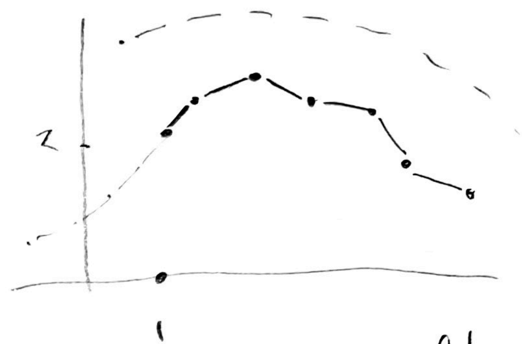
24 mars 2021

$$y'(2x-1) = 3y+2$$

1 ordens diff. likning.

$$y' = \frac{3y+2}{2x-1}$$



$y(1) = 2$ randbetingelse

separabel diff. likning.

$$\underset{\text{bare } y}{\frac{y'}{3y+2}} = \underset{\text{bare}}{\frac{1}{2x-1}}$$

$$\int \frac{y'}{3y+2}\,dx = \int \frac{1}{2x-1}\,dx$$

$u = 2x-1$

$u' = 2$

$du = 2\,dx \;,\; \frac{1}{2}du = dx$

$$\int \frac{1}{3y+2}\,dy = \int \frac{1}{2x-1}\,dx$$

$$= \int \frac{1}{u}\cdot\frac{1}{2}du$$

$$\frac{1}{3}\ln|3y+2| = \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln|2x-1| + c \quad \Big|\cdot 3$$

$$\ln|3y+2| = \frac{3}{2}\ln|2x-1| + c = \left(\ln|2x-1|^{3/2} + c\right)$$

$$e \qquad\qquad e$$

$$|3y+2| = e^c \cdot e^{\ln|2x-1|^{3/2}} = e^c |2x-1|^{3/2}$$

$$3y+2 = k\,(|2x-1|)^{3/2} \qquad k \in \mathbb{R}$$

$$y = \frac{k}{3}(|2x-1|)^{3/2} - \frac{2}{3} \qquad \text{(erstatter } \tfrac{k}{3} \text{ med } k \ldots\text{)}$$

$$y(x) = k\left(\sqrt{|2x-1|}\right)^3 - \frac{2}{3}.$$

$y(1)=2$ randbetingelsen gir

$$2 = k\left(\sqrt{|2-1|}\right)^3 - \frac{2}{3}$$

$$k = 2 + \frac{2}{3} = \frac{8}{3} \approx 2.66.$$

oppg.

$$y' - \frac{2}{y} = 1.$$

$$y' = 1 + \frac{2}{y} = \frac{y+2}{y}$$

$$\frac{y}{y+2}\,y' = 1$$

$$\int \frac{y}{y+2}\underbrace{y'\,dx}_{dy} = \int \frac{y}{y+2}\,dy = \int 1\,dx = x + c$$

$$\int \frac{y}{y+2}\, dy$$

$\int$ rasjonalt uttrykk

1) Polynom divisjon

- substitusjon

2) Delbrøksoppspalting

- substitusjon ..

polynomdivisjon:

$$\frac{y}{y+2} = \frac{y+2-2}{y+2}$$

$$= 1 - \frac{2}{y+2}$$

$$\int \frac{y}{y+2}\, dy = \int 1 - \frac{2}{y+2}\, dy = y - 2\ln|y+2| = x + C$$

$$x(y) = y - 2\ln|y+2| + C$$

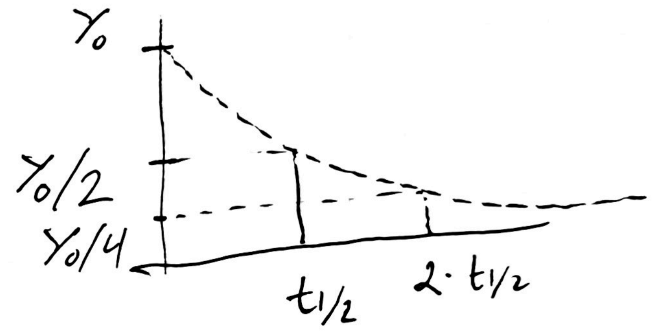
$y(x)$?

Radioaktiv nedbryting.

$Y(t)$ mengden av en radioaktiv isotop.

$k > 0$

$$Y' = -k \cdot Y$$



$$\int \frac{Y'}{Y} dt = \int -k \, dt$$

$$\int \frac{1}{Y} dY = -k \cdot t + C$$

$$\ln|Y| = -kt + C$$

$$Y = \pm e^{C} e^{-kt}$$

$$\underline{Y = Y_0 \cdot e^{-kt}}$$

$Y_0 \in \mathbb{R}$

Halveringstiden er tiden det tar før stoff-mengden halvers.

$$\frac{Y_0}{2} = Y_0 e^{-k t_{1/2}} \iff \frac{1}{2} = e^{-k \cdot t_{1/2}}$$

uavhengig av Y_0

$$\ln\left(\frac{1}{2}\right) = -k \cdot t_{1/2}$$

$$-\ln 2 = -k\, t_{1/2}$$

$$\underline{k \cdot t_{1/2} = \ln 2}\,, \quad k = \frac{\ln 2}{t_{1/2}}$$

antall protoner + antall nøytroner

$^{12}_{6}C$	99%
protoner	
$^{13}_{6}C$	1%
$^{14}_{6}C$	10^{-12}.

