17. mars 2021

16.4 Delvis integrasjon

$\int xe^x \, dx$?

$= xe^x - e^x + C$

$(xe^x)' = (x)' \cdot e^x + x \cdot (e^x)'$

$= e^x + xe^x$

$(xe^x)' - e^x = xe^x$

$(xe^x)' - (e^x)' = (xe^x - e^x)' = xe^x$

$(u \cdot v)' = u' \cdot v + u \cdot v'$ produktregelen for derivasjon.

$\int u' \cdot v \, dx + \int u \cdot v' \, dx = u \cdot v + C$

$\left(\int (u' \cdot v + u \cdot v') \, dx \right)$

$$\boxed{\int u' \cdot v \, dx = u \cdot v - \int u \cdot v' \, dx}$$

$$\int \underset{v}{x}\, \underset{u'}{e^x}\, dx = \underset{u}{(e^x+c)}\, \underset{v}{x} - \int \underset{v'}{1} \cdot \underset{u}{(e^x+c)}\, dx$$

$v = x$ | $u' = e^x$

$v' = 1$ | $u = e^x (+c)$

$$= xe^x + cx - \int e^x + c\, dx$$

$$= xe^x + cx - (e^x + c\cdot x) + k$$

$$\underline{= xe^x - e^x + k}$$

$= \underbrace{\text{antiderivert + konstant}}_{\text{klassen av alle antideriverte.}}$

$$\int \overset{v}{x^2}\, \overset{u'}{e^x}\, dx = \overset{v}{x^2}\, \overset{u}{e^x} - \int \overset{v'}{2x}\, \overset{u}{e^x}\, dx$$

$V = x^2$ | $u' = e^x$

$V' = 2x$ | velger $u = e^x$

$$= x^2 e^x - 2 \underbrace{\int x e^x\, dx}_{(xe^x - e^x)} + k$$

$$\int x^2 e^x\, dx = \underline{x^2 e^x - 2xe^x + 2e^x + C}$$

$$\int \underset{v}{x^n} \underset{u'}{e^x}\,dx = x^n e^x - n\int x^{n-1} e^x\,dx$$

$v' = nx^{n-1}$

$u = e^x$

$\int x^{14} e^x\,dx$ = bruk den rekursive formeln 14 ganger

(14-gradspolynom) $e^x + C$

oppg

$$\int \overset{v}{x} \overbrace{\sin(2x)}^{u'}\,dx = x\left(\frac{-\cos(2x)}{2}\right) - \int 1 \cdot \frac{-\cos(2x)}{2}\,dx$$

$$= -\frac{x}{2}\cos(2x) + \frac{1}{2}\int \cos(2x)\,dx$$

$$= \frac{-x}{2}\cos(2x) + \frac{1}{4}\sin(2x) + C$$

$v = x \quad v' = 1$

$u = \sin(2x)$

$u' = \frac{-\cos(2x)}{2}$

$$\int \ln|x|\,dx = \int \underset{u'}{1} \cdot \underset{v}{\ln|x|}\,dx$$

$u' = 1$ velger $u = x$
$v = \ln|x|$
$v' = 1/x$

delvis integrasjon

$$u \cdot v - \int u \cdot v'\,dx$$

$$= x \ln|x| - \int x \cdot \frac{1}{x}\,dx$$

$\left(x \cdot \frac{1}{x} = \frac{x}{x} = 1 \text{ for } x \neq 0 \right)$

$$= x \ln|x| - \int 1\,dx$$

$$= \underline{x \ln|x| - x + C}$$

oppg

$$\int (\ln|x|)^2\,dx = \int \underset{u'}{1} \cdot \overbrace{(\ln|x|)^2}^{v}\,dx$$

$\Big($ $u' = 1$
velger $u = x$
$v = (\ln|x|)^2$
$v' = 2\ln|x| \cdot \frac{1}{x}$

$$= x(\ln|x|)^2 - \int \underbrace{x \cdot 2\ln|x| \cdot \frac{1}{x}}_{x \cdot \frac{1}{x} = 1}\,dx$$

$$= x(\ln|x|)^2 - 2\underbrace{\int \ln|x|\,dx}_{(x\ln|x| - x)} = \underline{x(\ln(x|)^2 - 2x\ln|x| + 2x + C}$$

$$\int e^{2x} \sin 3x \, dx$$

$u' = e^{ax}$ $\quad u = \frac{1}{a} e^{ax}$

$v = \sin(bx)$ $\quad v' = b\cos(bx)$

$$\int \overset{u'}{e^{ax}} \overset{v}{\sin} bx \, dx$$

$z' = e^{ax}$

$z = \frac{1}{a} e^{ax}$

$w = \cos bx$

$w' = -b \sin bx$

$$= \underset{u}{\frac{1}{a} e^{ax}} \underset{v}{\sin bx} - \int \underset{u}{\frac{1}{a}} \overbrace{e^{ax}}^{z'} \underset{v'}{(b \overbrace{\cos bx}^{w})} \, dx$$

