Eksamen vår 2020

26 deloppgaver

$\sqrt{5} = 5^{1/2}$ $\quad \sqrt[5]{a^2} = a^{2/5}$

$(a \cdot b)^n = a^n \cdot b^n$ $\quad (a^{1/3})^3 = a^{3/3} = a$

$\frac{1}{(5^{3/2})} = (5^{3/2})^{-1} = 5^{-3/2} \dots$

$a^n \cdot b^n = (a \cdot b)^n$

$a^n \cdot a^m = a^{n+m}$

27 april 2021

$k : \frac{a}{b} = \frac{k}{(a/b)} \cdot \frac{b}{b} = \frac{k \cdot b}{a} = k \cdot \frac{b}{a}$

konjugatsetningen

$t^2 - a^2 = (t+a) \cdot (t-a)$

1b)

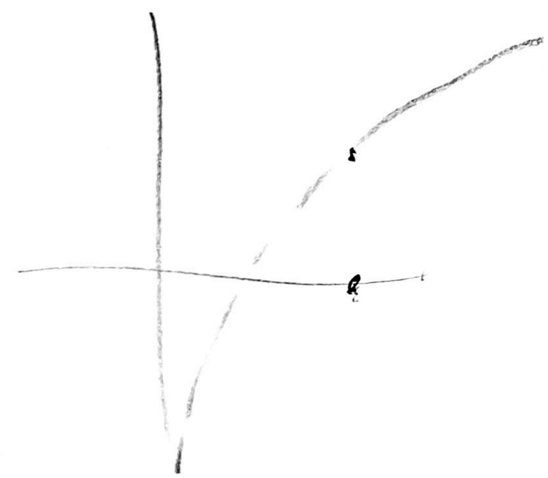
$$\frac{5(\underbrace{t^2-9}_{(t-3)(t+3)})}{t-3} \cdot \frac{3(t^2+2t)}{t+2}$$

$$= 5(t+3) \cdot 3 \cdot \frac{t(t+2)}{t+2} = \underline{15 \cdot t(t+3)}$$

$\ln(a^r) = r \ln a$

$\ln(a \cdot b) = \ln a + \ln b \Rightarrow \ln 1 = 0.$

$\ln\left(\frac{a}{b}\right) = \ln a \underbrace{- \ln b}_{\ln b^{-1}}$



$\ln\left(x \cdot (x^2 - x)^{-1}\right) = 1 = \ln e$

$\frac{x}{x^2 - x} = e$

$x > 0$ ✓

$x^2 - x > 0$ ✓

$\frac{x}{x(x-1)} = e$

$x \neq 0$

$\frac{1}{x-1} = e$

$x - 1 = \frac{1}{e} \Leftrightarrow \underline{\underline{x = 1 + \frac{1}{e}}} = \frac{e}{e} + \frac{1}{e}$

$= \frac{e+1}{e}$

2 b) $x^3 - 13x - 12 : x + 3 = \dots$

sjekke at det går opp.

RASKERE

$(x-a)$ er en faktor i polynom $P(x) \iff P(a) = 0.$

$x + 3 = x - (-3)$

Setter inn -3 :

$(-3)^3 - 13(-3) - 12$

$-27 + 39 - 12 = 0$

Så pol.div går opp.

c) $x^3 - 13x > 12$

$x^3 - 13x - 12 > 0$

Avgjøre fortegn

Forventer en rot til

$x^3 - 13x - 12$ (3.grad)

som er et lite heltall

Prøver oss frem...

$x = -1$ ✓

Så $x-(-1) = x+1$ en faktor i
$x^3 - 13x - 12$

$x^3 - 13x - 12 : x+1 = \underbrace{\text{2.gradsuttrykk...}}_{\text{faktoriser dette.}}$

Bruk fortegnsskjema på faktoriseringen
$x^3 - 13x - 12$ og avgjør når det er positivt.

3

a) $g(x) = \frac{1}{x^2} \cdot \ln(x)$ kvotientregel.

$= x^{-2} \cdot \ln(x)$ ... prod. regel ...

$(u \cdot v)' = u' \cdot v + u \cdot v'$

b) prod regel + kjerneregel.

