9 mars 2021

15.7

Repetisjon: Fundamentalteoremet i kalkulus

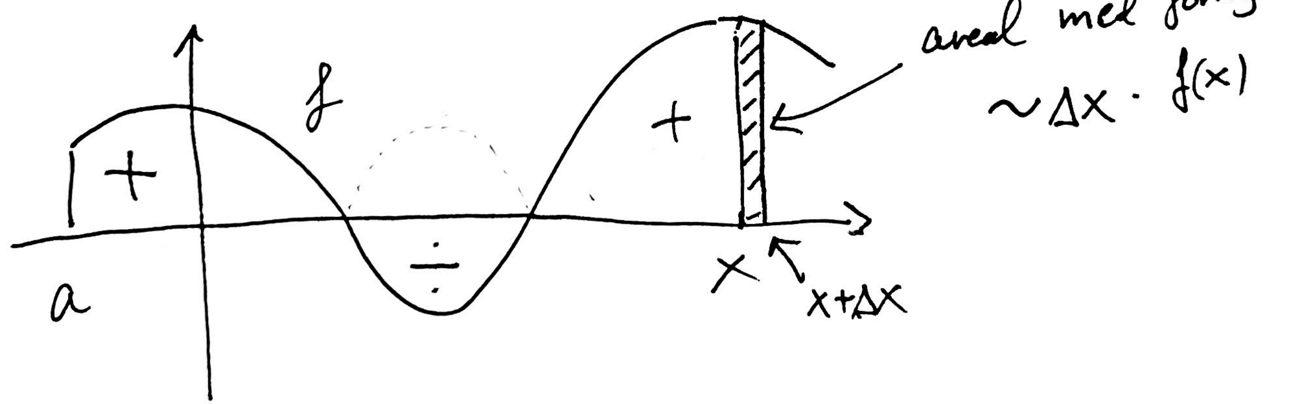
$$\left(\int_a^x f(t)\,dt \right)' = f(t) \qquad f \text{ kontinuerlig}$$

$\Rightarrow$ Kontinuerlige funksjoner har en antiderivert

$\Rightarrow$ $\int_a^b f(t)\,dt = F(b) - F(a) = F(x)\Big|_a^b$

hvor F er en antiderivert til $f(x)$ i $[a, b]$

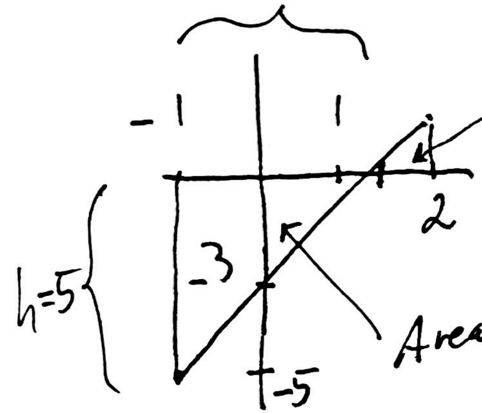
$\Delta x > 0$



$f(-1) = -5$

$$\int_{-1}^{2} \overbrace{2x - 3}^{f(x)}\,dx = x^2 - 3x \Big|_{-1}^{2} = (2^2 - 3\cdot 2) - ((-1)^2 - 3(-1))$$

$$= (4-6) - (4) = \underline{-6}$$



Areal A_1 $\frac{1}{4}$

Areal $A_2 = \frac{2{,}5 \cdot 5}{2} = \frac{25}{4}.$

$$\int_{-1}^{2} 2x-3\,dx = A_1 - A_2$$

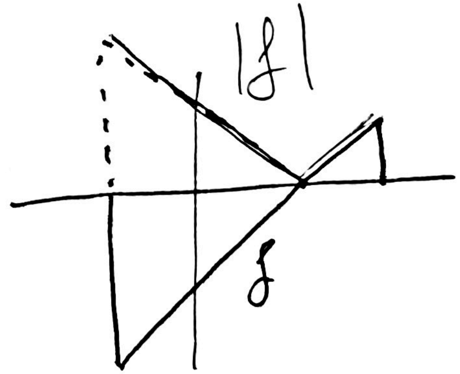
$$= \frac{1}{4} - \frac{25}{4} = \frac{-24}{4}$$

