int d \cdot x \, dx = d \int x \, dx$

$$= d \cdot \frac{x^2}{2} + c$$

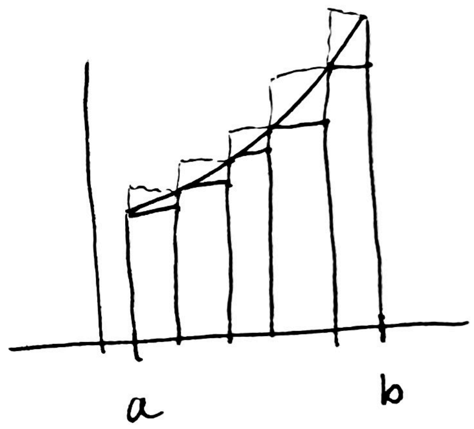
så $\quad \frac{d}{2}(b^2 - a^2) = F(b) - F(a)$

hvor F er en antiderivert til $d \cdot x$.



$$\int_a^b V(t)\,dt = \text{Forflytting fra tid } a \text{ til tid } b.$$

Hva er arealet mellom $f(x)$ og x-aksen



$$\text{nedre estimat} \leq \int_a^b f(x)\,dx \leq \text{øvre estimat}$$

Deler opp intervallet $[a, b]$ i n deler

$$a = x_0 < x_1 < x_2 < \dots < x_n = b$$

$$S_{øvre} = \sum_{i=1}^{n} \left(\max_{[x_{i-1}, x_i]} f(x) \right) (x_i - x_{i-1})$$

$$S_{nedre} = \sum_{i=1}^{n} \left(\min_{[x_{i-1}, x_i]} f(x) \right) (x_i - x_{i-1})$$

velg gjerne oppdelingen slik at $x_i - x_{i-1}$ er like

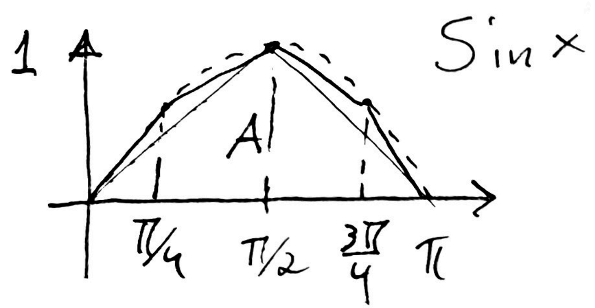
Store $x_i - x_{i-1} = \frac{b-a}{n}$.

$$x_i = a + \frac{b-a}{n} \cdot i$$

Resultat: Hvis $f(x)$ er kontinuerlig da vil

$$\lim_{n \to \infty} S^n_{øvre} = \lim_{n \to \infty} S^n_{nedre}$$

f kont : Definer $\int_a^b f(x)\,dx = \lim_{n \to \infty} S^n_{nedre}$.

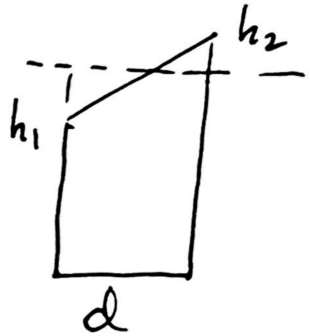


Hva $A = \int_0^{\pi} \sin x \, dx$?

$0 < A < \pi \cdot 1$

areal Δ-kant $\frac{\pi}{2} < A < \pi$

Arealet estimert ved de fire trapesene (▱) :



Bredde $\frac{\pi}{4} = d$

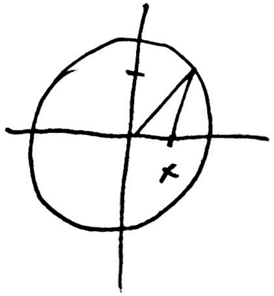
arealet $d \cdot \left(\frac{h_1 + h_2}{2}\right)$

Estimat $A_4 = \frac{\pi}{4} \cdot \frac{1}{2}\left(\left(0 + \sin\left(\frac{\pi}{4}\right)\right) + \left(\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{2}\right)\right) + \left(\sin\left(\frac{\pi}{2}\right) + \sin\left(\frac{3\pi}{4}\right)\right) + \left(\sin\left(\frac{3\pi}{4}\right) + 0\right)\right)$

