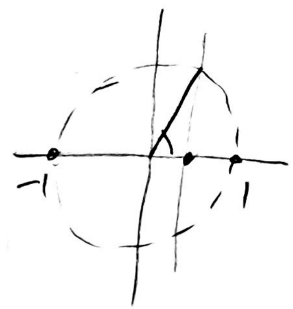
oblig 6 #7 variant. 5 mars 2021

Hva er $(\arccos x)'$?

$\arccos x$ def for $-1 \le x \le 1$ gir en vinkel mellom 0 og π rad.



$\cos(\arccos x) = x$

Deriverer og benytter kjerneregelen

$-\sin(\arccos x) \cdot (\arccos x)' = (x)' = 1$.

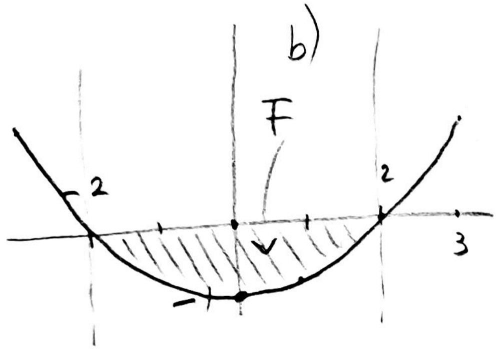
For vinkler v mellom 0 og π rad er $\sin(v) = \sqrt{1-\cos^2 v}$ (siden $\sin v \geq 0$)

$$\sin(\arccos x) = \sqrt{1-\cos^2(\arccos x)} = \sqrt{1-x^2}.$$

$$(\arccos x)' = \frac{1}{-\sqrt{1-x^2}} = \underline{\frac{-1}{\sqrt{1-x^2}}}$$

15.52 $\quad f(x) = \frac{1}{4}x^2 - 1 \qquad x \in [-3, 3]$

a)



c) $\int_{-2}^{2} f(x)\,dx$ tilnærme ved bruk av 8 rektangler.

$=$ - "arealet til flatestykket F".

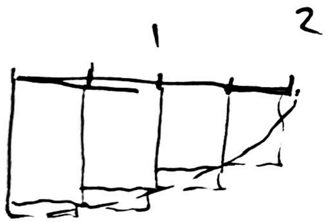
x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	-1	$\frac{-15}{16}$	$-\frac{3}{4}$	$\frac{-7}{16}$	0

$f(\frac{1}{2}) + f(1) + f(\frac{3}{2})$

$= \frac{-15-12-7}{16}$

$= \frac{-34}{16} = -2 - \frac{1}{8}$

tilsvarende



bredde

$$2 \cdot \frac{1}{2}\left[f(0) + f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2})\right] \leq \int_{-2}^{2} f(x)\,dx \leq 2\left(\frac{1}{2}\right)\left[f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2}) + f(2)\right]$$

$$-3 - \frac{1}{8} \leq \int_{-2}^{2} f(x)\,dx \leq -2 - \frac{1}{8}$$

d) Arealet er mellom $2 + \frac{1}{8}$ og $3\frac{1}{8}$.

(geogebra: antyder $2\frac{2}{3}$?)

$$(\ln x)' = \frac{1}{x} \qquad x > 0$$

$$(\ln(-x))' = \frac{1}{-x} \cdot (-x)' = \frac{-1}{-x} = \frac{1}{x} \qquad x < 0$$

$$\ln|x| = \begin{cases} \ln x & x > 0 \\ \ln(-x) & x < 0 \end{cases} \qquad \text{definert for alle } x \neq 0$$

$$(\ln|x|)' = \frac{1}{x} \qquad x \neq 0$$

oblig 6 #6

$$g(x) = e^{2x} \cdot \sin(3x)$$

$$g'(x) = (e^{2x})' \sin(3x) + (e^{2x}) \cdot (\sin 3x)'$$

$$= e^{2x} \cdot (2x)' \sin(3x) + e^{2x} \cdot (\cos(3x) \cdot (3x)')$$

$$= e^{2x} \cdot 2 \cdot \sin 3x + e^{2x}(3\cos(3x))$$

$$= \underline{e^{2x}(2\sin(3x) + 3\cos(3x))}$$

$g''(x)$ samme type derivasjon...

spørsmål: $\ln(3(x+1)) = \cancel{x \ln 3} + \underline{\ln 3}$?

