

23aug 2021 Lineære ligninger
Kap 2.

(1)

$$\begin{aligned} 2x + 5 &= 9 && \text{lineær ligning} \\ 2x^2 + 4x + 5 &= 9 && \text{kvadratisk} \\ x^3 &= -27 && \text{3.grads ligning.} \end{aligned}$$

Ligning er påstand.

Løsning til en ligning.

$2x + 5$ skal være like 9
Verdi til x som gir
påstanden sann.

Så $x = 2$ er en løsning til

$$2x + 5 = 9$$

frekket fra 5 på beggesider

$$2x + 5 = 9$$

$$2x + 5 - 5 = 9 - 5 = 4$$

$$2x = 4 \text{ ganger med } \frac{1}{2} \text{ på begge sider}$$

$$x = \frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 4 = 2.$$

$$3x + 7 = -6$$

$$(2) \quad 3x = -6 - 7 = -13 \quad | \cdot \frac{1}{3}$$

$$x = \frac{1}{3}3x = \frac{1}{3} \cdot (-13) = \underline{\underline{-\frac{13}{3}}} = -\left(\frac{12+1}{3}\right) = -4,333\dots$$

$= -4,\underline{3} = -4,\overline{3}$

$$\begin{array}{rcl} 4 & = & 3 \quad \text{galt!} \\ 3 & = & 3, \quad 2+1 = 3 \quad \text{sant.} \end{array}$$

$= 4,123123123\dots$
 $= 4,\underline{123}$

$$\boxed{\begin{array}{l} a = b \quad \Leftrightarrow a+c = b+c \\ \qquad \text{Samhets-} \\ \qquad \text{verdien} \\ a = b \quad \Leftrightarrow c \cdot a = c \cdot b, \quad c \neq 0. \end{array}}$$

$a+d = b$ legge til $-d$ på begge sider av likhets tegn
 $c = a+d-d = b-d$ så $a+d=b \Leftrightarrow a=b-d$
 \Rightarrow Flytter d over på andre siden
 av likhetsregnet og snur fortegn

$$ax + b = 0$$

x variabel
 a, b parametere.

(Velger verdier for a, b ,
 $a = 2, b = -3$ gir)

$$2x - 3 = 0$$

$$ax + b = 0 \Leftrightarrow ax = -b \Leftrightarrow x = \frac{-b}{a} \quad a \neq 0.$$

$$a=0 : \quad 0 \cdot x + b = 0 \Leftrightarrow b = 0$$

$$ax + b = 0 \quad \text{(løsning)} \quad \begin{aligned} &\text{alle } x \text{ hvis } b = 0 \\ &\text{ingen } x \text{ - } b \neq 0 \end{aligned}$$

$$0 \cdot x + 2 = 0 \Leftrightarrow 2 = 0 \quad \text{ingen løsning.}$$

$$0 \cdot x + 0 = 0 \quad \begin{aligned} &0 = 0 \\ &(\text{for alle } x) \end{aligned} \quad \text{alle } x \text{ er en løsning!}$$

$$a) \quad 4x + 17 = 5$$

Forslag $x = -3$: setter inn:

$$4(-3) + 17 = -12 + 17 = 5 \quad \checkmark$$

$$4x = 5 - 17 = -12 \quad | \cdot \frac{1}{4}$$

$$\frac{1}{4}x = x$$

$$\underline{x = \frac{1}{5}}$$

$$b) \quad -3x + 1 = 2x \quad | \cdot \frac{1}{2}$$

$$| = 2x + 3x$$

$$1 = (2+3)x = 5x \quad | \cdot \frac{1}{5}$$

$$x = \frac{-12}{4} = -3$$

c)

$$(x-1) = \frac{7}{2}$$

$$x = \frac{7}{2} + 1 = \frac{7}{2} + \frac{2}{2} = \frac{9}{2}$$

$$= 3,5 + 1 = \underline{4,5}$$

d)

$$c) \quad 2(x-1) = 7$$

$$Y = 30 + 5X$$

$$100 = 30 + 5X$$

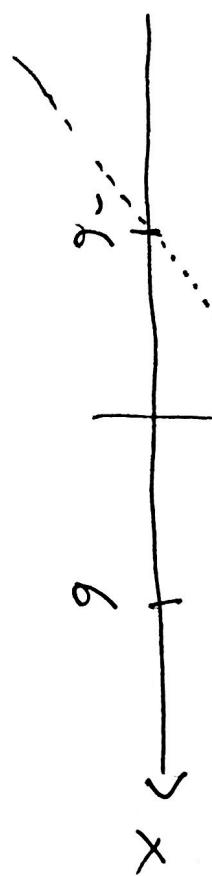
$$70 = 5X \quad | \frac{1}{5}$$

$$\underline{X = 14}$$

Grafen til

$$Y = ax + b$$

er en linje

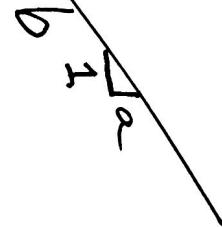


(5)