$(x+3)^3$ gange ut og derivere ; Tungvint

$((x+3)^3)' = 3(x+3)^2 \cdot (x+3)'$ enkelt.

c) $\frac{1}{2x} y' = e^{-y}$ $y(0) = 1$

Løsning $y = \ln(x^2 + e)$

$\frac{y'}{e^{-y}} = 2x$

separabel

$\left(\frac{1}{e^{-y}} = (e^{-y})^{-1} = e^{y} \right)$

$\int e^{y} \underbrace{y' dx}_{dy} = \int 2x\, dx = x^2 + C$

$e^{y} = x^2 + C$

$y(0) = \ln(C) = 1$

så $C = e$

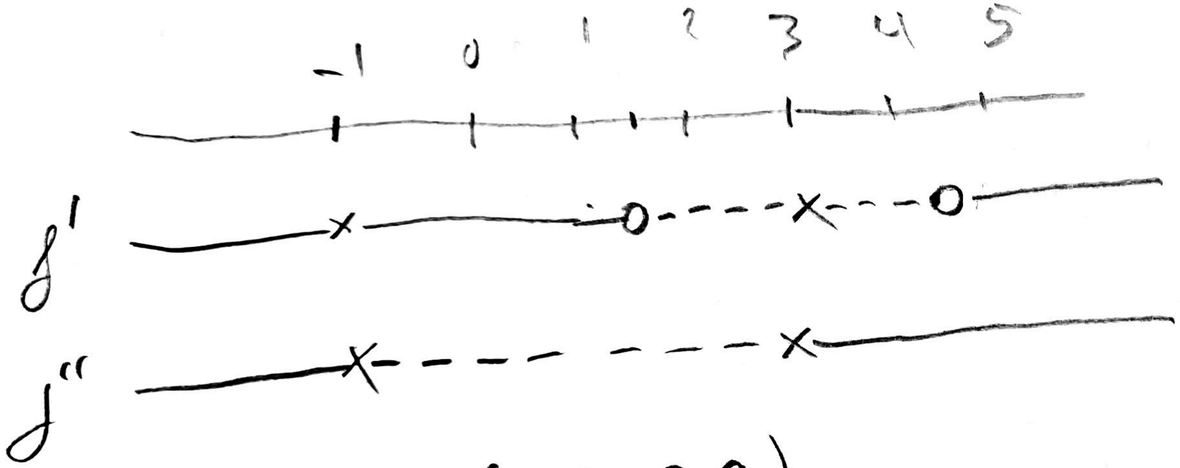
6 To vertikale $x = -1$ og $x = 3$.

skrå asymptote. $y = \frac{x}{2} + 3$

$y(0) = 3$ ✓

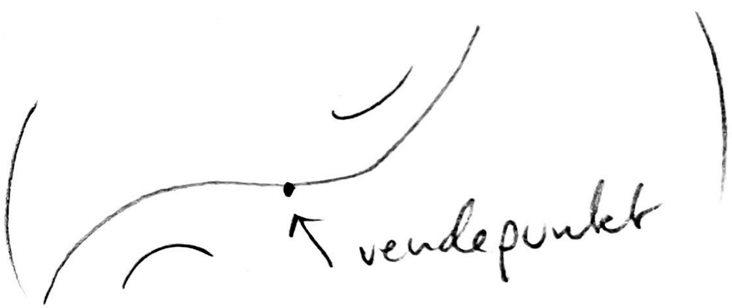
stigningstall $\frac{\Delta y}{\Delta x}$

$\frac{3}{6} = \frac{1}{2}$.



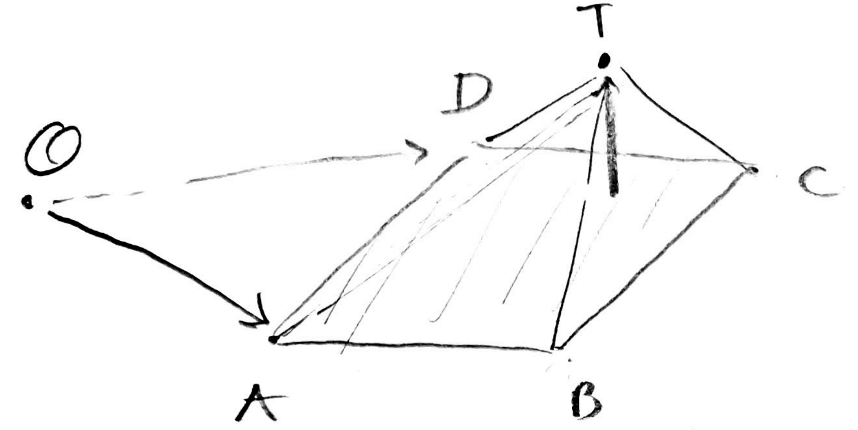
c) Toppunkt $(1.6, 2.9)$

bunnpunkt : $(4.4, 5.8)$



ingen vendepunkt.

