16 mars 2021

16.2 Substitusjon, variabelbytte, variabelskifte

$F'(x) = f(x)$ F er da en antiderivert til $f(x)$

$\int f(x)\,dx = F(x) + C$ ubestemt integral av $f(x)$

$$F(u(x))' = F'(u(x)) \cdot u'(x) \quad \text{kjerneregelen}$$
$$= f(u(x)) \cdot u'(x)$$

Så $F(u(x))$ er en antiderivert til $f(u(x)) \cdot u'(x)$

$$\int f(u(x)) \cdot u'(x)\,dx = F(u(x)) + C$$
$$= \int f(u)\,du$$

$$\int \underbrace{3x^2}_{u'(x)} \cdot (\underbrace{4+x^3}_{u(x)})^5 \, dx = \int u^5 \, du$$

$$= \frac{u^6}{6} + C$$

$$= \frac{(4+x^3)^6}{6} + C$$

$u(x) = 4+x^3$

$u'(x) = 3x^2$

$$\int x e^{-x^2} dx$$

$u(x) = -x^2$

$\frac{u'(x)}{-2} = \frac{-2x}{-2}$

$\frac{-1}{2} \cdot u' = x$

$$\int \frac{-1}{2} u' e^u \, dx$$

$$= \frac{-1}{2} \int e^u \, du$$

$$= \frac{-1}{2} e^u + C = \frac{-1}{2} e^{-x^2} + C$$

$$\int e^{x^2}\,dx$$

$u = x^2 \quad , \quad x = \sqrt{u}$

$(x \geq 0)$

$u' = 2x$

$\frac{du}{dx} = 2x$

$\underline{du} = 2x\,dx = \underline{u'\,dx}$

$dx = \frac{1}{u'}\,du$

$$= \int e^u \frac{1}{u'}\,du$$

$$= \int e^u \frac{1}{2x}\,du$$

$$= \int e^u \cdot \frac{1}{2\sqrt{u}}\,du \quad ??$$

$u = \cos x$

$u' = -\sin x$

oppg

$$\int \underbrace{\sin x}_{-u'}\ \underbrace{e^{\cos x}}_{e^u}\,dx$$

$$= \int -u' e^u\,dx \underset{\text{subshitusjon}}{=} -\int e^u\,du = -e^u + C = \underline{\underline{-e^{\cos x} + C}}$$

oppg. $\displaystyle\int \frac{e^x}{1+e^x}\,dx = \int \frac{u'}{u}\,dx$

$\displaystyle = \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln|1+e^x| + C$

$u = 1+e^x$

$u' = e^x$

$\Big(\int \underbrace{f(x)}_{\text{integranden}}\,dx$ integranden "det som skal integreres" $\Big)$

$\displaystyle\int \frac{1}{1+e^{-x}}\,dx = \int \frac{1}{1+e^{-x}} \cdot \frac{e^x}{e^x}\,dx$

$\displaystyle = \int \frac{e^x}{e^x + e^{-x} \cdot e^x}\,dx = \int \frac{e^x}{1+e^x}\,dx$ som tidligere.

$$\int x\sqrt{x+1}\,dx$$

$x+1 = u$ så $x = u-1$

$u' = \frac{du}{dx} = 1$

så $du = dx$

$$= \int (u-1)\sqrt{u}\,du$$

$$= \int u^{3/2} - u^{1/2}\,du$$

$$= \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + c$$

$$= \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + c$$

$$= \frac{2}{5}(x+1)^{2}\sqrt{x+1} - \frac{2}{3}(x+1)\sqrt{x+1} + C$$

$$\left(\frac{1}{5/2} \cdot \overset{=1}{\left(\frac{2}{2}\right)} = \frac{2}{5}\right)$$