$$= \underline{-6}$$

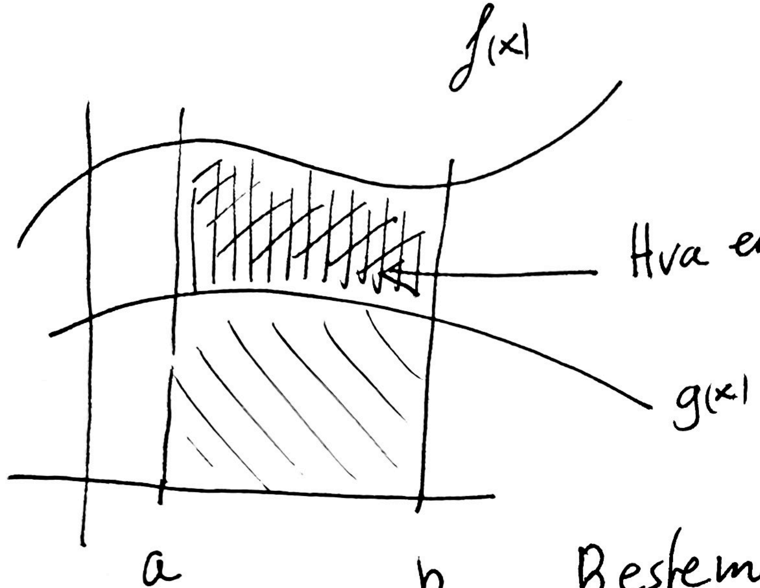
Hva er arealet mellom grafen til $f(x) = 2x-3$ og x-aksen avgrenset av $x = -1$ og $x = 2$?

$$A = A_1 + A_2 = \frac{1}{4} + \frac{25}{4} = \frac{26}{4} = \frac{13}{2} = \underline{6.5}$$

$$= \int_a^b |f(x)|\,dx$$



$f(x) \geq g(x) > 0$



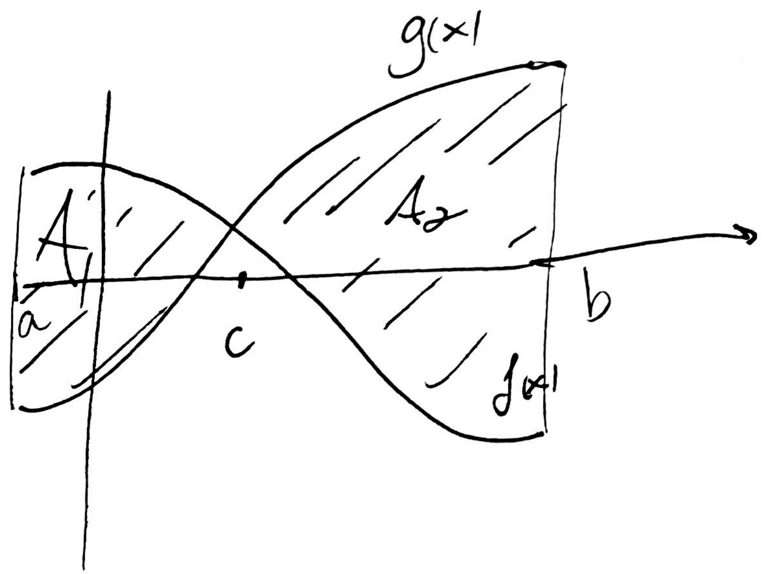
Hva er arealet? $\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