7



a) $\vec{AD} = \vec{BC}$

$\vec{OD} = \vec{OA} + \underbrace{\vec{AD}}$

$\vec{OD} = \vec{OA} + \underbrace{\vec{CB}}_{(\vec{OB} - \vec{OC})}$

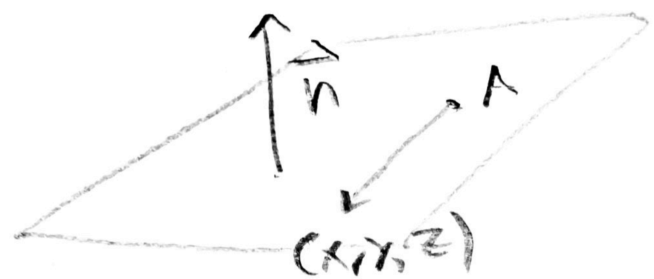
<u>Gir D</u>

b) Areal $|\vec{AB} \times \vec{AD}| = |\vec{AB} \times \vec{BC}|$

c) Volum pyramide $= (\text{Areal } ABCD) \cdot \underline{\text{høyde}} \cdot \frac{1}{3}$

$= \frac{1}{3} (\vec{AB} \times \vec{BC}) \cdot \vec{AT}$

d) $\vec{AB} \times \vec{BC} = \vec{n}$ normalvektor til planet gjennom A, B og C.



(x,y,z) i planet

$\Updownarrow$

$\overrightarrow{A(x,y,z)} \cdot \vec{n} = 0$

$[x,y,z] \cdot \vec{n} = \vec{OA} \cdot \vec{n} \ldots$

8. Benytter

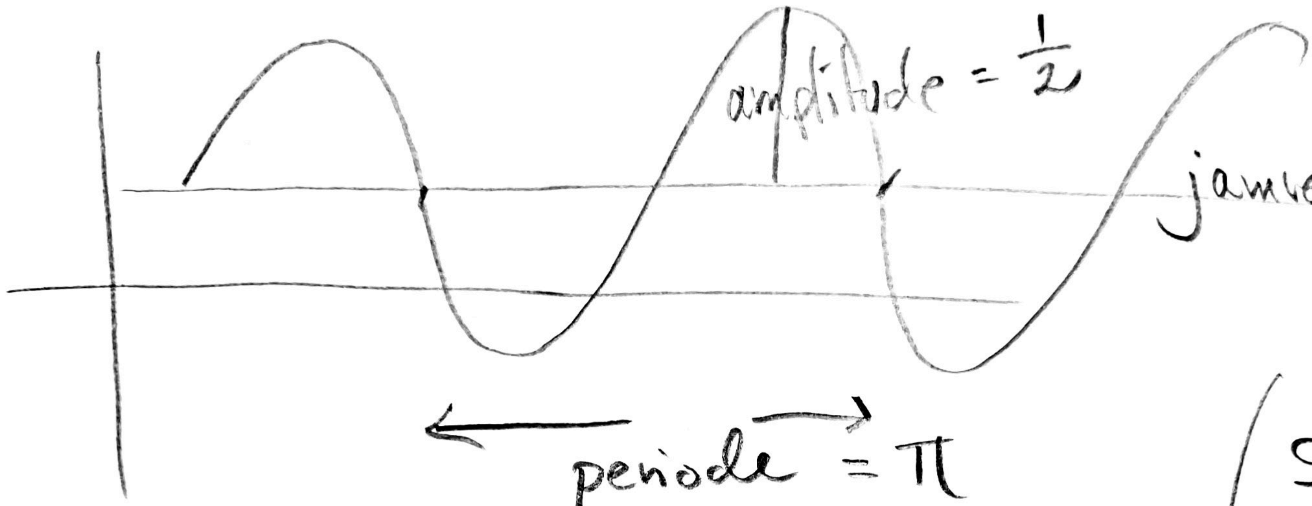
$\cos(2x) = \cos^2 x - \sin^2 x$

$\frac{1}{2}(1-\cos(2x)) = \frac{1}{2}(1-(\cos^2 x - \sin^2 x))$

$\overset{\text{Pyt.}}{=} \frac{1}{2}(\sin^2 x + \sin^2 x) = \sin^2 x$ ✓

benyttet $1-\cos^2 x = \sin^2 x$.

