oblig 7 # 1c) $\ln|x| > 2$

Variasjon av

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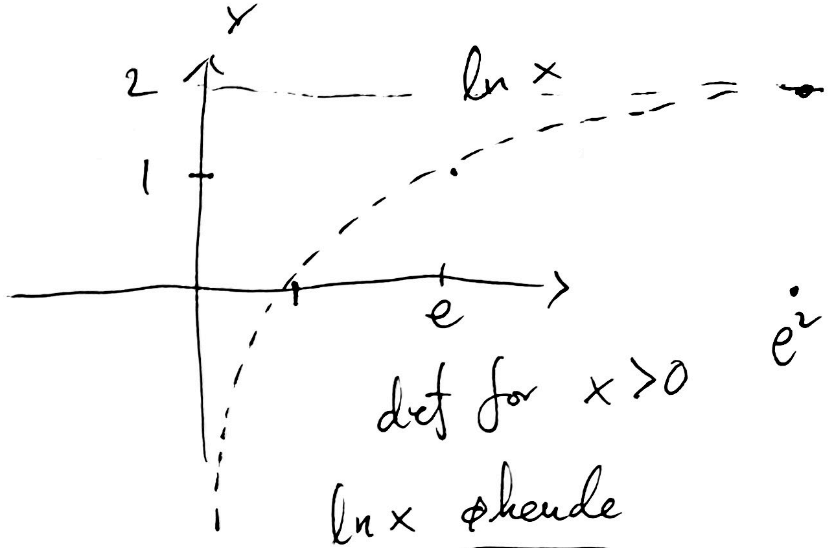
$$e^{\ln y} = e^2$$

$$y = e^2$$

Så $\ln y > 2$ for $y > e^2$.



def for $x > 0$

ln x økende funksjon.

$(\ln x)' = \frac{1}{x} > 0$

$x > 0$

Løsningen er alle x

$$|x| > e^2 \iff$$

$x < -e^2$ eller $x > e^2$.

$x \in \underline{\langle -\infty, -e^2\rangle \cup \langle e^2, \infty\rangle}$

Variant av 1d)

$$\sin\left(x-\frac{\pi}{4}\right) > \cos\left(x+\frac{\pi}{4}\right)$$

$$\sin(\underbrace{x-\frac{\pi}{4}}_{u}) - \cos\left(x+\frac{\pi}{4}\right) \underline{> 0}$$

så $x = u + \frac{\pi}{4}$ $\quad u + \frac{\pi}{4} + \frac{\pi}{4}$

$$\sin(u) - \underbrace{\cos\left(u+\frac{\pi}{2}\right)}_{-\sin u}$$

$u = x - \frac{\pi}{4}$ ✓

$x + \frac{\pi}{4} = u + \frac{\pi}{2}$

$\sin x = \cos\left(\frac{\pi}{2} - x\right)$

$= 2\sin(u)$

$2\sin(u) > 0$

setter inn $-x$ for x

$\sin(-x) = \cos\left(\frac{\pi}{2} + x\right)$

$-\sin x$

$$u \in \bigcup_{n\in\mathbb{Z}} \langle 0 + 2\pi\cdot n,\ \pi + 2\pi\cdot n\rangle$$

$x = u + \frac{\pi}{4}$

Løsningene er $\quad x \in \bigcup_{n\in\mathbb{Z}} \left\langle \frac{\pi}{4} + 2\pi\cdot n,\ \frac{5\pi}{4} + 2\pi\cdot n\right\rangle$

Variant av oppg 3a oblig 7

Hint: anvend ln på begge sider.