$\underline{\ln 3} + \underline{\ln(x+1)}$

$x \ln 3 = \ln 3^x$

nyversjon: $\ln(3^{x+1}) = (x+1)\ln 3$

$= x \cdot \ln 3 + \ln 3$

1f variant: Deriver $f(x) = \ln(3x^3 \cdot 10^{x^2})$

$= \ln(3) + \ln(x^3) + \ln(10^{x^2})$

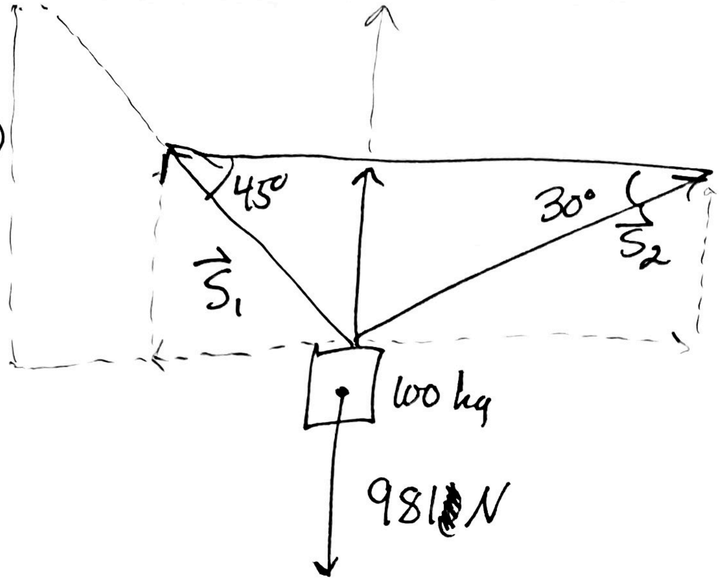
$f(x) = \ln 3 + 3\ln x + x^2 \ln 10$

$f'(x) = (\ln 3)' + (3\ln x)' + (x^2 \ln 10)'$

$0 \quad 3(\ln x)' + 2x \cdot \ln(10)$

$= \frac{3}{x} + 2x \cdot \ln(10) = \underline{\frac{3}{x} + 2\ln(10) \cdot x}$

12.200



vertikale krefter

$$|\vec{S}_2| \sin(30°) + |\vec{S}_1| \sin(45°) = 981\,N$$

horisontale krefter

$$|\vec{S}_1| \cos 45 = |\vec{S}_2| \cos(30°)$$

$$\frac{1}{\sqrt{2}} \qquad \frac{\sqrt{3}}{2}$$

$$|\vec{S}_1| = \frac{\sqrt{3}}{2} \cdot \sqrt{2}\, |\vec{S}_2|$$

$$= \sqrt{\frac{3}{2}}\, |\vec{S}_2|$$

setter inn for $|\vec{S}_1|$:

$$|\vec{S}_2| \cdot \left(\frac{1}{2}\right) + \sqrt{\frac{3}{2}}\, |S_2| \cdot \frac{1}{\sqrt{2}} = 981\,N$$

