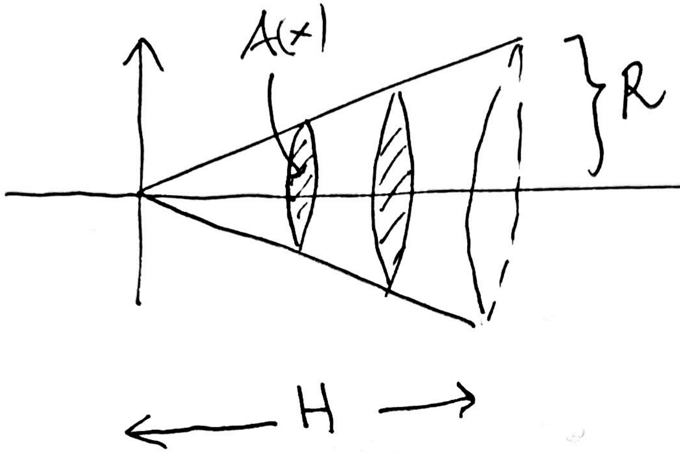
15 mars 2021

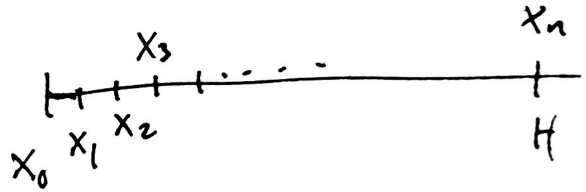
16.1 Volum.



Kjegle

Volumet til disken $\sim A(x) \cdot \Delta x$

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i)\,\Delta x_i$$



$\Delta x_i = x_i - x_{i-1}$ bredde.

$$V = \int_0^H A(x)\,dx$$

$y = \frac{R}{H} \cdot x$

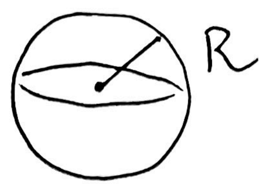
Areal $\pi \cdot y(x)^2 = \pi \left(\frac{R}{H}\right)^2 \cdot x^2$

$$V = \int_0^H \pi \frac{R^2}{H^2} x^2\,dx$$

$$= \frac{\pi R^2}{H^2} \int_0^H x^2\,dx = \frac{\pi R^2}{H^2} \left(\frac{x^3}{3}\right)\Big|_0^H$$

$$= \frac{\pi R^2}{H^2} \left(\frac{H^3}{3} - 0\right) = \underline{\underline{\frac{\pi R^2 \cdot H}{3}}}$$

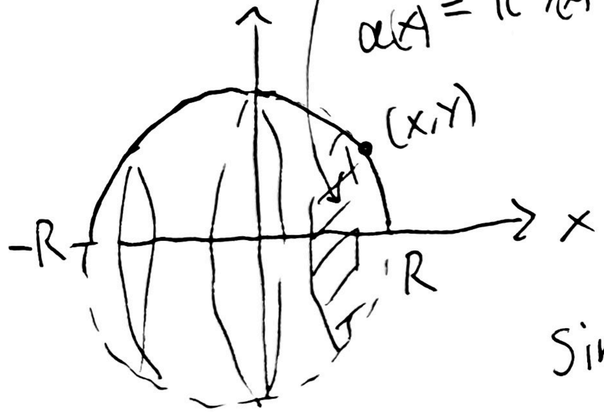
Volum til en kule



Volumet til en kule

$$V = \underline{\frac{4\pi}{3} R^3} \sim \underline{4{,}18879\ldots} \cdot R^3$$

Tverrsnitt arealet $a(x) = \pi y_{(x)}^2$



"Rotasjonslegemet til sirkelen er kulen"

