

13 april
2021

19.3 Sum av sannsynligheter.

Sannsynlighetsmodell.

Utfallsrom

S
 ω utfall

Hendelser: delmenger av S .

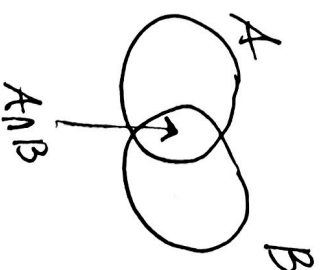
resultatet av et
stokastisk forsøk.

$P: \{ \text{Hendelser} \} \rightarrow [0, 1]$

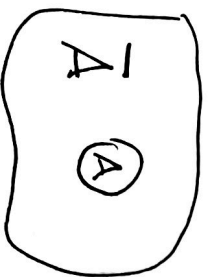
$$P(\emptyset) = 0$$

$$P(S) = 1$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



S



$$P(\bar{A}) = 1 - P(A)$$

$$\bar{A} = S \setminus A$$

$$S - A$$

Gitt $P(A) = 0.6$ Hva er $P(A \cup B)$?

$$P(A \cap B) = 0.2$$

$$P(\bar{B}) = 0.6$$

$$P(B) = 1 - P(\bar{B}) = 0.4$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ 0.6 + 0.4 - 0.2 = \underline{\underline{0.8}}$$

Gitt

$$P(A) = 0.2$$

Hva kan

$$P(B) = 0.3$$

$$P(A \cup B) \text{ være?}$$

$$0.3 \leq P(A \cup B) \leq 0.5$$

$$0 \leq P(A \cap B) \leq 0.2$$

oppg.

$$P(A) = 0.6$$

Hva

kan da

$$P(B) = 0.5$$

$$P(A \cap B)$$

og

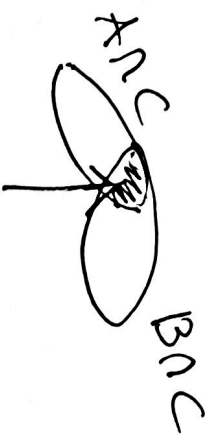
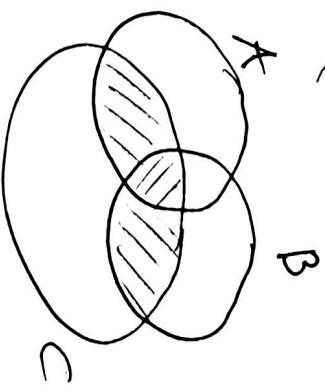
$$P(A \cup B) \text{ være?}$$

$$0.1 \leq P(A \cap B) \leq 0.5$$

$$0.6 \leq P(A \cup B) \leq 1$$



$$\begin{aligned}
 P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \\
 P((A \cup B) \cup C) &= P(A \cup B) + P(C) - \underbrace{P((A \cup B) \cap C)}_{[P(A \cap C) + P(B \cap C)]} \\
 &= P(A) + P(B) + P(C) - P(A \cap B) - [P(A \cap C) + P(B \cap C)] + P(A \cap B \cap C) \\
 &= P(A) + P(B) + P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \quad \checkmark \\
 &= (A \cap C) \cup (B \cap C)
 \end{aligned}$$



$$\begin{aligned}
 &(A \cap C) \cap (B \cap C) \\
 &= A \cap B \cap C
 \end{aligned}$$

$$P(A) = 0.8$$

$$P(B) = 0.9$$

Tallene gir ikke mening.
ikke mulig å realisere.

$$P(A \cap B) = 0.3$$

$$P(A \cup B) = 0.8 + 0.9 - 0.3 = 1.4 \text{ ikke mulig}$$

sidan $P(H) \leq 1$
for alle hendelser H .

Testoppgave.

Sannsynlighet for å ha sykdom K er 0.7%
L er 0.6%

enten K eller L er 1%
(eller begge)

Hva er sannsynligheten for å ha begge sykdommene?

Hendelser

K : har sykdom K
 L : _____ L
 $K \cup L$: har sykdom K, L eller begge
 $K \cap L$: har begge sykdommene.