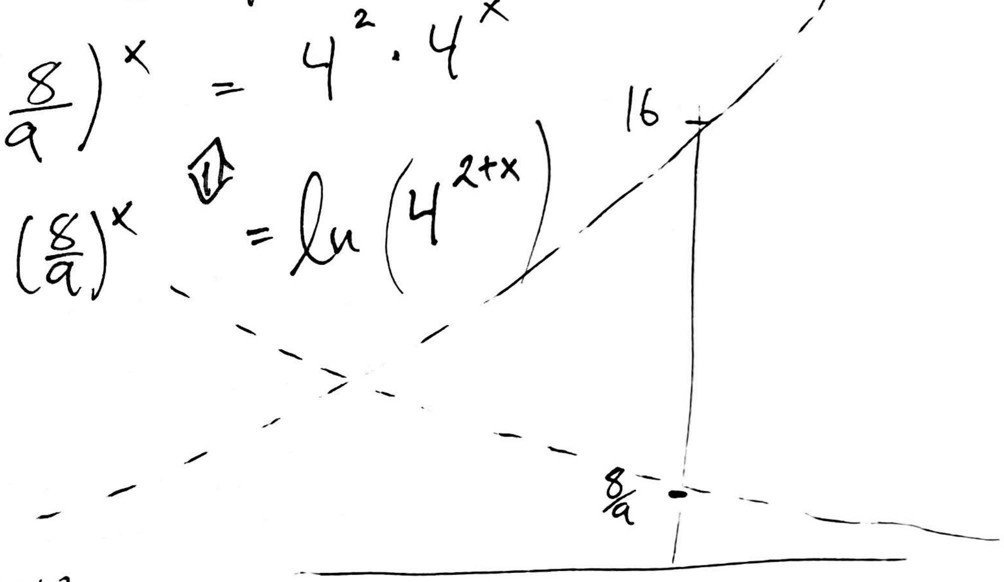
$$8^x\left(\frac{1}{3^x}\right)^2 = 4^{x+2}$$

$$8^x\frac{1}{3^{2x}} = 8^x\frac{1}{9^x} = 4^{x+2}$$

$$\left(\frac{8}{9}\right)^x = 4^2\cdot 4^x$$

$$\Updownarrow$$

$$\ln\left(\frac{8}{9}\right)^x = \ln\left(4^{2+x}\right)$$



$$\left(\ln 4 = \ln(2\cdot 2) = \ln 2 + \ln 2 = 2\ln 2\right)$$

$$\ln 4 = \ln 2^2 = 2\ln 2$$

$$\ln\left(\frac{8}{9}\right)^x = \ln 4^{x+2}$$

$$x\ln\left(\frac{8}{9}\right) = (x+2)\ln 4$$

$$-2\ln 4 = x\left(\ln 4 - \ln\left(\frac{8}{9}\right)\right) = x(\ln 4 - \overbrace{\ln 8}^{\ln 4+\ln 2} + \ln 9) = x(\ln 9 - \ln 2)$$

$$-\underbrace{2\ln 4}_{2\cdot 2\cdot\ln 2}$$

$$x = \frac{-4\ln 2}{\ln 9 - \ln 2} = \underline{-1{,}84338}$$

altemativt. gjøre alle eksponent funksjonene om til formen $e^{\cdots}$

$$8^x 3^{-2x} = e^{\ln 8 \cdot x} \, e^{-2x \cdot \ln 3} = e^{\ln 4 (x+2)}$$

$$e^{\ln 8 \cdot x - 2x \ln 3} = e^{\ln 4 (x+2)}$$

e^u er en økende funksjon så likningen er ekvivalent til at eksponentene er like

$$\underline{\ln 8 \cdot x - 2x \cdot \ln 3 = \ln 4 (x+2)} \quad \ldots$$

$$\left(\begin{aligned} (2^4)^2 &= 2^8 \\ \neq 2^{4^2} &= 2^{16} \ldots \end{aligned} \right)$$

$$\ln(2^{2x}) = 2x \ln 2.$$

$$\ln(2^{x^2}) = x^2 \ln 2$$

$$\ln((2^x)^2) = \ln(2^{2x}) = 2x \ln 2.$$

Variant av oblig 6 #6

$$y' = \frac{x}{y^2} \qquad y(3) = -4.$$

$$= x \cdot \frac{1}{y^2}$$

1. orders separabel diff likning

$$y^2 y' = x$$

$$\int y^2 \underbrace{y' dx}_{dy} = \int x \, dx$$

$$\int y^2 \, dy = \int x \, dx$$

$$\frac{y^3}{3} = \frac{1}{2}x^2 + c \qquad \cdot 6$$

$$\underline{2y^3 - 3x^2 = c}$$

Variant til oppg 4a) oblig 7.