Sirkel i x-y-planet med senter origo og radius R

$$x^2 + y^2 = R^2 \quad , \quad y_{(x)}^2 = R^2 - x^2$$

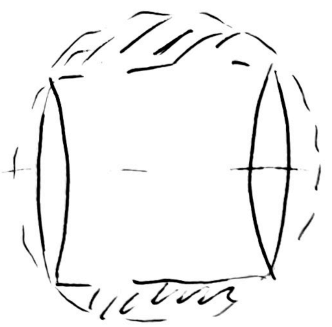
y positiv : $y_{(x)} = \sqrt{R^2 - x^2}$

$$V_{kule} = \int_{-R}^{R} a(x)\,dx = \int_{-R}^{R} \pi\left(R^2 - x^2\right) dx \quad \left(= 2\int_0^R \pi\left(R^2 - x^2\right) dx\right)$$

$$= \pi\left(R^2 x - \frac{x^3}{3}\right)\Big|_{-R}^{R} = \pi\left[\left(R^3 - \frac{R^3}{3}\right) - \left(-R^3 - \frac{(-R)^3}{3}\right)\right]$$

$$= \pi\left[\frac{2R^3}{3} - \left(-\frac{2R^3}{3}\right)\right]$$

$$= \underline{\underline{\frac{4\pi R^3}{3}}}$$



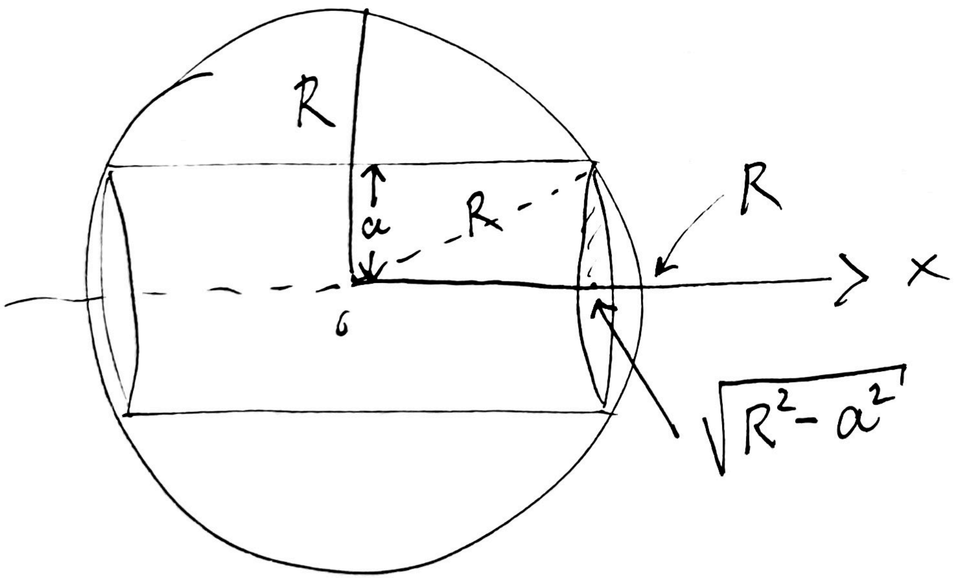
R radius

Borer ut en sylinder med radius $a \leq R$ (symmetrisk i kula)

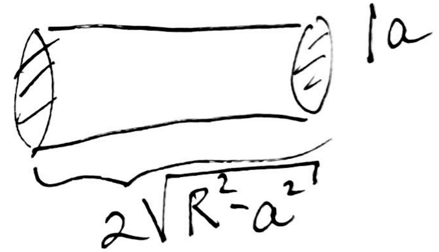
Beskriv volumet til den uthula sylinderen. $V(R,a)$

$V(R,R) = 0$

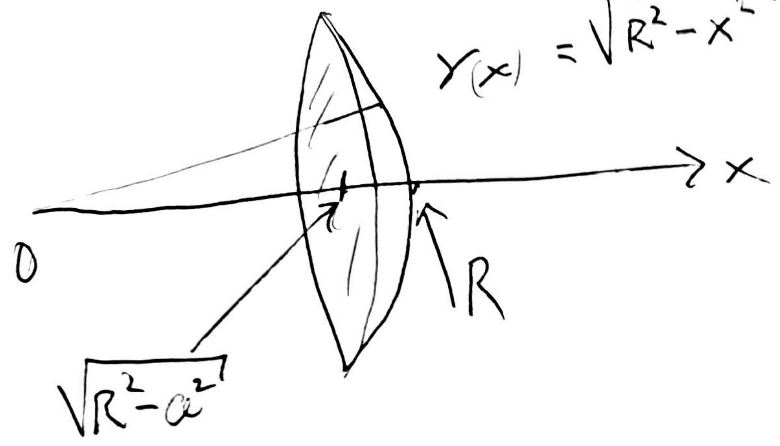
$$V(R,0) = \frac{4\pi}{3} R^3.$$



Volum til sylinderen



er $2\pi a^2 \sqrt{R^2 - a^2}$



$$y_{(x)} = \sqrt{R^2-x^2}$$

$$V_{Topp} = \int_{\sqrt{R^2-a^2}}^{R} \pi\, y_{(x)}^2 \, dx$$

$$= \pi \int_{\sqrt{R^2-a^2}}^{R} R^2 - x^2 \, dx$$

$$V_{topp} = \pi \left(R^2 x - \frac{x^3}{3} \right) \Big|_{\sqrt{R^2-a^2}}^{R}$$