Utfallsrommet
 $S = \{ \text{personene} \}$
 K personer med sykdom K
 $L \dots$

$$P(K \cup L) = P(K) + P(L) - P(K \cap L)$$

$$1\% = 0.7\% + 0.6\% - ?$$

$$P(K \cap L) = (0.7 + 0.6 - 1)\% = \underline{0.3\%}$$

Variant,
 Antak

1% som har sykdom K eller L , men ikke begge.
 Hva er sannsynligheten for å velge ut en person
 som har begge sykdommene?

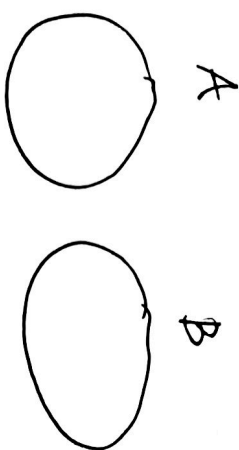
Diagram: Two overlapping circles, K and L . The intersection is shaded with a cross-hatch pattern. An arrow points from the word "Personer" to this shaded intersection.

$$P(K \cup L) = P(K) + P(L) - 2P(K \cap L)$$

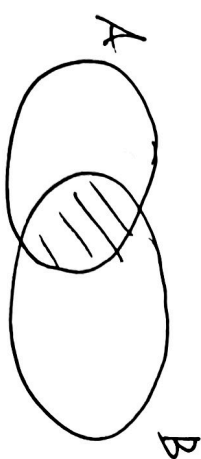
$$P(K \cup L \setminus K \cap L) = P(K) + P(L) - 2P(K \cap L)$$

$$1\% = 0.7\% + 0.6\% - 2P(K \cap L)$$

$$\frac{0.3\%}{2} = 0.15\% \quad \text{här är begreppet symmetriskt.}$$

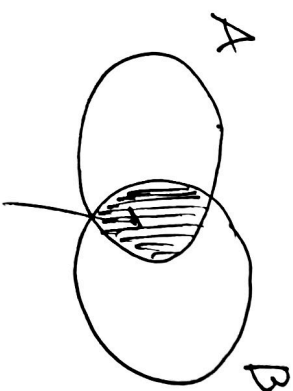


$$P(A \cup B) = P(A) + P(B) \quad \text{disjunkte}$$



$$= A \cup (B \setminus A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

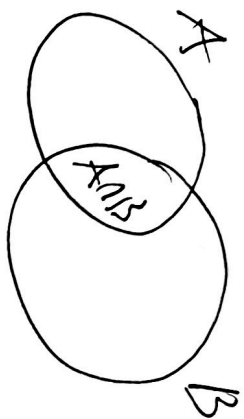


symmetriskt

$$P(A \cup B \setminus A \cap B) = (P(A) + P(B) - P(A \cap B)) - P(A \cap B)$$

$$= P(A) + P(B) - 2P(A \cap B)$$

Exempel



skriver som en disjoint union

$$P(A \cup B) = P(A) + P(B \setminus A \cap B)$$

$$B = (B \setminus A \cap B) \cup (A \cap B)$$

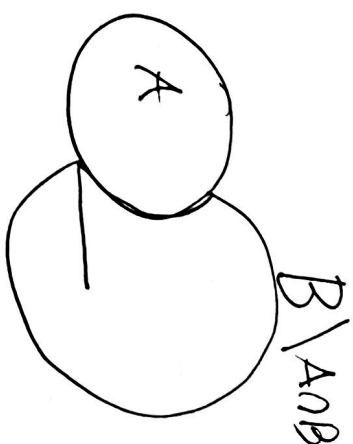
disjoint union

$$P(B) = P(B \setminus A \cap B) + P(A \cap B)$$

$$\text{Så } P(B \setminus A \cap B) = P(B) - P(A \cap B)$$

setter inn

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$S = \{\text{elevene}\}$
 Stokastisk fors k:
 tr kker et en elev.