$= \int_a^b f(x) - g(x)\,dx$

Bestemte <u>integral er lineære</u>

$\int_a^b f(x) + g(x)\,dx = \int_a^b f(x) + \int_a^b g(x)\,dx$

for integrerbare funksjoner $f(x), g(x)$



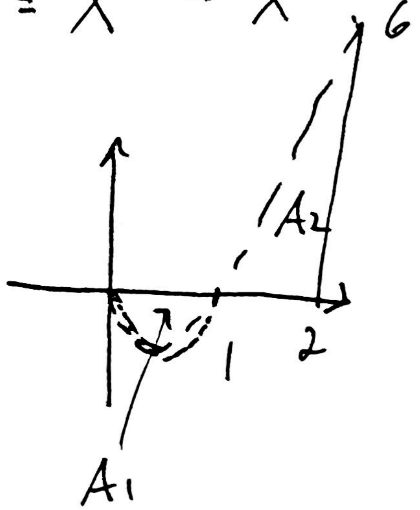
$A_1 = \int_a^c f(x) - g(x)\,dx$ $\quad f \geq g$

$A_2 = \int_c^b g(x) - f(x)\,dx$ $\quad g \geq f$

$A_1 + A_2 = \int_a^b \underbrace{|f(x) - g(x)|}_{\text{absolutt verdien}}\,dx$

absolutt verdien til differansen mellom $f(x)$ og $g(x)$.

$f(x) = x^3 - x$



Arealet mellom $f(x)$ og x-aksen i $[0, 2]$.

$$f(x) = x(x^2-1) = x(x-1)(x+1)$$

$$F(x) = \frac{x^4}{4} - \frac{x^2}{2}$$

$$A_1 = -\int_0^1 f(x)\,dx$$

$$A_2 = \int_1^2 f(x)\,dx$$

Arealet er $A = A_1 + A_2$.

$F(0) = 0$

$$A_1 = -F(x)\Big|_0^1 = -\left(\overbrace{\frac{1}{4} - \frac{1}{2}}^{F(1)}\right) = \underline{\frac{1}{4}}$$

$$A_2 = F(x)\Big|_1^2 = \underbrace{\left(\frac{2^4}{4} - \frac{2^2}{2}\right)}_{F(2)} + \underbrace{\frac{1}{4}}_{-F(1)} = 4 - 2 + \frac{1}{4} = 2 + \frac{1}{4} = 2.25$$

$$A = A_1 + A_2 = \frac{1}{4} + 2 + \frac{1}{4} = 2 + \frac{1}{2} = \underline{2.5}$$

$$\int_0^2 f(x)\,dx = F(x)\Big|_0^2 = 2 = \underset{\text{over x-akse}}{A_2} - \underset{\text{under x-aksen}}{A_1} \quad (= 2.25 - 0.25 = 2)$$

$f(x) = x^3 + 2x^2 - x + 5$

$g(x) = 2x^2 + 5$

$(f(x) - g(x)) = x^3 - x$

Arealet mellom $f(x)$ og $g(x)$ i intervallet $[0, 2]$

$$A = \int_0^2 |f(x) - g(x)|\,dx = -\int_0^1 \overbrace{f(x) - g(x)}^{\text{negativ}}\,dx + \int_1^2 \underbrace{f(x) - g(x)}_{\text{positiv}}\,dx$$

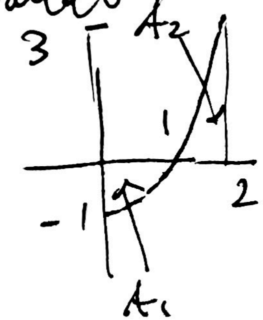
$$= \frac{1}{4} + 2 + \frac{1}{4} = 2 + \frac{1}{2} = \underline{2.5}$$

Oppg.

Finn arealet mellom $f(x)$ og $g(x)$ i intervallet $[0, 2]$.