b)

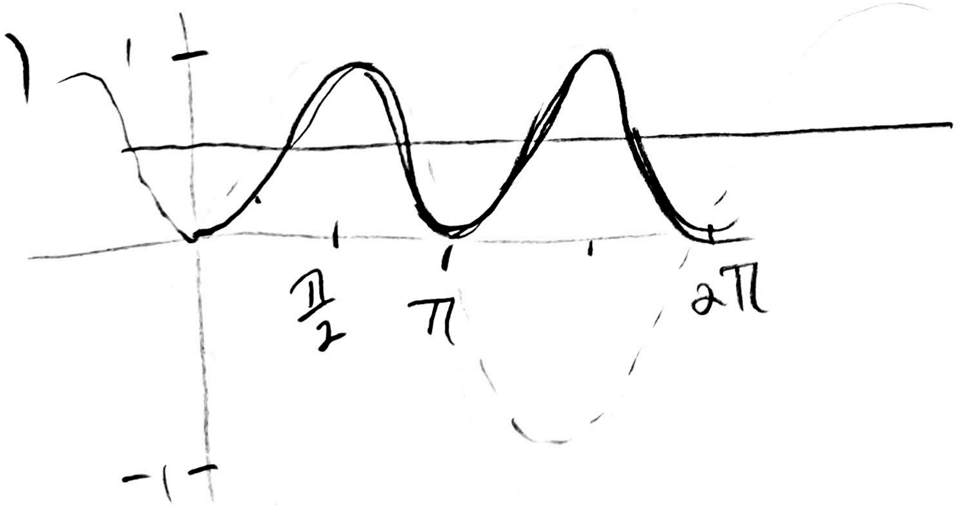


jamvektslinje $= \frac{1}{2}$

$$\begin{pmatrix} \sin(k(x-c_1)) \\ \sin(kx - c_2) \end{pmatrix}$$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

c)



d) Verdimengde

$[0,1]$

e) $\int \sin^2 x \, dx = \int \frac{1}{2}(1-\cos 2x)\, dx$

$= \frac{1}{2}\Big[\underbrace{\int 1\, dx}_{x} - \underbrace{\int \cos \underbrace{2x}\, dx}_{\int \cos u \cdot \frac{1}{2} du}\Big]$