$$= \pi \left(\left(R^3 - \frac{R^3}{3} \right) - \left(R^2 \sqrt{R^2-a^2} - \frac{1}{3} \sqrt{R^2-a^2} \left(R^2 - a^2 \right) \right) \right)$$

$$= \pi \left(\frac{2R^3}{3} - \sqrt{R^2-a^2} \left(\underbrace{R^2 - \frac{1}{3} R^2}_{2R^2/3} + \frac{a^2}{3} \right) \right)$$

$$V_{topp} = \pi \left(\frac{2R^3}{3} - \frac{2R^2}{3} \sqrt{R^2-a^2} - \frac{a^2}{3} \sqrt{R^2-a^2} \right)$$

$$V = V_{kule} - V_{sylinder} - 2V_{topp}$$

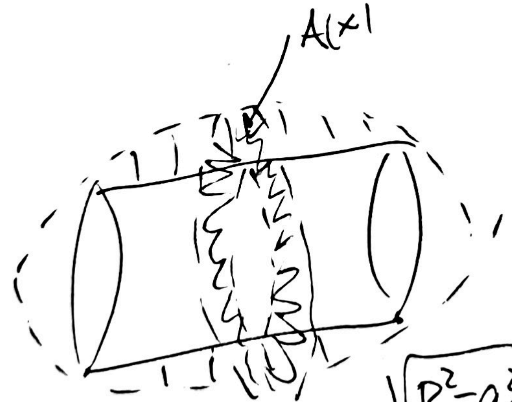
$$= \frac{4\pi R^3}{3} - 2\pi a^2\sqrt{R^2-a^2} - 2\cdot\pi\left(\frac{2\cdot R^3}{3} - \frac{2R^2}{3}\sqrt{R^2-a^2} - \frac{a^2}{3}\sqrt{R^2-a^2}\right)$$

$$= 2\pi\sqrt{R^2-a^2}\left(-a^2 + \frac{a^2}{3} + \frac{2R^2}{3}\right)$$

$$\boxed{V = \frac{4\pi(R^2-a^2)\sqrt{R^2-a^2}}{3}}$$

test $V(R,0) = \frac{4\pi R^3}{3}$ ✓ $\quad V(R,R) = 0$ ✓

Alternativt



$$V = \int_{-X}^{X} \overbrace{\pi(R^2-x^2) - \pi a^2}^{A(x)}\, dx$$

$y(x) = \sqrt{x}$

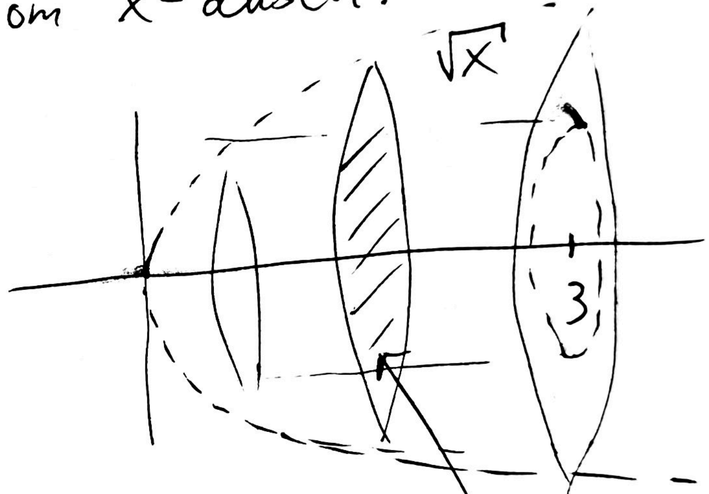
Finn volumet til rotasjonslegemet avgrenset av

$x = 0$ $\quad x = 3$ og legemet som fremkommer ved å

rotere $y(x)$ om x-aksen.