C14 metoden

Halveringstid på ca $t_{1/2} = 5700$ år

$$^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta + \nu$$

Andelen $^{14}_{6}C$ i atmosfæren er nokså stabil.

Samme andel ^{14}C i levende skapninger.

Vi finner levninger av en mammut i en myr i Sibir. Andelen C14 til mammuten er $\frac{1}{3}$ av andelen i atmosfæren. Hvor gammel er mammuten?

$$Y = Y_0 e^{-kt} \qquad k = \frac{\ln 2}{t_{1/2}}$$

$$\frac{1}{3} Y_0 = Y_0 e^{-kt}$$

$$\frac{1}{3} = e^{-k \cdot t}$$

$$\ln(1/3) = \ln e^{-kt} = -kt$$

$$-\ln 3.$$

$$k \cdot t = \ln 3$$

$$\frac{\ln 2}{t_{1/2}} \cdot t = \ln 3$$

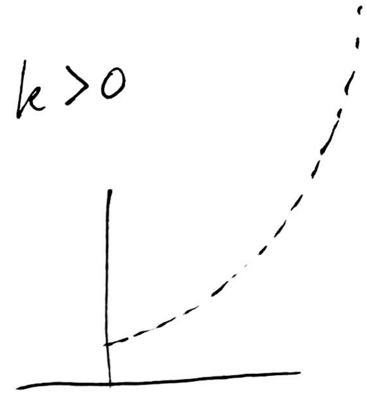
tiden $t = t_{1/2} \cdot \frac{\ln 3}{\ln 2} \approx \underline{9034 \text{ år}}$

Eksponensiell vekst

$$y' = k y$$

$$y = y_0 e^{kt}$$

$k > 0$



Logistisk diff. likning.

$$y' = \underset{\substack{\uparrow \\ \text{vekstrate}}}{k}\, y \left(1 - \frac{y}{\underset{\substack{\uparrow \\ P \text{ bæreevne.}}}{P}}\right)$$

$$\int \frac{y'}{y(1 - y/P)}\, dt = \int k\, dt$$

$$\int \frac{1}{y(1-\frac{y}{P})} = k \cdot t + c$$

Delbrøksoppspalting

$$\frac{1}{y\left(1-\frac{y}{p}\right)} \cdot \frac{p}{p} = \frac{p}{y(p-y)}$$

$$= \frac{A}{y} + \frac{B}{p-y} = \frac{A(p-y) + By}{y(p-y)}$$

$$p = A(p-y) + By = A \cdot p + y(B-A)$$

$p - y = u$

$u' = -1$

Så $A = B = 1$

$$\int \frac{1}{y\left(1-\frac{y}{p}\right)} dy = \int \frac{1}{y} + \frac{1}{p-y} \, dy = \ln|y| - \int \frac{du}{u}$$

$$= \ln|y| - \ln|p-y| = \ln\left|\frac{y}{p-y}\right|.$$

$$e^{\ln\left|\frac{y}{p-y}\right|} = e^{(kt+c)}$$

$$\left|\frac{y}{p-y}\right| = e^{c} \cdot e^{kt}$$

$$\frac{y}{p-y} = K e^{kt}$$

$$y = (P-y)Ke^{kt} = P \cdot Ke^{kt} - y(Ke^{kt})$$

$$y(1+Ke^{kt}) = PKe^{kt}$$

$$y(t) = P\frac{Ke^{kt}}{1+Ke^{kt}} = P\left(\frac{K}{e^{-kt}+K}\right)$$

Hvis $y(0) = P_0$ $\qquad \frac{P_0}{P-P_0} = Ke^0 = K$

$$K = \frac{P_0}{P-P_0}$$

$$y(t) = P\frac{P_0}{(P-P_0)e^{-kt}+P_0}$$

18april Luftmotstanden viser seg å være proporsjonal til fart kvadrert : $\ell \cdot V^2$.

ℓ konstant.

$$x$$

$$x' = V \quad \text{fart}$$

$$x'' = V' = a \quad \text{aksellerasjon}$$

Newtons andre lov $\quad m \cdot a = \Sigma$ krefter

$$m \cdot V' = m \cdot g - \ell \cdot V^2.$$

Når legemet faller vil farten stabilisere seg.

$V' = 0$: $\quad m \cdot g - \ell \cdot V^2 = 0$

så stabil fart er: $\quad V_{st} = \sqrt{\frac{m \cdot g}{\ell}}$.