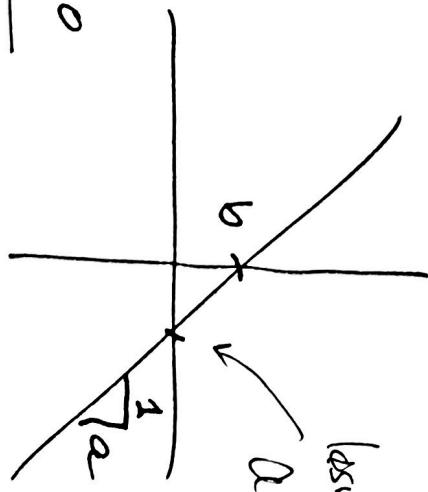
$$Y = ax + b$$

$$ax + b = 0$$

(lösning til
 $ax + b = 0$)



$$a > 0$$



$$a < 0$$

(lösning til
 $ax + b = 0$)

$$ax + b = 0$$

Løs likningene

b) $3x - 7 = 5x + 2$

$$3x - 5x = 2 + 7$$

$$(3-5) \cdot x = 9$$

$$-2x = 9$$

$$x = \frac{9}{-2} = \frac{-9}{2} = -4,5$$

(6)

a) $3,714x - 5,223 = 0$

$$x = \frac{5,223}{3,714}$$

$$\approx 1,406$$

Kvadratsetningen

$$(a+b)^2 = a^2 + \underbrace{2ab} + b^2$$

keyssleddet

(7)

$$\begin{aligned} (a+b)(a+b) &= (a+b) \cdot a + (a+b) \cdot b \\ &= a \cdot a + b \cdot a + a \cdot b + b \cdot b \quad \left. \begin{array}{l} \text{transitivitet} \\ \text{bwkes} \\ \text{3 ganger} \end{array} \right\} \\ &= a^2 + a \cdot b + a \cdot b + b^2 \\ &= \underline{\underline{a^2 + 2ab + b^2}} \end{aligned}$$

$$\begin{aligned} (a-b)^2 &= a^2 - 2ab + b^2 \\ (a+(-b))^2 &= a^2 + 2 \cdot a \cdot (-b) + (-b)^2 = a^2 - 2ab + b^2 \\ (2x-3)^2 &= (2x)^2 + 2(2x)(-3) + (-3)^2 \\ (2+3)^2 &= 5^2 = 25 \\ &= 2^2 + 2 \cdot (2)(3) + 3^2 \\ &= \underline{\underline{4 + 12 + 9 = 25}} \end{aligned}$$

(8)

$$(3-4)^2 = (-1)^2 = 1$$

$$(a-4)^2 = a^2 + (-4)^2 + 2 \cdot a(-4)$$

$$= a^2 + 16 - 8a$$

$$= \underline{a^2 - 8a + 16}$$

$$(3x + \underbrace{\sqrt{2}y}_a)^2 = (3x)^2 + 2(3x)(\sqrt{2}y) + (\sqrt{2}y)^2$$

$$= 9x^2 + 6\sqrt{2}xy + 2y^2.$$

$$(a+b+c)^2 = \dots \quad a^2 + b^2 + c^2 + 2(ab + ac + bc)$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^3 = (a+b)^2 \cdot (a+b) \dots$$

Konjugatsætning
(3. kvadratsætning)

$$b^2 - a^2 = (b+a)(b-a)$$

$$(b+a)(b-a) = (b+a) \cdot b + (b+a)(-a)$$

$$= b^2 + ab + \underbrace{b(-a)}_{-a \cdot b} - a^2$$

$\underbrace{0}_{\text{korrigerede koeffiserne}} \quad \text{kan skrives til}$

$$(x-3)(x+3) = x^2 - 9 = x^2 - 3^2$$

$$= (20)^2 - 3^2 = (20+3)(20-3)$$

$$391 = 400 - 9$$

differance av
kvadratene til
2 heltall.

$$391 = \underline{\underline{23 \cdot 17}}$$