$$\int_{-1}^{1} \underset{u'}{3x^2} \cdot \underset{u^4}{(1+x^3)^4}\, dx$$

Substitusjon

$(1+x^3)' = 3x^2$

$u = 1+x^3$

$u(-1) = 0$

$u(1) = 2$

$$\int_{-1}^{1} u' \cdot u^4 \, dx = \int_{u(-1)}^{u(2)} u^4 \, du$$

$$= \int_0^2 u^4 \, du = \frac{u^5}{5} \Big|_0^2 =$$

$$\frac{2^5}{5} = \frac{32}{5} = \underline{6{,}4}$$

Delvis integrasjon.

$$(u \cdot v)' \underset{\text{produktregelen}}{=} u' \cdot v + u \cdot v'$$

$$C + u \cdot v = \int (u \cdot v)' \, dx = \int u' \cdot v \, dx + \int u \cdot v' \, dx$$

$$\int u' \cdot v \, dx = u \cdot v - \int u \cdot v' \, dx$$

$u = -\cos x$

$u' = \sin x$

$v' = 1$

$$\int \underset{v}{(1+x)} \underset{u'}{\sin x} \, dx$$

$$= (1+x) \cdot (-\cos x) - \int \overset{v'}{1} \cdot \overbrace{(-\cos x)}^{u} \, dx$$

$$= (1+x)(-\cos x) + \sin x + c$$

$$= \underline{-x \cos x - \cos x + \sin x + c}$$

$$\int \overset{v}{x}\, \overset{u'}{e^{-x}}\, dx = \underline{-xe^{-x} - e^{-x} + c}$$

$$\int \overset{v}{(2x+1)}\, \underset{u'}{e^{-3x+1}}\, dx$$

$v = 2x+1$

$v' = 2$

$u' = e^{-3x+1}$

$u = \frac{-1}{3} e^{-3x+1}$

$$= \frac{-1}{3} e^{-3x+1}(2x+1) - \int 2 \cdot \left(\frac{-1}{3}\right) e^{-3x+1}\, dx$$

$$= \frac{-(2x+1)}{3} e^{-3x+1} + \frac{2}{3}\left(-\frac{1}{3} e^{-3x+1}\right) + c$$

$$= \left(\frac{-6x-3}{9} - \frac{2}{9}\right) e^{-3x+1}$$

$$= \underline{\frac{-(6x+5)}{9} e^{-3x+1}}$$

$$I_{2n} = \int \sin^{2n} x \, dx = \int (1-\cos^2 x)\sin^{2n-2}(x)\, dx$$

$$= I_{2n-2} - \int \underbrace{\cos x}_{v} \cdot \underbrace{\cos(x)\sin^{2n-2} x}_{u'} \, dx$$

$$v' = -\sin x \qquad u = \frac{\sin^{2n-1}(x)}{2n-1}$$

$$= I_{2n-2} - \frac{\sin^{2n-1}(x)\cdot\cos(x)}{2n-1} + \underbrace{\int -\sin x \cdot \frac{\sin^{2n-1} x}{2n-1}\, dx}_{\frac{-1}{2n-1} I_{2n}}$$

$$\left(1+\frac{1}{2n-1}\right) I_{2n} = I_{2n-2} - \frac{\sin^{2n-1} x \cos x}{2n-1} + C$$

$$I_{2n} = \left(\frac{2n-1}{2n}\right) I_{2n-2} - \frac{\sin^{2n-1} x \cos x}{2n} + C$$

$$I_0 = \int 1\, dx = x + C.$$