$2x = u$

$2dx = du$

$dx = \frac{1}{2} du$

$= \frac{1}{2}\Big[x - \frac{1}{2}\sin(2x)\Big] + C$

$\int \sin^2 x\, dx = \frac{x}{2} - \frac{1}{4}\sin(2x) + C$

f) Delvis integrasjon

$\int \overset{u'}{\sin x} \cdot \overset{v}{\sin x}\, dx$

$\sin x = u'$

$-\cos x = u$

$$\int u' \cdot v \, dx = u \cdot v - \int u \cdot v' \, dx$$

$$\int \sin^2 x \, dx = -\cos x \sin x - \int (-\cos x) \cos x \, dx$$

$$= -\cos x \sin x + \int \underbrace{\cos^2 x}_{1-\sin^2 x} \, dx$$

$$= -\cos x \sin x + \int 1 \, dx - \int \sin^2 x \, dx$$

flytter over

$$2\int \sin^2 x \, dx = -\cos x \sin x + x + C$$

$$\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin x \cos x}{2} + C$$

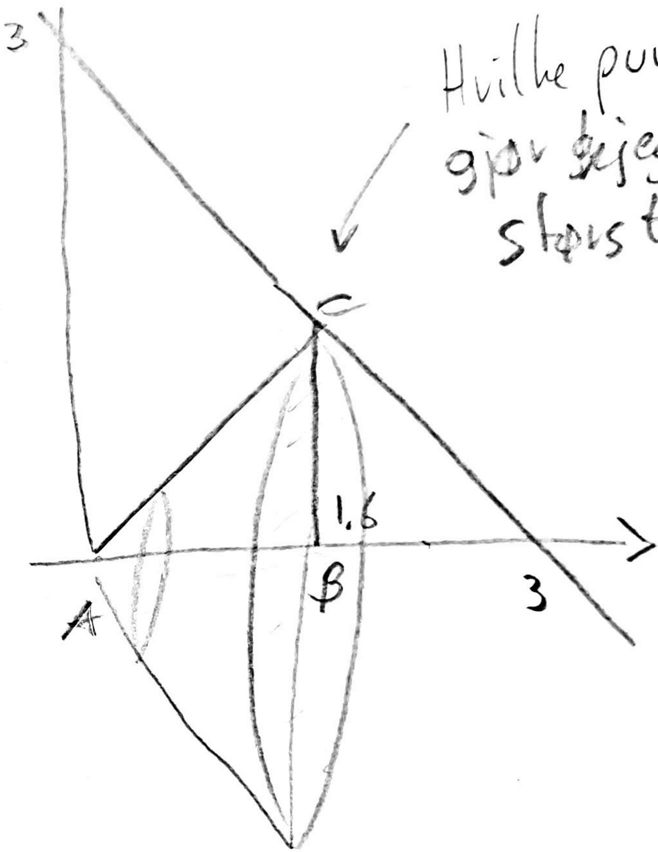
Dette er det samme som i e)

fordi $\sin(2x) = 2\sin x \cos x$

Kont. ex 2018

#5

$f(x) = 3 - x$

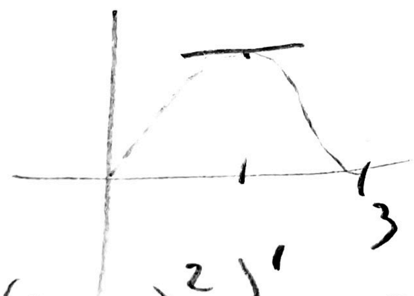


Hvilke punkt C gjør kjeglen størst?

Volum kjegle

$$V = \frac{1}{3} \underset{\uparrow \text{ høyde}}{x} \cdot \underbrace{\pi f(x)^2}_{\text{areal grunnflate}}$$

$$\underline{V(x) = \frac{\pi}{3} x (3-x)^2}$$



$$V'(x) = \frac{\pi}{3} \left(x \cdot (3-x)^2 \right)' = \frac{\pi}{3} \left(1 \cdot (3-x)^2 + x \cdot 2(3-x)\underbrace{(3-x)'}_{-1} \right)$$

$$= \frac{\pi}{3} (3-x)(3 - x - 2x)$$

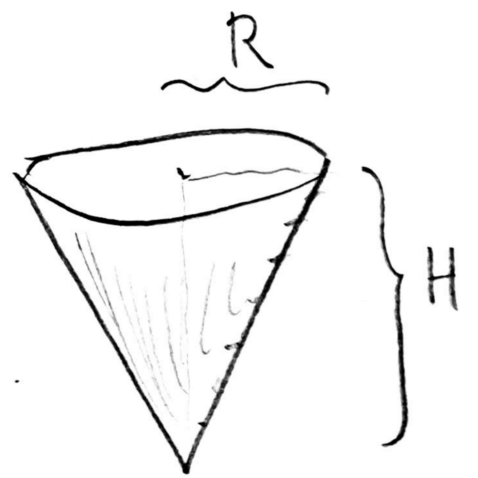
$$V'(x) = \frac{\pi}{3} (3-x)(3-3x) = \underline{\pi (3-x)(1-x)}$$

$V'(x) = 0$ når $x = 1$ og $x = 3$.

Toppunkt for $x = 1$.

$$V(1) = \frac{\pi}{3}(1)(3-1)^2 = \underline{\frac{4\pi}{3}}$$

Volumet er størst for $x = 1$, da er $C(1,2)$ og volumet til kjeglen er $V(1) = \underline{\frac{4\pi}{3}}$