$= \frac{\pi}{8}\left(0 + \frac{1}{\sqrt{2}} \cdot 2 + 1 \cdot 2 + \frac{1}{\sqrt{2}} \cdot 2 + 0\right)$

$= \frac{\pi}{8}\left(2 \cdot \sqrt{2} + 2\right) = \frac{\pi}{8} \cdot 2\left(\sqrt{2} + 1\right) = \frac{\pi}{4}\left(1 + \sqrt{2}\right)$

$\sim 1.896\ldots$



$x^2 + y^2 = 1$

$y = \sqrt{1-x^2}$

$\sqrt{1-x^2}$

Geogebra:

lowersum(...) (nedresum(..))

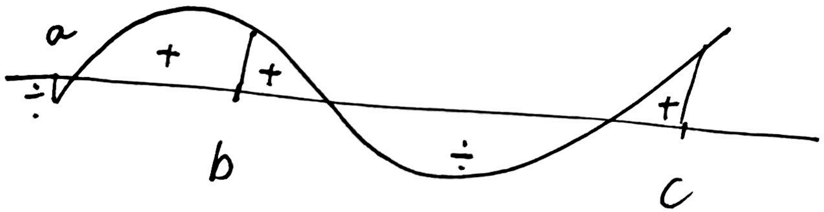
uppersum(...) (øvresum (...))

Dere kan også bruke sequence (følge) når dere skal estimere integraler (se video)

Estimere: $2\int_{-1}^{1} \sqrt{1-x^2}\,dx$

gir tilnærma verdier til π.

Nedre sum tilnærming med rektangler gir ~ 3.137.



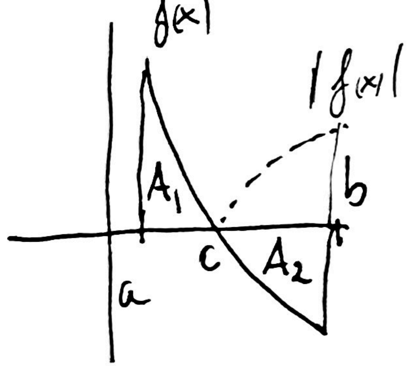
$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

$$= \int_a^c f(x)\,dx$$

$a < b$ definere vi $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

(siden $\int_a^b f(x)\,dx + \int_b^a f(x) = \int_a^a f(x) = 0 \ldots$)

$\int_a^b f(x)\,dx = A_1 - A_2$ areal med fortegn mellom $f(x)$ og x-aksen



$$\int_a^b |f(x)|\,dx = A_1 + A_2$$

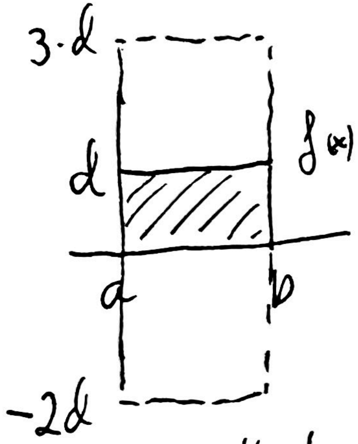
$$= \underbrace{\int_a^c f(x)\,dx}_{f(x) \geq 0} + \underbrace{\int_c^b -f(x)\,dx}_{f(x) \leq 0}$$

Arealet mellom $f(x)$ og x-aksen.

$$= \int_a^c f(x)\,dx - \int_c^b f(x)\,dx$$

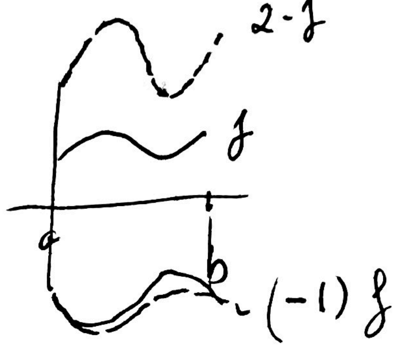
$$\int_a^b k f(x)\,dx = k \cdot \int_a^b f(x)\,dx$$

k reelt tall.



f(x) konstant

vi illustrer egenskapen
med k = 3 og -2
og en konstant
funksjon.



(k = -1 og 2)