$$= \frac{1}{a} e^{ax} \sin bx - \frac{b}{a} \underbrace{\int \overset{z'}{e^{ax}} \overbrace{\cos bx}^{w} \, dx}_{\frac{1}{a} e^{ax} \cos bx - \int \frac{1}{a} e^{ax} (-b \sin bx) dx}$$

$$\int e^{ax} \sin bx \, dx = \frac{1}{a} e^{ax} \sin bx - \frac{b}{a^2} e^{ax} \cos bx - \frac{b^2}{a^2} \int e^{ax} \sin bx \, dx$$

flytter over

$$\left(1 + \frac{b^2}{a^2}\right) \int e^{ax} \sin bx \, dx = \frac{1}{a} e^{ax} \sin bx - \frac{b}{a^2} e^{ax} \cos bx + C$$

Så

$$\int e^{ax} \sin bx \, dx = \frac{1}{1 + (b^2/a^2)} \left[\frac{1}{a} e^{ax} \sin bx - \frac{b}{a^2} e^{ax} \cos bx \right] + C$$

$$\int e^{2x}\sin 3x\,dx = \frac{1}{1+9/4}\left[\frac{1}{2}e^{2x}\sin 3x - \frac{3}{4}e^{2x}\cos 3x\right] + C$$

$a=2$
$b=3$

$u = x^2$
$u' = 2x$

$\int f(u(x))\cdot \underbrace{u'\,dx}$

$\int f(u)\,du$

tenk på det som
$u'dx = \frac{du}{dx}dx = du$

$$\int x^3 e^{x^2}\,dx$$

$$= \int \underset{u'}{2x}\,\underset{\left(\frac{1}{2}\cdot u e^{u}\right)}{\left(\frac{1}{2}x^2 e^{x^2}\right)}\,dx$$

$$= \int \frac{1}{2} u e^{u}\,du = \frac{1}{2}\int u e^{u}\,dx$$

delvis integrasjon

$$= \frac{1}{2}\left(ue^{u} - e^{u}\right) + C$$

$$= \frac{x^2}{2}e^{x^2} - \frac{e^{x^2}}{2} + C$$

$$\int 3x\, e^{2x+1}\, dx = 3\int x\, e^{2x} \cdot e^{1}\, dx \qquad u' = e^{2x}$$

$$= 3e \int \underbrace{x}_{v}\, \underbrace{e^{2x}}_{u'}\, dx \qquad u = \frac{1}{2} e^{2x}.$$

$$= 3e\left(\frac{x}{2} e^{2x} - \int 1 \cdot \frac{1}{2} e^{2x}\, dx \right)$$

$$= 3e\left(\frac{x}{2} e^{2x} - \frac{1}{2}\left(\frac{1}{2} e^{2x} \right)\right) + c$$

$$= 3e\left(\frac{x}{2} e^{2x} - \frac{1}{4} e^{2x} \right) + c$$

$$= \frac{3e}{4}(2x - 1)\, e^{2x}$$

$$\int \underbrace{(3x-5)}_{v}\, \underbrace{e^{-x}}_{u'}\, dx = u \cdot v - \int v' u\, dx \qquad v = 3x-5$$

$$= -(3x-5)e^{-x} - \int 3(-e^{-x})\, dx \qquad v' = 3 \qquad u' = e^{-x} \qquad u = -e^{-x}$$

$$= -(3x-5)e^{-x} + 3\int e^{-x}\, dx = -(3x-5)e^{-x} - 3e^{-x} + C$$

$$= (-(3x-5) - 3)\, e^{-x} + c = \underline{(-3x+2)\, e^{-x} + c}$$

oppg. $\int x^{15} \ln |x^{12}|\, dx$

$\ln a^b = b \ln a$

prøver $x^3 = u$ $\quad x^{12} = x^{3 \cdot 4} = (x^3)^4 = u^4$

$u' = 3x^2 \ldots$ $\quad x^{15} = (x^3)^5 = u^5$

$= 3u^{2/3} \ldots$

$\ln |x^{12}| = 12 \ln |x|$

prøver noe annet :

" $\ln |x|^{12}$ "

$V = \ln |x|, \quad V' = \frac{1}{x}$

$u' = x^{15} \quad u = \frac{x^{16}}{16}$

$12 \int \underset{u'}{x^{15}} \underbrace{\ln |x|}_{V} dx$

$12 \left[\frac{x^{16}}{16} \ln |x| - \int \overbrace{\frac{x^{16}}{16} \cdot \frac{1}{x}}^{\frac{1}{16} x^{15}} dx \right]$

$12 \left[\frac{x^{16}}{16} \ln |x| - \frac{1}{16} \left(\frac{x^{16}}{16} \right) \right] + C$

$= 12 \left[\frac{x^{16}}{16} \ln |x| - \frac{3}{64} x^{16} + C \right.$

$= \frac{3}{4} x^{16} \ln |x| - \frac{3}{64} x^{16} + C$