$$\frac{a}{b} \cdot \frac{b}{a} = 1$$

$$\frac{1}{a/b} = \left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

$$u = 1 - x^2$$
$$u' = -2x$$
$$x = -\frac{1}{2} \cdot u'$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx =$$
$$= \int -\frac{1}{2} \cdot u' \frac{1}{\sqrt{u}}\,dx$$
$$= -\frac{1}{2} \int \frac{1}{\sqrt{u}}\,du$$
$$= -\frac{1}{2} \int u^{-1/2}\,du$$
$$= -\frac{1}{2} \cdot \frac{u^{1/2}}{1/2} + c$$
$$= -u^{1/2} + c = \underline{-\sqrt{1-x^2} + c}$$

oppg $\int \underbrace{\cos x}_{u'} \underbrace{\sin^5 x}_{u^5}\, dx$

prøver med $\sin x = u$

$u' = \cos x$

$= \int u' \cdot u^5\, dx$

$= \int u^5\, du = \frac{u^6}{6} + C$

$= \frac{\cos^6 x}{6} + C$

$u = \sin x$

$\int \cos^3 x \sin^5 x\, dx = \int \underbrace{\cos x}_{u'} \cdot \underbrace{\underbrace{\cos^2 x}_{1-\sin^2 x}}_{1-U^2} \cdot \underbrace{\sin^5 x}_{U^5}\, dx$

$= \int (1-U^2)\, U^5\, dU$

$= \int U^5 - U^7\, dU = \frac{U^6}{6} - \frac{U^8}{8} + C$

$= \frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + C$

$$\int \frac{1}{x} \ln|x^3| \, dx = \int \frac{1}{x} \cdot 3 \ln|x| \, dx$$

$u = \ln|x|$

$u' = \frac{1}{x}$

$$= 3 \int \underbrace{\frac{1}{x}}_{u'} \underbrace{\ln|x|}_{u} \, dx$$

$$= 3 \int u \, du = 3 \, \frac{u^2}{2} + c$$

$$= \underline{\frac{3}{2} \left(\ln|x|\right)^2 + c}$$

$u = \cos x$

$u' = -\sin x$

oppg. $\int \frac{\sin x}{\cos x} \, dx = \int \tan x \, dx$

$$= \int \frac{-u'}{u} \, dx = - \int \frac{1}{u} \, du$$

$$= -\ln|u| + c = \underline{-\ln|\cos x| + c}$$

16.3 Substitusjon og bestemte integral.

$$\int_a^b f(u(x)) \cdot u'(x)\,dx = F(u(x))\Big|_a^b$$

$$= F(u(b)) - F(u(a))$$

$$= \int_{u(a)}^{u(b)} f(u)\,du$$

$u = 1 + x^2$

$u' = 2x$

$\frac{1}{2}u' = x$

$$\int_0^2 x(1+x^2)^8\,dx$$

$$\int_0^2 \frac{1}{2}u' \cdot u^8\,dx = \frac{1}{2}\int_{u(0)}^{u(2)} u^8\,du = \frac{1}{2}\int_1^5 u^8\,du$$

$$\frac{1}{2} \cdot \frac{u^9}{9}\Big|_1^5 = \frac{1}{18}(5^9 - 1) = \frac{1}{18}(1953124)$$

$$\approx 108506{,}94\ldots$$

$$u' = \frac{du}{dx}$$

$$\big(F(u)\big)' = F'(u) \cdot u' \qquad \boxed{u'\,dx = du} \qquad F' = f$$

$$\frac{d\,F(u(x))}{dx} = \frac{dF}{du} \cdot \frac{du}{dx} = F'(u_x) \cdot u'$$

$$\int f(u(x))\, \underbrace{u'(x)\,dx}_{du} = \int f(u(x))\, du$$

$$u'\,dx = du$$

$$\frac{du}{dx}\,dx = du$$

oppgave

$$\int_0^{\pi/2} \sin x \cdot \cos^3 x \, dx$$

$u = \cos x$

$u' = -\sin x$

$$\int_0^{\pi/2} -u' \cdot u^3 \, dx = -\int_{u(0)}^{u(\pi/2)} u^3 \, du$$

$$\left(\int_a^b f(x)dx = -\int_b^a f(x)dx \right)$$

$$= \int_1^0 -u^3 \, du \quad \left(= -\int_0^1 -u^3 \, du \right)$$

$$= -\int_1^0 u^3 \, du = -\frac{u^4}{4} \Big|_1^0 = -\frac{1}{4}(0 - 1^4) = \underline{\underline{\frac{1}{4}}}$$