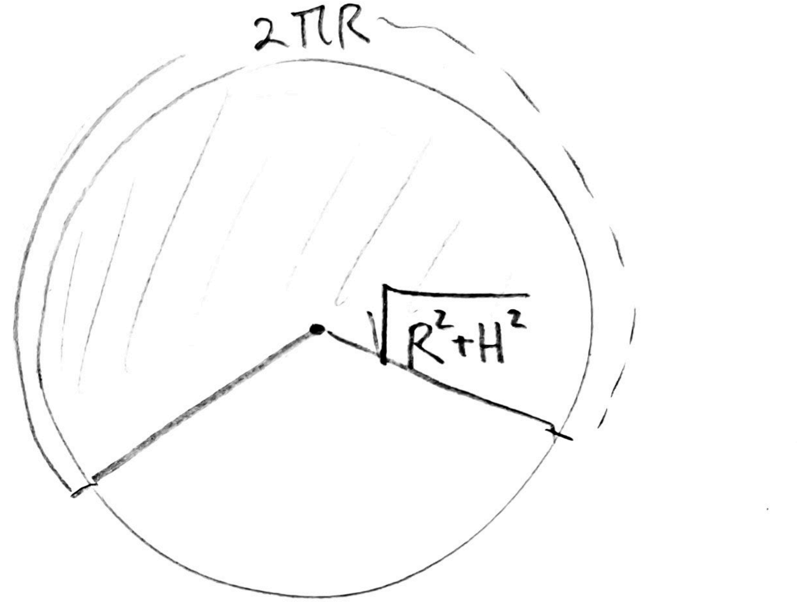


Åpen kjegle

$$V = \frac{\pi}{3} H R^2$$

overflaten:

$$A = 2\pi R\sqrt{R^2+H^2}$$



$V = 1$. **når er A minst mulig?** (Litt vanskelig oppgave)

$$H = \frac{3V}{\pi} \cdot \frac{1}{R^2}$$

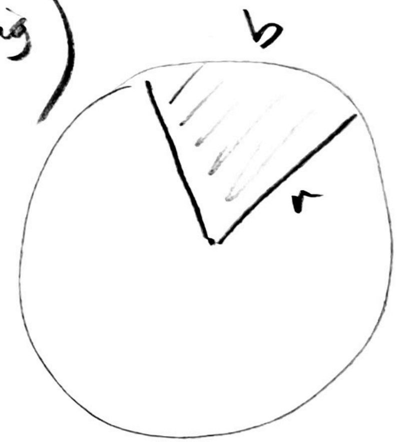
$$A(R) = 2\pi R\sqrt{R^2 + \left(\frac{3V}{\pi} \cdot \frac{1}{R^2}\right)^2}$$

> 0

$A(R) \to \infty$ hvis $R \to 0^+$ eller $R \to \infty$

$A(R)$ minst når A^2 er minst

$$A^2 = 4\pi^2 R^2\left(R^2 + \left(\frac{3V}{\pi} \cdot \frac{1}{R^2}\right)^2\right)$$



$$A = \frac{1}{2} b \cdot r$$

$$(A^2)' = 0 \ ?$$

$$= 4\pi^2 \left[R^4 + \left(\frac{3V}{\pi}\right)^2 \cdot \frac{1}{R^2} \right]'$$

$$= 4\pi^2 \left[4R^3 + \left(\frac{3V}{\pi}\right)^2 \cdot \frac{-2}{R^3} \right] = 0$$

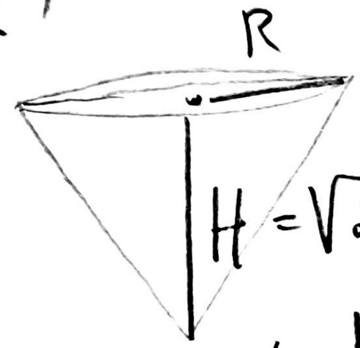
$$4R^3 = \left(\frac{3V}{\pi}\right)^2 \frac{2}{R^3}$$

$$2R^6 = \left(\frac{3}{\pi}\right)^2 \cdot \underbrace{\left(\frac{\pi}{3} \cdot H \cdot R^2\right)^2}_{V}$$

$$2R^6 = H^2 \cdot R^4$$

$$2 = \frac{H^2 \cdot R^4}{R^6} = \left(\frac{H}{R}\right)^2$$

$$\underline{\frac{H}{R} = \sqrt{2}}$$



$H = \sqrt{2} R$

A minst!