$$|\vec{S}_2| \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = 981\,N$$

$$|\vec{S}_2| = \frac{2 \cdot 981\,N}{1+\sqrt{3}}$$

$$|\vec{S}_1| = \sqrt{\frac{3}{2}} \cdot \frac{2}{1+\sqrt{3}} \cdot 981\,N$$

oblig 6 #4

$$(x^3 e^{-2x})' = (x^3)' e^{-2x} + x^3 (e^{-2x})'$$

$$= 3x^2 e^{-2x} + x^3 \big(e^{-2x} \cdot \underbrace{(-2x)'}_{-2}\big)$$

$$= \underline{e^{-2x}(3x^2 - 2x^3)}$$

15.41 a) $\int 4 e^{2x+1} \, dx = 4 \int e^{2x+1} \, dx$

lineær substitusjon

$$\int f(ax+b) \, dx = \frac{F(ax+b)}{a} + C$$

$$4 \int e^{2x+1} dx = 4 \cdot \frac{e^{2x+1}}{2} + C = \underline{2 e^{2x+1} + C}$$

$xe^x = x^3$

$x(e^x - x^2) = 0$

$x = 0$

$x \sim -0{,}703$

$e^{x-1} = x^2$ tre snittpunkt $x \sim -0{,}477$

1

$3{,}51$

$\int_0^1 e^{x^2}\,dx$ numerisk integrasjon

e^{x^2} har ingen elementær antiderivert..

$\lg = \log = \log_{10}$

$\ln = \log_e$

$\int \frac{1}{x}\,dx = \ln|x| + C = \frac{\log x}{\log(e)} + C$

$\ln x = \frac{\log x}{\log(e)}$

$\int \log(e) \cdot \frac{1}{x}\,dx = \log(e)\ln|x| + C$

$= \underline{\log|x| + C}$

Variant av oblig 6 #2.3

$\log|x-5| = 2$

$\log|2x+4| = 1$

$10^{\log|2x+4|} = 10^1$

$|2x+4| = 10$

$2x+4 = 10$ eller $2x+4 = -10$

$\underline{\underline{x = 3}}$ $\underline{\underline{x = -7}}$

$$\left(\cos^2 x\right)' = \left(\left(\cos x\right)^2\right)^2 \quad \text{kjerneregel}$$

$$= 2\cos x \cdot \underbrace{\left(\cos x\right)'}_{-\sin x}$$

$$= \underline{-2\sin x \cos x} \quad = -\sin 2x.$$

Merk:

$$\cos^2 x - \underbrace{\sin^2 x}_{1-\cos^2 x} = \cos 2x$$

$$2\cos^2 x - 1 = \cos 2x$$

$$\cos^2 x = \frac{1}{2}\left(1+\cos(2x)\right)$$

Vi kan også bruke denne trig identiteten for å derivere $\cos^2 x$.

$$\left(\cos^2 x\right) = \left(\frac{1}{2}\left(1+\cos 2x\right)\right)' = \underbrace{\left(\frac{1}{2}\right)'}_{0} + \left(\frac{1}{2}\cos(2x)\right)'$$