$y = \sqrt{x}$

$y^2 = x$



$A(x)$ tverrsnitt-arealet

$A(x) = \pi(\sqrt{x})^2 = \pi x.$

$$V = \int_0^3 A(x)\,dx$$

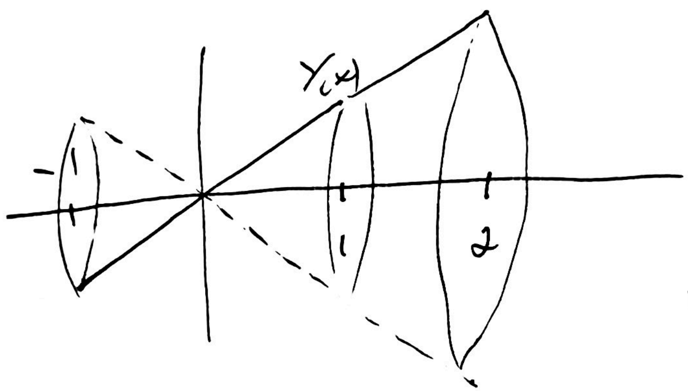
$$= \int_0^3 \pi \cdot x\,dx$$

$$= \pi \left.\frac{x^2}{2}\right|_0^3 = \pi \frac{9}{2} = \underline{\underline{\frac{9\pi}{2}}}$$

oppgave

$Y(x) = x$ $\quad a = -1 \quad b = 2$

Volum til rotasjonslegemet som frem kommer ved å rotere $Y(x)$ om x-aksen fra $a=-1$ til $b=2$.



$V_{storkjegle} = \frac{2 \cdot \pi \cdot 2^2}{3} = \frac{8\pi}{3}$

$V_{litenkjegle} = \frac{1 \cdot \pi \cdot 1^2}{3} = \frac{\pi}{3}$

$V = V_{storkjegle} + V_{litenkjegle} = \frac{8\pi}{3} + \frac{\pi}{3}$

$V = \underline{3\pi}$

$A(x) = \pi \cdot Y(x)^2 \; (= \pi \, |Y(x)|^2)$

$V = \int_{-1}^{2} \pi \cdot (x)^2 dx \quad = \pi \int_{-1}^{2} x^2 dx \quad = \pi \frac{x^3}{3} \Big|_{-1}^{2}$

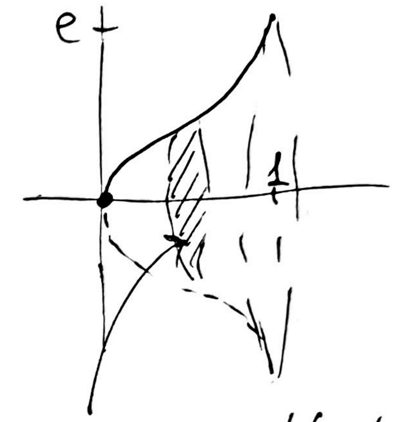
$= \pi \left(\frac{2^3}{3} - \frac{(-1)^3}{3} \right) = \pi \left(\frac{8}{3} - \frac{-1}{3} \right)$

$= \pi \left(\frac{9}{3} \right) = \underline{\underline{3\pi}}$ ✓

Oppg 4

Prøveeksamen 2021.

$f(x) = \sqrt{x}\, e^{x^2} \qquad x \in [0,1]$



$A(x) = \pi \cdot |f(x)|^2$

areal til snittet i x

$$V = \int_0^1 \underbrace{\pi f(x)^2}_{\text{areal}} \, dx$$

$$= \pi \int_0^1 (\sqrt{x})^2 (e^{x^2})^2 \, dx$$

$$V = \pi \int_0^1 x\, e^{2x^2} \, dx$$

$$= \pi \left(\frac{1}{4} e^{2x^2}\right)\Big|_0^1$$

$$= \frac{\pi}{4}\left(e^2 - 1\right)$$

$$\approx \underline{5{,}01795..}$$

$$(e^{2x^2})' = e^{2x^2} \cdot (2x^2)'$$

$$= 4x\, e^{2x^2}$$

Så $\left(\frac{1}{4} e^{2x^2}\right)' = \frac{1}{4}\left(e^{2x^2}\right)' = x e^{2x^2}.$

15.81 I dag $F_0 = 50\,000$ barn/år

Fødselstallene øker med 1.2% hvert år.

a) Fødselstallet om 30 år.