$f(x) = x^2 + e^{-x^2} + 3x + 2$

$g(x) = e^{-x^2} + 3x + 3$



$f(x) - g(x) = x^2 - 1$
$= (x-1)(x+1)$

$$A = \int_0^2 |f(x) - g(x)|\,dx$$
$$-\int_0^1 x^2 - 1\,dx + \int_1^2 x^2 - 1\,dx$$

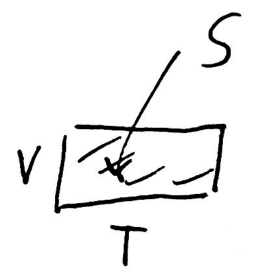
$$A_1 = -\int_0^1 x^2 - 1\,dx = -\left(\frac{x^3}{3} - x\right)\Big|_0^1 = -\left(\frac{1}{3} - 1\right) = \underline{\tfrac{2}{3}}$$

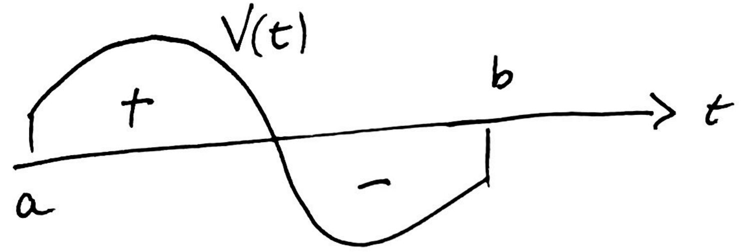
$$A_2 = \int_1^2 (x^2-1)\,dx = \frac{x^3}{3} - x \Big|_1^2 = \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 1\right) = \frac{2}{3} + \frac{2}{3} = \underline{\frac{4}{3}}$$

$$A = A_1 + A_2 = \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = \underline{2}$$

15.8 Samla resultat.

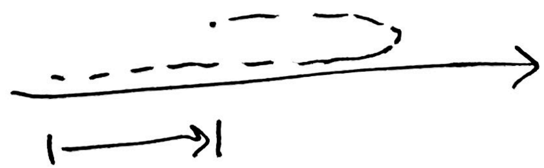
Fart × tid = forflytting

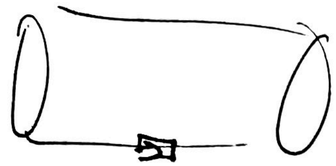




$$\text{Forflytting} = \int_a^b V(t)\,dt$$

Total distanse tilbakelagt $\int_a^b |V(t)|\,dt$

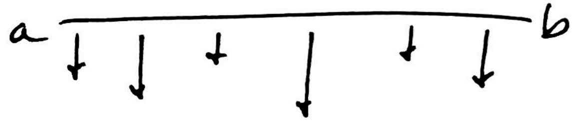




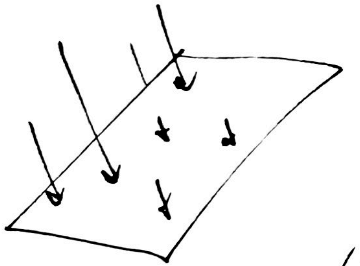
sensor $V(t) = \frac{\text{Volom}}{\text{tid}}$ ved tiden t.

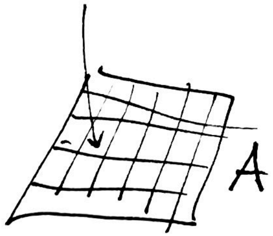
Totalt volum $V = \int_a^b V(t)\,dt$

$K = \frac{\text{kraft}}{\text{lengde}}$



Total kraft $= \int_a^b K\,dx$





P : Trykk $= \frac{\text{Kraft}}{\text{Areal}}$

$\iint_A P\,dA = \text{Kraft}$

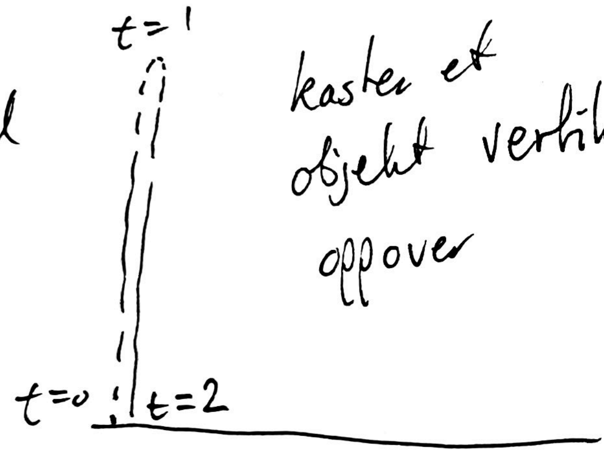
Til orientering

Matte2000.