2016 5e) Rett linje parametrisert ved

$$l : \begin{cases} x = 1+t \\ y = 3-8t \\ z = 1+kt \end{cases} \qquad t \in \mathbb{R}$$

Hva må k være for at l skal skjære planet

Linja går gjennom $(1,3,1)$ $(t=0)$ og har retningsvektor $[1,-8,k]$,

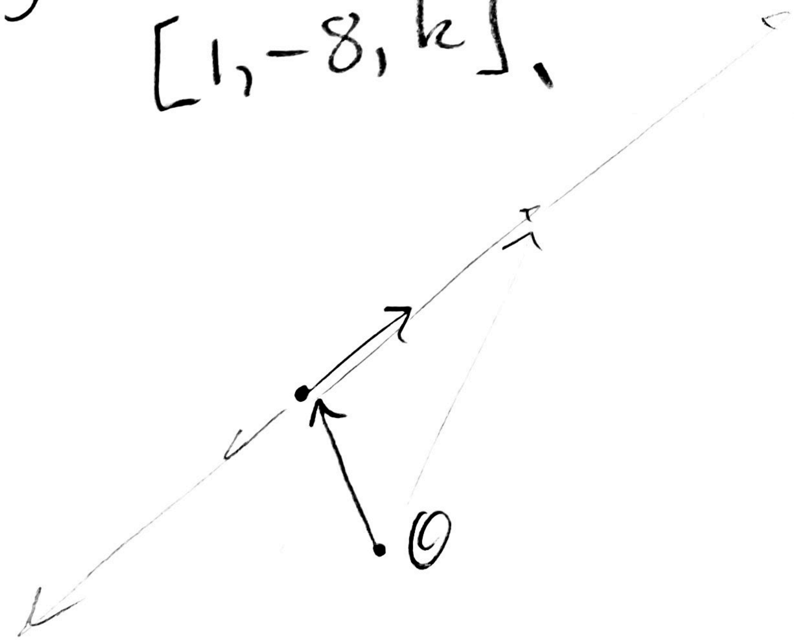
$(2,-5,0)$

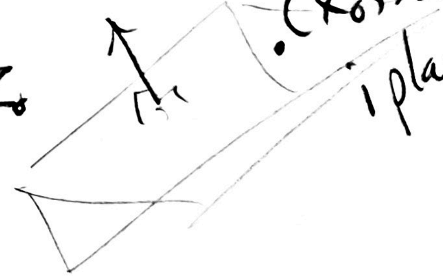
$$ax + by + cz = d$$

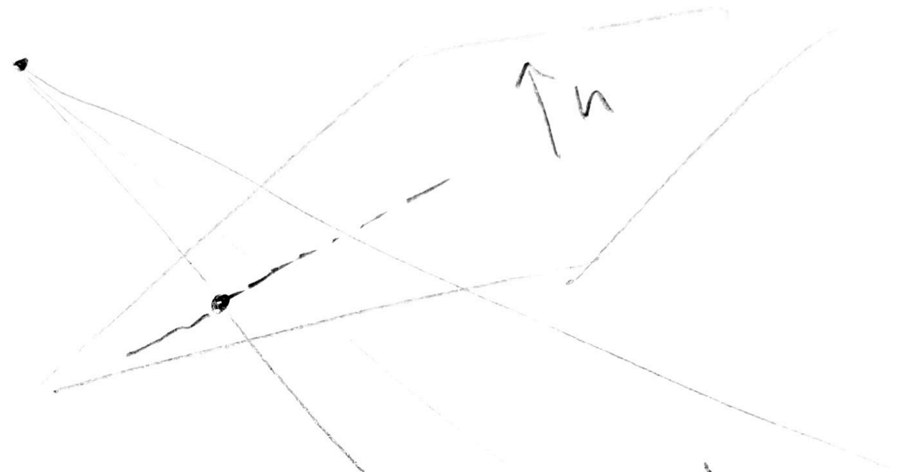
$[a,b,c]$ normalvektor

$$d = ax_0 + by_0 + cz_0$$

(x_0, y_0, z_0) i planet







$$\left(a(1+t) + b(3-8t) + c(1+kt) = d \right.$$

$\left. \text{gir } t \text{ og da punktet på linjen som møter planet.} \right)$

$z=0 \qquad z = 1 + k \cdot t = 0$

$x=2 \qquad x = 1 + t = 2 \qquad t = 1$

setter inn for z:

$1 + k \cdot 1 = 0$

Så $k = -1$.

$\left(y = -5 \qquad y = 3 - 8t = -5 \right)$

$t = 1$