Hvis $V_{st} = 60 m/s$ (~ 220 km/t) så er $\frac{\ell}{m} = \underline{0{,}0027 \; m^{-1}}$

Vi løser differensialligningen

29 april

$$mV' = mg - \ell \cdot V^2$$

$$V' = g - \left(\frac{\ell}{m}\right) \cdot V^2$$

$$\frac{V'}{g - \frac{\ell}{m} \cdot V^2} = 1$$

alternativt: $\quad \dfrac{\frac{m}{\ell} \cdot V'}{\frac{mg}{\ell} - V^2} = 1$

Delbrøksoppspalting : $\quad \frac{m}{\ell} V' \dfrac{1}{\left(\sqrt{\frac{mg}{\ell}} - V\right)\left(\sqrt{\frac{mg}{\ell}} + V\right)} = 1$

$$\frac{m}{l} V' \left(\frac{1}{\left(\sqrt{\frac{mg}{l}} - V\right)} + \frac{1}{\sqrt{\frac{mg}{l}} + V} \right) \frac{1}{2\sqrt{\frac{mg}{l}}} = 1$$

integrerer

$$\frac{m}{l} \frac{1}{2\sqrt{\frac{mg}{l}}} \int \frac{1}{\sqrt{\frac{mg}{l}} - V} + \frac{1}{\sqrt{\frac{mg}{l}} + V}\, dV = \int 1\, dt$$

$$= \frac{1}{2\sqrt{\frac{m\cdot g}{l} \cdot \frac{l}{m}}} \left[-\ln\left|\sqrt{\frac{mg}{l}} - V\right| + \ln\left|\sqrt{\frac{mg}{l}} + V\right| \right] = t + c$$

$$= \frac{1}{\underbrace{2\sqrt{\frac{m\cdot g}{l} \cdot \frac{l^2}{m^2}}}_{2\sqrt{\frac{gl}{m}}}} \ln\left| \frac{\sqrt{\frac{mg}{l}} + V}{\sqrt{\frac{mg}{l}} - V} \right| = t + c$$

$$\left| \frac{\sqrt{\frac{mg}{l}} + V}{\sqrt{\frac{mg}{l}} - V} \right| = e^{2\sqrt{\frac{gl}{m}}\cdot t + c}$$

$$= e^{c} \cdot e^{2\sqrt{\frac{gl}{m}}\, t}$$

Lineær likning i variabel V

$$\frac{\sqrt{\frac{mg}{l}} + V}{\sqrt{\frac{mg}{l}} - V} = K \cdot e^{2\sqrt{\frac{gl}{m}}\cdot t}$$

$$\sqrt{\frac{mg}{l}} + V = \left(\sqrt{\frac{mg}{l}} - V\right) K e^{2\sqrt{\frac{gl}{m}}\cdot t}$$

$$V\left(1 + K e^{2\sqrt{\frac{gl}{m}}\, t}\right) = \sqrt{\frac{mg}{l}} \left(K e^{2\sqrt{\frac{gl}{m}}\cdot t} - 1\right)$$

Så $$\underline{V(t) = \sqrt{\frac{mg}{l}} \cdot \frac{K e^{2\sqrt{\frac{gl}{m}}\cdot t} - 1}{K e^{2\sqrt{\frac{gl}{m}}\cdot t} + 1}}$$

Randbetingelsen $V(0) = 0$ gir : $K = 1$.

Da er løsningen

$$V(t) = \sqrt{\frac{mg}{l}} \cdot \frac{e^{2\sqrt{\frac{gl}{m}}t} - 1}{e^{2\sqrt{\frac{gl}{m}}t} + 1} = \underline{\sqrt{\frac{mg}{l}}\left(1 - \frac{2}{e^{2\sqrt{\frac{gl}{m}} \cdot t} + 1}\right)}$$

Hvor lang tid tar det før vi oppnår 90% av stabil fart? Da ~~må~~ $1 - \frac{2}{e^{2\sqrt{\frac{gl}{m}}t} + 1} = \frac{9}{10} = 90\%$

Så $\frac{2}{e^{2\sqrt{\frac{gl}{m}}} + 1} = \frac{1}{10}$

så $e^{2\sqrt{\frac{gl}{m}} \cdot t} = 20 - 1 = 19.$

$$t_{90\%\,\text{fart}} = \frac{\ln 19}{2\sqrt{\frac{g \cdot l}{m}}}$$

$\frac{l}{m}$ som tidligere

$$\sim \frac{2.94}{2\sqrt{9.8\,m/s^2 \cdot 0.027\,m^{-1}}} \sim \underline{9\,\text{sekund}}$$

Posisjonen $S(t)$ er gitt ved $S'(t) = V(t)$

$$S(t) = \int V(t)\,dt$$

$$= \sqrt{\frac{mg}{l}}\left(t + \sqrt{\frac{m}{gl}} \ln\left(1 + e^{-2\sqrt{\frac{gl}{m}}t}\right)\right) + C$$

Rand betingelsen $S(0) = H$ gir

$$S(t) = \sqrt{\frac{mg}{l}} \cdot t + \frac{m}{l}\left(\ln\left(1 + e^{-2\sqrt{\frac{gl}{m}}t}\right) - \ln 2\right) + H$$