P Effekt $= \dfrac{\text{Energi}}{\text{Tid}}$ Watt W

Energi $= \int_a^b P \, dt$

Eksempel



kaster et objekt vertikalt oppover

$V_0 = 10 m/s$

$V(t) = 10 m/s - (10 m/s^2) \cdot t$

$(V'(t) = -10 m/s^2)$

$S(t) = \int_0^t V(t) \, dt$

$= 10 m/s \cdot t - (5 m/s^2) t^2$

Når toppen når $V(t) = 0$
$t = 1$

$\int_0^2 V(t) \, dt = S(2) - S(0) = 0 \text{ m}$.
= tilbake der vi startet.

$S(1) = \underline{5 m}$

Distanse tilbakelagt: $\int_0^2 |V(t)| \, dt = \int_0^1 V(t) \, dt + \left(-\int_1^2 V(t) \, dt\right)$

$5 m + 5 m = \underline{10 m}$

$\vec{V}(t) = [2+t, 3-t]$

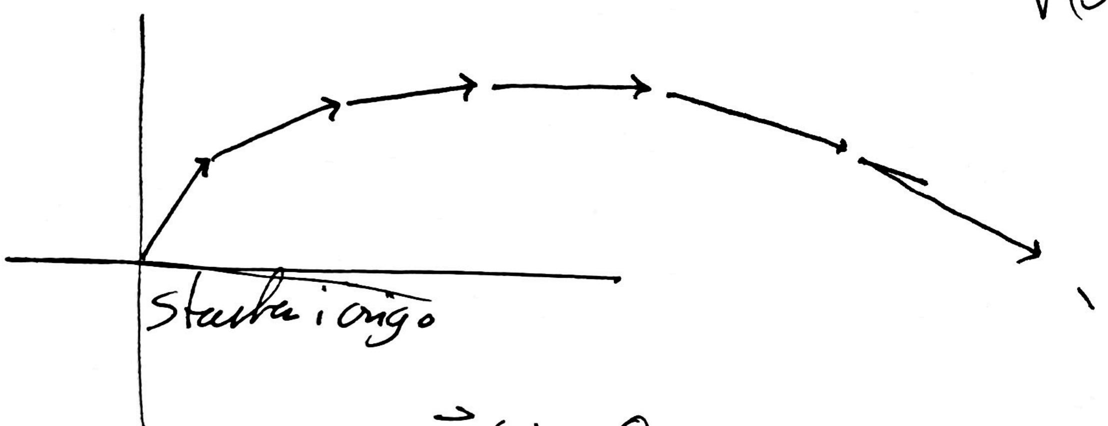
Eksempel

$\vec{V}(0) = [2,3]$

$\vec{V}(1) = [3,2]$

$\vec{V}(2) = [4,1]$

$\vec{V}(3) = [5,0]$



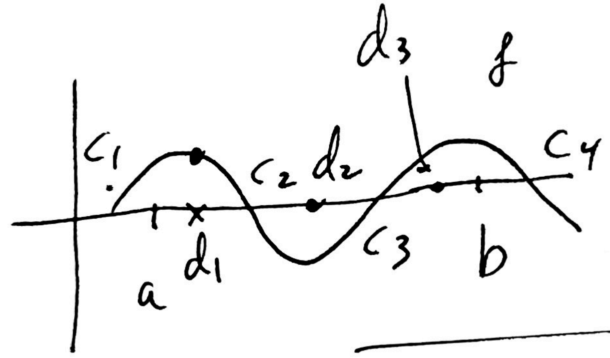
Tegnet i geogebra...