$$F(t) = F_0\,(1{,}012)^{t}$$

t tiden i år

nå: $t = 0$

$$F(30) = F_0\,(1{,}012)^{30}$$

$$= 71\,513 \text{ barn/år}$$

b) $F(0)\cdot 1\text{år} + F(1)\cdot 1\text{år} + \dots + F(29)\cdot 1\text{år}$

$$F_0\left(1 + (1{,}012) + (1{,}012)^2 + \dots (1{,}012)^{29}\right) = F_0\,\frac{(1{,}012)^{30} - 1}{1{,}012 - 1}$$

$$= 1\,792\,755$$

$$\approx 1{,}79 \text{ millioner}$$

c) $\dfrac{1{,}792755 \text{ mill}}{30\text{ år}} = 59\,759 \text{ barn/år}.$

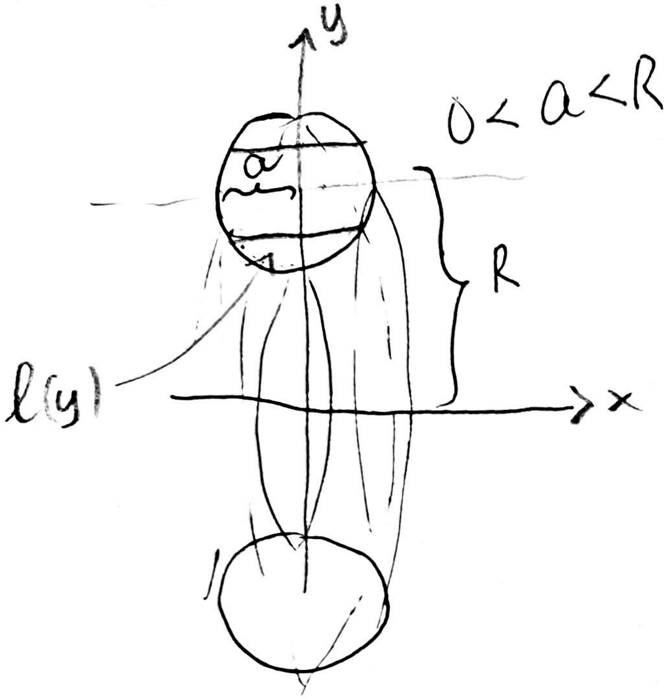
$F(t) = F_0 \, (1{,}012)^t$ $\qquad$ $t \geq 0$ reell tid (ikke bare hele år)

Barn født fra $t=0$ til $t=30$

$$\int_0^{30} F_0 \, (1{,}012)^t \, dt = F_0 \int_0^{30} e^{\ln 1{,}012 \cdot t} \, dt$$

$$= F_0 \left. \frac{e^{\ln(1{,}012) \cdot t}}{\ln(1{,}012)} \right|_0^{30} = F_0 \frac{1}{\ln(1{,}012)} \left((1{,}012)^{30} - 1 \right)$$

$$= 1{,}803490 \text{ mill barn} \sim \underline{1{,}80 \text{ mill barn}}$$



$0 < a < R$

TORUS.

$a < R$

$\pi a^2 \cdot 2\pi R$?