$$\int_0^2 \frac{x^2}{1+x^3} \, dx$$

, finner det ubestemte integralet:

$u = 1+x^3$

$u' = 3x^2$

$\frac{1}{3}u' = x^2$

$$\int \frac{x^2}{1+x^3} dx = \int \frac{1}{3} u' \cdot \frac{1}{u} dx$$

$$= \frac{1}{3} \int \frac{1}{u} du = \frac{1}{3} \ln|u| + C$$

$$= \frac{1}{3} \ln|1+x^3| + C$$

$$\int_0^2 \frac{x^2}{1+x^3}\,dx = \left[\frac{1}{3} \ln|1+x^3| \Big|_0^2 \right]$$

$$= \frac{1}{3}\left(\ln(1+2^3) - \ln(1+0)\right)$$

$$= \frac{1}{3}\ln(9) = \frac{1}{3}\ln(3^2) = \frac{2}{3}\ln(3)$$

$$\sim 0.73241$$

Alternativt

$$\int_0^2 \frac{x^2}{1+x^3}\,dx = \int_0^2 \frac{1}{3} u' \cdot \frac{1}{u}\,dx$$

$$= \int_{u(0)}^{u(2)} \frac{1}{3} \cdot \frac{1}{u}\,du = \frac{1}{3}\int_1^9 \frac{1}{u}\,du$$

$$= \frac{1}{3}\ln|u| \Big|_1^9 = \frac{1}{3}(\ln 9 - \ln 1)$$

$$= \underline{\frac{2}{3}\ln 3}$$

16.31 c) $\int_0^3 4x\, 2^{x^2+3}\, dx$

$u = x^2+3$

$u' = 2x$

$= \int_0^3 2u'\, 2^u\, dx$

$= 2\int_{u(0)}^{u(3)} 2^u\, du = 2\int_3^{12} 2^u\, du$

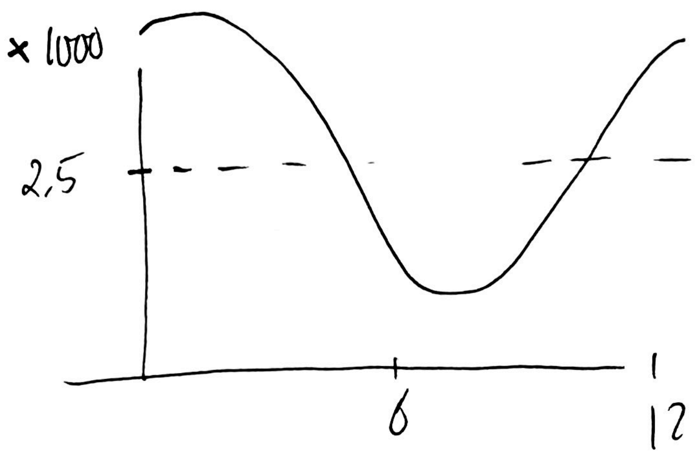
$\left(2^u = \left(e^{\ln 2}\right)^u = e^{u \cdot \ln 2} \right)$

$= 2\int_3^{12} e^{(\ln 2)\cdot u}\, du = 2 \left. \frac{e^{(\ln 2)\cdot u}}{\ln 2} \right|_3^{12}$

$= \frac{2}{\ln 2}\left(2^{12} - 2^3\right) = \frac{2}{\ln 2}(4096 - 8) = \frac{2}{\ln 2}(4088)$

$\sim \underline{11795,47..}$

16.34



$$S(x) = 2500 - 1500\sin\left(\frac{\pi x}{6} - \frac{2\pi}{3}\right) \frac{kWh}{mnd.}$$

$$x \in [0, 12]$$

I løpet av året går vi gjennom 1 periode for sin.