$$= \frac{1}{2}\left(\cos(2x)\right)'$$

$$= \frac{1}{2}\left(-\sin(2x)\cdot(2x)'\right) = \underline{-\sin(2x)}$$

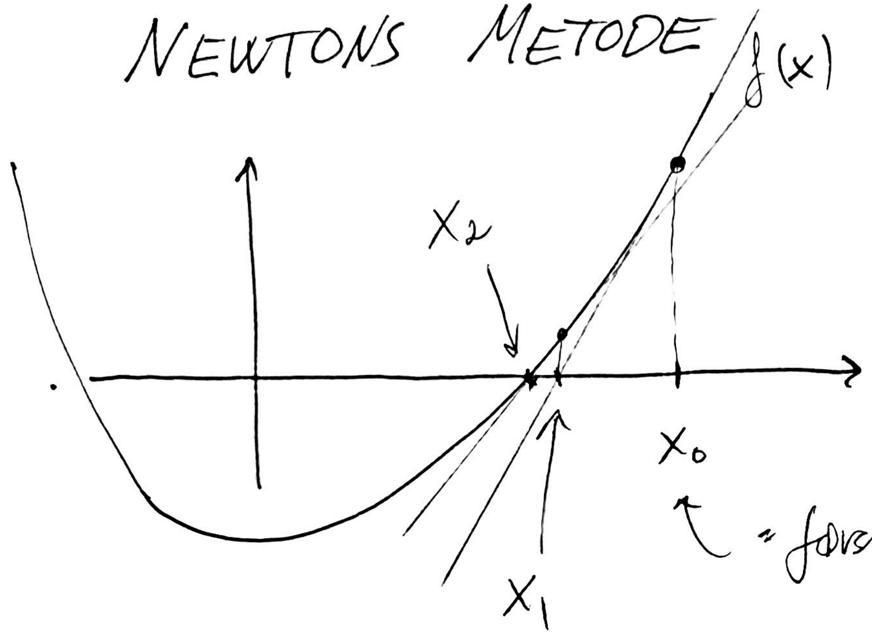
5 mars

NEWTONS METODE (IKKJE PENSUM) Til orientering

(kommer i Matte 1000)

Hvordan estimere løsninger til $f(x) = 0$?

(finne nullpunkt)



Likning for tangentlinjen i $(x_n, f(x_n))$

$$y = f'(x_n)(x - x_n) + f(x_n)$$

Tangentlinjen treffer x-aksen når $y = 0$:

$$0 = f'(x_n)(x - x_n) + f(x_n)$$

Løser ut for x, og lar løsningen være "neste x"

Følger tangentlinjen og finner ut hvor den treffer x-aksen.

$$-f(x_n) = f'(x_n)(x - x_n)$$

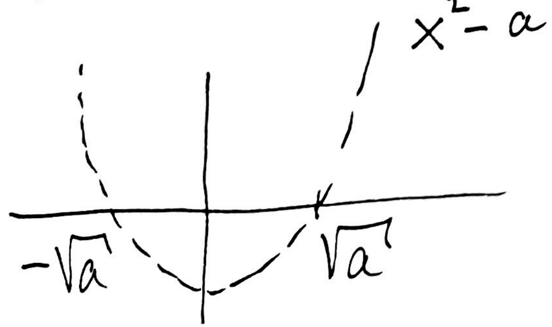
$$-\frac{f(x_n)}{f'(x_n)} = x - x_n$$

$$(x=)\ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Rekursiv formel

Starter med $x_0 \to x_1 \to x_2 \to x_3 \to \dots$

Bruker dette til å estime kvadrat røtter:



$$f(x) = x^2 - a$$

$$f'(x) = 2x$$

så $x_{n+1} = x_n - \frac{x_n^2 - a}{2x_n} = \frac{x_n^2 + a}{2x_n}$

$$= \frac{2x_n^2}{2x_n} - \frac{x_n^2 - a}{2x_n}$$

$a = 2$ starter med $x_0 = 2$ $(\sqrt{2} = 1.41421\ldots)$

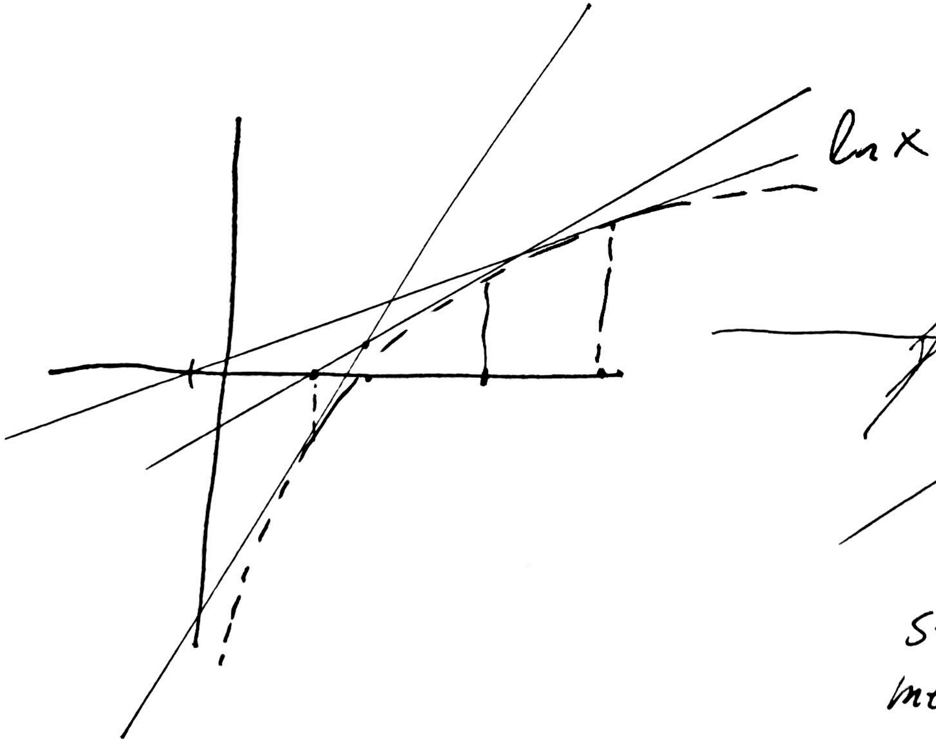
$$x_0 = 2$$

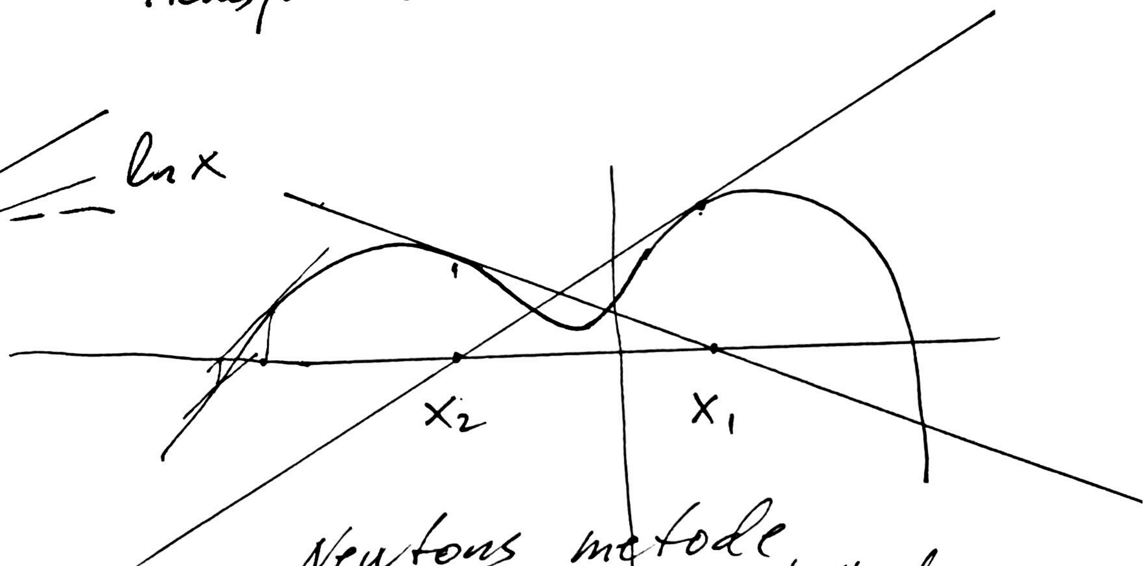
$$x_1 = \frac{2^2+2}{2\cdot 2} = \frac{3}{2}$$

$$x_2 = \frac{(3/2)^2+2}{2\cdot 3/2} = \frac{9/4+2}{3} = \frac{17}{12} \approx 1.41667$$

Brukte geogebra til å gjenta prosedyren med flere iterasjoner.

Illustrer hva som kan gå galt med Newtons metode





Newtons metode sender oss frem og tilbake mellom x_1 og x_2