$\vec{S}(0) = 0$

$\vec{S}(t)$

$\left(\vec{S}(t)\right)' = \vec{V}(t) = [2+t, 3-t]$

$\vec{S}(t) = \left[2t + \frac{t^2}{2}, 3t, - \frac{t^2}{2}\right]$ $(\vec{S}(0)=0)$



Hvordan finne areal til regionen mellom grafen til $f(x)$ og x-aksen, uten å tegne grafen ...

f finn nullpunkt

c_1, c_2, c_3, c_4

liggen i $[a, b]$.

Anta f er kont.

f kan bare skifte fortegn i nullpunktene.

$f(x)$ samme fortegn, $\Rightarrow x \in [a, c_2]$ — $f(x) \geq 0$

$x \in [c_2, c_3]$ — " — $f(x) \leq 0$

$x \in [c_3, b]$ — " — $f(x) \geq 0$

Sjekker fortegnene:

$$\left.\begin{array}{l} f(d_1) > 0 \\ f(d_2) < 0 \\ f(d_3) > 0 \end{array}\right\} \text{gir}$$

Areal mellom $f(x)$ og x-akse

$$\int_a^{c_2} f(x)\,dx + \int_{c_2}^{c_3} -f(x)\,dx + \int_{c_3}^{b} f(x)\,dx \ldots$$

15.42 $\int 2\sin\left(\underbrace{\frac{\pi}{3}x - \frac{\pi}{6}}_{u(x)=ax+b}\right)dx$

c)

$$\int f(ax+b)\,dx = \frac{F(ax+b)}{a} + c$$

$$F'(x) = f(x)$$

Fordi: $\left(\frac{1}{a}F(ax+b)\right)' = \frac{1}{a}F'(ax+b)\cdot\underbrace{(ax+b)'}_{a}$

$$= f(ax+b)$$

$$\int \sin x\,dx = -\cos x + c$$

lineær substitusjon

$$\int 2\sin\left(\frac{\pi}{3}x - \frac{\pi}{6}\right)dx = 2\left(\frac{-\cos\left(\frac{\pi}{3}x - \frac{\pi}{6}\right)}{\pi/3}\right) + c$$

$$= \underline{\frac{-6}{\pi}\cos\left(\frac{\pi}{3}x - \frac{\pi}{6}\right) + c}$$

#7 Vis at $(\arctan x)' = \dfrac{1}{1+x^2}$

$$\tan(\arctan x) = x \qquad x \in \mathbb{R}$$

Deriverer begge sider

$$\tan'(\arctan x) \cdot (\arctan x)' = (x)' = 1$$

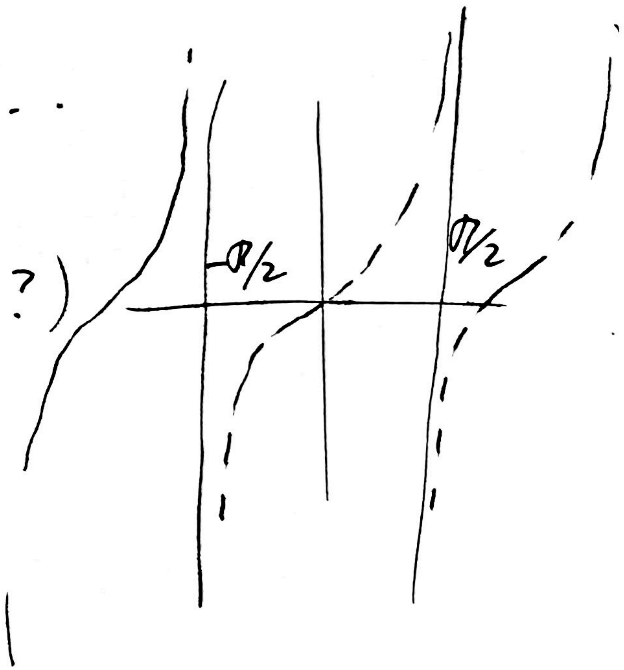
$$\underbrace{(1+\tan^2(\arctan x))}_{(1+x^2)} \cdot (\arctan x)' \overset{?}{=} 1$$