Så $\int_0^{12} \sin\left(\frac{\pi x}{6} - \frac{2\pi}{3}\right)dx = 0$

a) Strømforbruk

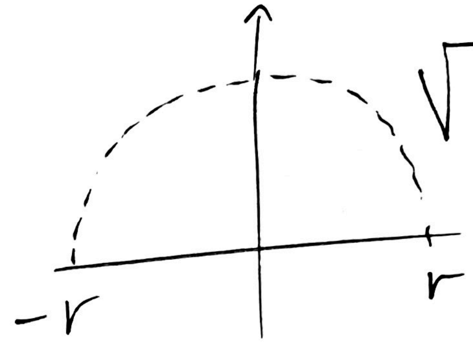
$$2500 \frac{kWh}{mnd} \cdot 12 mnd$$

$= 25000$ kwh + 5000 kwh

$= \underline{30\ 000 \text{ kWh}}$

b) Gjennomsnittlig strømforbruk per måned er $\underline{2500 \text{ kWh/mnd}}$

16.12 Volum til kule
gjennomgått på mandag



$\sqrt{r^2-x^2} = y$

$r^2 - x^2 = y^2$

$x^2+y^2=r^2$

$|[x,y]|^2 = r^2$

oppg.

$$\int \underbrace{x^3}_{\frac{1}{4}u'} (\underbrace{1+x^4}_{u})^5 \, dx \qquad u = 1+x^4, \quad u' = 4x^3, \quad \tfrac{1}{4}u' = x^3$$

$$= \frac{1}{4}\int u' u^5 \, dx$$

$$= \frac{1}{4}\int u^5 \, du = \frac{1}{4}\cdot\frac{u^6}{6} + C$$

$$= \frac{1}{24}(1+x^4)^6 + C$$

$$\left(\text{test}: \left(\frac{1}{24}(1+x^4)^6\right)' = \frac{1}{24}\, 6(1+x^4)^5 \cdot \overbrace{(1+x^4)'}^{4x^3} = \frac{24}{24}(1+x^4)^5 \cdot x^3 \ \checkmark\right)$$

$$\int (2+x)^7 \cdot x^2 \, dx$$

prøver med: $2+x=u$, $x=u-2$

$u'=1$

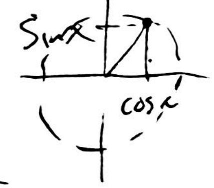
$du=dx$

$$= \int u^7 \underbrace{(u-2)^2}_{u^2-4u+4} du$$

$$= \int u^7(u^2-4u+4)\,du = \int u^9 - 4u^8 + 4u^7 \, du$$

$$= \frac{u^{10}}{10} - 4\frac{u^9}{9} + 4\frac{u^8}{8} + c$$

$$= \frac{(2+x)^{10}}{10} - \frac{4(2+x)^9}{9} + \frac{(2+x)^8}{2} + c$$



Opg

Pyt: $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$$\int \sin^3 x \, dx$$

$u = \cos x$

$u' = -\sin x$

$$\int \sin x(1-\cos^2 x)\,dx = \int \sin x \, dx - \int \underbrace{\cos^2 x}_{u^2} \cdot \underbrace{\sin x}_{(-u')} \, dx$$

$$= -\cos x - \left(-\int u^2 u' \cancel{dx}\right)$$

$$= -\cos x + \int u^2 du = -\cos x + \frac{u^3}{3} + c = -\cos x + \frac{\cos^3 x}{3} + c$$