12.04
2021

19 Sannsynlighetsregning

Sannsynlighetsmodell

<u>Utfallsrommet</u> S (mengde)

elementene i S kalles for <u>utfall</u>

Eks Kaste mynt : Utfallsrom : $S = \{\text{Mynt}, \text{Krone}\}$

Terning — : $S = \{1, 2, 3, 4, 5, 6\}$

Et stokastisk forsøk resulterer i et <u>element i S</u>
utfall.

En <u>hendelse</u> er en delmengde as S.

Eks : Terning : $H_{\geq 3} = \{3, 4, 5, 6\}$ S

$H_{odd} = \{1, 3, 5\}$ $H_{even} = \{2, 4, 6\}$, $\emptyset$

(Ikke alle delmengder av S behøver å være hendelser)

Sannsynlighetsmål P

P : Hendelser $\longrightarrow [0,1]$

Uniform sannsynlighetsmodell.

S endelig.
#element i S $= n$

Alle utfall er hendelser.
Samme sannsynlighet for hvert utfall

$P(s)$ like for alle $s \in S$.

$$P(\{s\}) = \frac{1}{n} = \frac{1}{\#\text{elementer i } S}.$$

H hendelse $\quad P(H) =$ Sum av sannsynlighet for hvert av utfallene i H

$$= \frac{\#H}{\#S} = \frac{\#H}{S}$$

Rettferdig terning

$$P(\{1\}) = P\{2\} = \cdots = P\{6\} = \frac{1}{6}.$$

$$P(H_{\geq 3}) = P(\{3,4,5,6\}) = 4 \cdot \frac{1}{6} = \underline{\frac{2}{3}}$$

$$P(H_{odd}) = P(\{1,3,5\}) = \frac{3}{6} = \underline{\frac{1}{2}}$$

Egenskaper til sannsynlighetsmål P

$$P(\emptyset) = 0$$

$$P(S) = 1$$

A, B disjunkte : $P(A \cup B) = P(A) + P(B)$

$A \cap B = \emptyset$

A, B hendelser

$(A, B \subset S)$

$$P(H) = \lim_{n \to \infty} \frac{\#\text{utfallet er i } H}{\#\text{antall forsøk}, n}.$$

n antall stokastiske forsøk

$$P(\bar{A}) = 1 - P(A)$$

$A \cup \bar{A} = S$

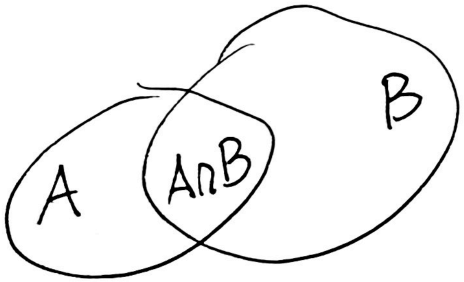
$A \cap \bar{A} = \emptyset$

A hendelse

$\bar{A} = S - A$
komplementet

$$P(A \cup \bar{A}) = P(A) + P(\bar{A})$$
$$= P(S) = 1$$

så $P(\bar{A}) = 1 - P(A)$ ✓



$A \setminus B = A - (B \cap A)$

$A = \underbrace{(A \cap B) \cup (A \setminus B)}_{\text{disjunkt}}$

$P(A) = P(A \cap B) + P(A \setminus B)$

så $P(A \setminus B) = P(A) - P(A \cap B)$.

disjunkt
union

$A \cup B = \underbrace{A \setminus B \cup B}_{\text{disjunkt}}$

$$P(A \cup B) = P(\underbrace{(A \setminus B) \cup B}_{\text{disj}}) = \underbrace{P(A \setminus B)}_{P(A) - P(A \cap B)} + P(B)$$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Til orientering.

(Samlingen av hendelser : $\emptyset$, S

tellbare snitt av hendelser er hendelser
— unioner)

(Stokastisk funksjon $S \xrightarrow{f} \mathbb{R}$

$f^{-1}(a,b)$
er hendelser.)

<u>Eks.</u> Kaster en mynt 3 ganger
(alternativt: kaster 3 mynter i en gang)

Anta hver mynt er rettferdig $P(\{K\}) = \frac{1}{2} = P(\{M\})$

Utfallsrommet $S = \{ \begin{matrix} MMM & MMK \\ MKM & MKK \\ KMM & KMK \\ KKM & KKK \end{matrix} \}$ $2^3 = 8$ utfall

uniform sannsynlighetsmodell, hvert av utfallene har sannsynlighet $\frac{1}{8}$.