$$\underline{(\arctan x)'} = \underline{\frac{1}{1+x^2}}$$

$$\frac{\sin x}{e^{2x}} = \frac{1}{e^{2x}} \sin x$$
$$= e^{-2x} \cdot \sin x \quad \ldots$$

$$10^{-2x} = \left(e^{\ln 10}\right)^{-2x}$$
$$= e^{-2\ln 10\, x} \quad \ldots$$

#10 gjennomgått tidligere. (mandag 8.03?)



$\tan(x-a)+b$

Forskyver grafen til $\tan x$

a til høgre og b oppover...

oblig 6 #11 Deriver

$\left(\ln(a/b) = \ln(a \cdot b^{-1}) = \ln a + \ln b^{-1} = \ln a - \ln(b) \right)$

$$f(x) = \sqrt{3e^{-5x^2}} + \ln\left(x^{15}/(1-x)^3\right) + \frac{1}{(4^x)^3}$$

$$f(x) = \left(3e^{-5x^2}\right)^{1/2} + \ln\left(x^{15}\cdot(1-x)^{-3}\right) + \frac{1}{4^{3\cdot x}}$$

$$= \sqrt{3}\, e^{-5x^2/2} + \ln(x^{15}) + \ln(1-x)^{-3} + (4^{3x})^{-1}$$

$$= \sqrt{3}\, e^{-\frac{5}{2}x^2} + 15\ln x + (-3)\ln(1-x) + 4^{-3x}$$

$$f'(x) = \sqrt{3}\, e^{-\frac{5}{2}x^2}\left(-\frac{5}{2}x^2\right)' + 15\cdot\frac{1}{x} + (-3)\frac{1}{1-x}(1-x)' + \left(e^{-3\ln 4\cdot x}\right)'$$

$$= \sqrt{3}\, e^{-\frac{5}{2}x^2}(-5x) + \frac{15}{x} + \frac{3}{1-x} + (-3\ln 4)\, e^{-3\ln 4\cdot x}$$

$$f'(x) = -\sqrt{3}\cdot 5\cdot x\, e^{-\frac{5}{2}x^2} + \frac{15}{x} + \frac{3}{1-x} - 3\ln 4\cdot\frac{1}{4^{3x}}$$

$$\left(\sqrt{e^{2x}}\right)' = \frac{1}{2\sqrt{e^{2x}}} \cdot \underbrace{\left(e^{2x}\right)'}_{2e^{2x}}$$

$$= \frac{e^{2x}}{\sqrt{e^{2x}}} = \frac{e^{2x}}{\left(e^{2x}\right)^{1/2}} = \frac{e^{2x}}{e^{x}}$$

$$= e^{2x} \cdot e^{-x} = e^{2x-x}$$

$$= \underline{e^{x}}$$

$$\sqrt{e^{2x}} = e^{2x/2} = e^{x}$$

$$\left(\sqrt{e^{2x}}\right)' = \left(e^{x}\right)' = e^{x} \quad \checkmark$$

$$\frac{e^{\ln 10 \cdot 2x} A + e^{\ln 10 \cdot 2x} \cdot B}{e^{\ln 10 \cdot 4x}}$$

$$= \frac{10^{2x} \cdot (A+B) \cdot 10^{-2x}}{10^{4x} \cdot 10^{-2x}} = \underline{\frac{A+B}{10^{2x}}}$$

$$\frac{e^{2x}}{e^x} = e^x$$

$$e^{2x} = (e^x)^2$$

1) $$\frac{(e^x)^2}{e^x} = e^x \checkmark$$

2) $$e^{2x} \cdot \frac{1}{e^x} = e^{2x} \cdot (e^x)^{-1}$$

$$= e^{2x} \cdot e^{-x} = e^{2x + (-x)} = e^x \checkmark$$