$$l(R+y) = l(R-y)$$

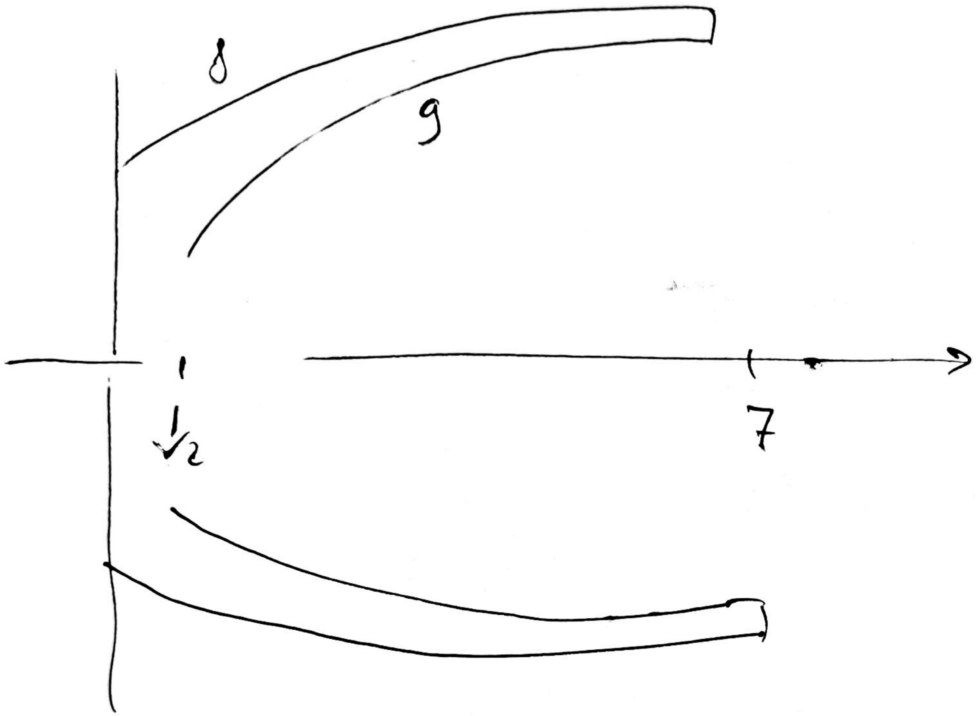
$$\Delta V = 2\pi\left(l(R-y)(R-y)\Delta y + l(R+y)(R+y)\Delta y\right)$$

$$= 2\pi \cdot 2\, l(R+y) \cdot R \Delta y$$

$$V = \int_0^a 2\pi \cdot 2\, l(R+y) \cdot R\, dy = 2\pi \cdot 2 \cdot R \overbrace{\int_0^a l(R+y)\, dy}^{\pi a^2/2}$$

$$= \underline{2\pi^2 a^2 R}$$

16.13



$g(x) = \sqrt{x - \frac{1}{2}} + 1 \qquad x \in [\frac{1}{2}, 7]$

$f(x) = \sqrt{x+1} + 1 \qquad x \in [0, 7]$

a) Volumet glasset rommer:

$$V = \int_{1/2}^{7} \pi g^2(x)\, dx$$

$$= \pi \int_{1/2}^{7} \left(\left(\sqrt{x - \tfrac{1}{2}}\right) + 1\right)^2 dx$$

$$V = \pi \int_{1/2}^{7} x - \frac{1}{2} + 1 + 2\sqrt{x - \frac{1}{2}}\, dx$$

$$= \pi \left(\frac{x^2}{2} + \frac{x}{2} + 2\, \frac{(x - \frac{1}{2})^{3/2}}{3/2 \cdot 1} \right) \Bigg|_{1/2}^{7}$$

$$= \pi \left(\frac{49 + 7 - 1/4 - 1/2}{2} + \frac{4}{3}\left(\left(\frac{13}{2}\right)^{3/2} - 0\right)\right) = \pi\left(28 - \frac{3}{8} + \frac{4}{3}\left(\frac{13}{2}\right)^{3/2}\right) \text{ cm}^3$$

$$V \sim 156.20 \text{ cm}^3 = \underline{0{,}1562 \text{ Liter}}$$

b) Volum til materialet i glasset er

$$\underbrace{\int_0^7 \pi \cdot f^2\,dx}_{\text{ytre volum}} - \underbrace{V = \int_{1/2}^7 \pi g_{(x)}^2\,dx}_{\text{volumet glassetromme}}$$

$$\text{ytre volum} = \pi \int_0^7 \left(\sqrt{x+1}+1\right)^2 dx = \pi \int_0^7 x+1+1+2\sqrt{x+1}\,dx$$

$$= \pi\left(\frac{x^2}{2} + 2x + 2\,\frac{(x+1)^{3/2}}{3/2 \cdot 1}\right)\Big|_0^7$$

$$= \pi\left(\frac{49}{2} + 14 + \frac{4}{3}\left(8^{3/2} - 1\right)\right)$$

$$\approx 211{,}54\,cm^3.$$

Volum til glassmaterialet i glasset er da $55{,}34(168)\ cm^3$.

$$\left(\begin{array}{l} 1dm = 10cm \\ (1dm)^3 = 1000\,cm^3 \\ = 1\,\text{Liter} \end{array}\right)$$

Egenvekten til glasset $2{,}5\,kg/dm^3$.

vekten til glasset : $2.5\,kg/dm^3 \cdot 0{,}05534 \approx 0.1383\,kg$

$= 138.3\,gram$