P_{nj}	F_{nj}
P_{ng}	F_{ng}

P Pass
 F Fail

Gitt
 en klassen:

$$P(j) = 60\% \Rightarrow P(g) = 1 - 60\% = 40\%$$

$$P(p) = 80\% \Rightarrow P(f) = 20\%$$

$$J = g$$

$$\bar{P} = F$$

Hendelser

$\{j\}$ jente
 $\{g\}$ gutt

$\{P\}$ sk r p  eks m
 $\{F\}$ skrytke

$$j \cap g = \emptyset$$

$$j \cup g = S$$

$$P \cap F = \emptyset$$

$$P \cup F = S.$$

$$P(p_{nj}) = 40\%$$

However $P(p_{ng})$?

$$(p_{nj}) \cup (p_{ng}) = P(\underbrace{j \cup g}_S) = P$$

$$(p_{nj}) \cap (p_{ng}) = \emptyset$$

$$\text{So } P(p_{nj}) + P(p_{ng}) = P(p)$$

$$0.4 \quad ? \quad 0.8$$

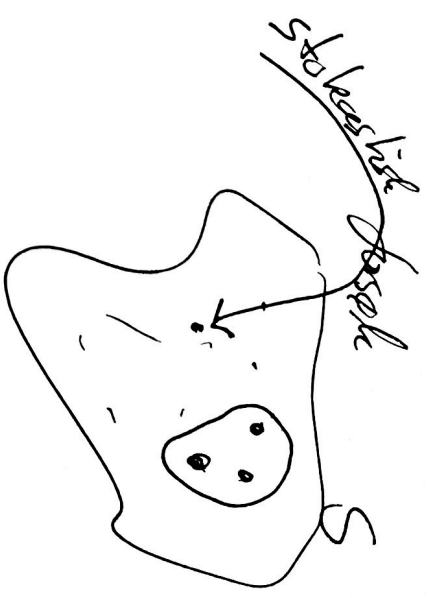
$$\text{So } P(p_{ng}) = 0.4.$$

$$P(f_{ng}) + \underbrace{P(p_{ng})}_{0.4} = P(g) = 0.4 \quad \text{So } P(f_{ng}) = 0$$

$$P(f_{nj}) = P(j) - P(p_{nj}) = 0.6 - 0.4 = \underline{0.2}$$

$$P(f_{lj}) = \frac{P(f_{nj})}{P(j)} = \frac{0.2}{0.6} = \frac{1}{3}.$$

delmængde
 $H \subset S$
 udfaldsrummet
 udfald.
 element
 x
 mængde



Højde til en person
 Stokastisk forsøg: måle højde

$S = \{1\text{cm}, 2\text{cm}, \dots, 300\text{cm}\}$

$$P(H) = \lim_{n \rightarrow \infty} \frac{\# \text{ gunstige udfald for } H}{\# \text{ stokastiske forsøg, } n}$$

Sandsynlighed for å ha syttem K

$$P(K) = 3\%$$

K eller L $P(K \cup L) = 4\%$
men ikke begge

å ha både syttem K og L $P(K \cap L) = 1\%$

Hva er sandsynligheten for å ha syttem L? $P(L)$?

$$P(K \cup L) = P(K) + P(L) - P(K \cap L)$$

$$\underbrace{P(K \cup L)}_{4\%} \setminus \underbrace{P(K \cap L)}_{1\%} = P(K) + \underbrace{P(L)}_{3\%} - 2 \underbrace{P(K \cap L)}_{1\%}$$

$$P(L) = -P(K) + 2 \cdot P(K \cap L) + P(K \cup L) \setminus P(K \cap L) \\ = -3\% + 2\% + 4\% = 3\%$$

$$\underline{\underline{P(L) = 3\%}}$$

Klasse 20 elever

8 jenter og 12 gutter.

2 personer trebles

tilfeldig t for å spille tennis

2 jenter skal spille ?

Hva er sannsynligheten at

2 gutter

er godt og en jente

Antall ordna utvalg: $20 \cdot 19$

$8 \cdot 7$

$12 \cdot 11$

To gutter

$8 \cdot 12$

énen hver

$12 \cdot 8$

$$P(jj) = \frac{7 \cdot 6}{20 \cdot 19} = \frac{14}{95} = \frac{14}{95}$$

$$P(gg) = \frac{11 \cdot 12}{20 \cdot 19} = \frac{33}{95} = \frac{33}{95}$$

$$P(gj, jg) = \frac{2 \cdot 8 \cdot 12}{20 \cdot 19} = \frac{48}{95} = \frac{48}{95}$$

Summe er lik 1