$H_{MMM} = \{MMM\}$ $\quad H_{KKK} = \{KKK\}$

$H_{MKK} = \{MKK, KMK, KKM\}$ $\quad H_{MMK} = \{MMK, MKM, KMM\}$

$P(H_{MMM}) = \frac{1}{8} = P(H_{KKK})$

$P(H_{KMM}) = P(H_{MKK}) = \frac{3}{8}$.

Alternativ modell $S = \{ M_0, M_1, M_2, M_3 \}$

$$P(M_0) = P(M_3) = \frac{1}{8}, \qquad P(M_1) = P(M_2) = \frac{3}{8}$$

M_i = utfallet: i mynt
$3-i$ kron.

$$P\{K\} = \frac{3}{4} = Y$$

$$P(\{M\}) = \frac{1}{4} = X$$

" urettferdig mynt "

Eksempel:

$$\left.\begin{aligned} P(M_0) &= \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64} \\ P(M_1) &= 3\left(\frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4}\right) = \frac{27}{64} \\ P(M_2) &= 3\left(\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4}\right) = \frac{9}{64} \\ P(M_3) &= \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{64} \end{aligned}\right\} \text{Summen er lik 1.}$$

oppg. Rettferdig mynt.

Kaster den 5 ganger.

Hva er sannsynligheten for å få kron 1 eller 0 ganger.

K_0 kron 0 ganger: mynt 5 ganger: $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2^5} = \frac{1}{32}$.

$$P(K_0) = \frac{1}{32}.$$

K_1 kron 1 gang: $5\left(\frac{1}{2}\right)^5 = \frac{5}{32}$.

$$P(K \leq 1) = P(K_0) + P(K_1) = \frac{1}{32} + \frac{5}{32} = \frac{6}{32} = \frac{3}{16}$$

$S = \{ MMMMM, MMMMK, \ldots \}$ 32 muligheter.

$K_1 = \{ MMMMK, \ldots, KMMMM \}$ 5 muligheter.

uniform sannsynlighetsmodell

$$P(K_1) = \frac{\# K_1}{\# S} = \frac{5}{32} \ldots$$

Kaster 2 terninger.

$$S = \{ (1,2), (1,1), (1,3), \ldots, (6,5), (6,6) \}$$

$$\#S = 6 \cdot 6 = 36.$$

Rettferdig terning: <u>uniform</u> sannsynlighet:

H_2 = Summen er 2 $\{(1,1)\}$

H_3 = — 3 $\{(1,2), (2,1)\}$

H_4 = — $\{(1,3), (2,2), (3,1)\}$

H_5 = — $\{(1,4), (2,3), (3,2), (4,1)\}$

H_6 = — $\{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

H_7 = — $\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$$P(H_2) = P(H_{12}) = \frac{1}{36}$$

$$P(H_3) = P(H_{11}) = \frac{2}{36} = \frac{1}{18}$$

$$P(H_4) = P(H_{10}) = \frac{3}{36} = \frac{1}{12}$$

$$P(H_5) = P(H_9) = \frac{4}{36} = \frac{1}{9}$$

$$P(H_6) = P(H_8) = \frac{5}{36}$$

$$P(H_7) = \frac{6}{36} = \frac{1}{6}.$$

$$P(H_{\leq 4}) = P(\underbrace{H_2 \cup H_3 \cup H_4}_{\text{disj}}) = \frac{1}{36} + \frac{1}{18} + \frac{1}{12} = \frac{1+2+3}{36} = \frac{6}{36}$$

$$= \underline{\frac{1}{6}.} \quad \left(= P(H_7)\right)$$

$$P(H_7) = \frac{\#\text{ganger summen blir 7}}{\#\text{kast av 2terning}}.$$

oppgave

$$P(H_{oddsum}) = P(H_3 \cup H_5 \cup H_7 \cup H_9 \cup H_{11})$$

$$P(H_{jevnsum}) = P(H_2 \cup H_4 \cup H_6 \cup H_8 \cup H_{10} \cup H_{12})$$

$$P(H_{oddsum}) = P(H_3) + P(H_5) + P(H_7) + P(H_9) + P(H_{11}) = \frac{2}{36} \cdot 2 + \frac{4}{36} \cdot 2 + \frac{1}{6}$$

$$= \frac{4+8+6}{36} = \frac{18}{36} = \underline{\underline{\frac{1}{2}}}$$

$$P(H_{jevnsum}) = P(\overline{H_{odd}}) = 1 - P(H_{oddsum}) = \underline{\underline{\frac{1}{2}}}$$