onsdag 25.08 2021

Andregradsformelen

Faush

oppsummering

Kvadratsetning $(a+b)^2 = a^2 + 2ab + b^2$

Konjugatsetning $b^2 - a^2 = (b+a)(b-a)$

Faktorisere

$$x^3 - 4x = x(x^2-4) = x(x^2-2^2)$$
$$= x(x+2)(x-2)$$

Rasjonalt uttrykk

$$\frac{x^3-4x}{x^2+4x+4} = \frac{x(x+2)(x-2)}{(x+2)^2} = \frac{x+2}{x+2} \cdot \frac{x(x-2)}{x+2}$$
$$= \frac{x(x-2)}{x+2}, \quad x \neq -2.$$

Fullføre kvadrat

$$x^2+6x+c = \underbrace{(x+3)^2}_{x^2+6x+9} - 9 + c$$

$$x^2+bx+c = \left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

$ax^2 + bx + c$ 2.grads uttrykk $a \neq 0$

2.gradslikning $ax^2 + bx + c = 0$

(2) $b = 0$ $\quad ax^2 + c = 0$

$x^2 + \frac{c}{a} = 0$

$\frac{c}{a} > 0$ ingen løsning

$\frac{c}{a} = 0 \;:\; x = 0$

$x^2 = \frac{-c}{a}$ $\quad x = \pm\sqrt{\frac{-c}{a}}$

$\frac{c}{a} < 0$ så $-\frac{c}{a} > 0$,

$x^2 + \frac{c}{a} = x^2 - \left(-\frac{c}{a}\right) = x^2 - \left(\sqrt{\frac{-c}{a}}\right)^2$

$= \left(x + \sqrt{\frac{-c}{a}}\right)\left(x - \sqrt{\frac{-c}{a}}\right)$

$x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$.

$$ax^2+bx+c=0 \quad \Big| \ \frac{1}{a}$$

③ $x^2+\frac{b}{a}x+\frac{c}{a}=0$ reduseres til tilfellet $a=1$

$a=1$ $x^2+bx+c=0$ Fullfører kvadratet

$$\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c=0$$

$$\left(x+\frac{b}{2}\right)^2=\frac{b^2}{4}-c=\frac{b^2-4c}{4}$$

$$x+\frac{b}{2}=\pm\sqrt{\frac{b^2-4c}{4}}=\frac{\pm\sqrt{b^2-4c}}{2}$$

$$x=\frac{-b\pm\sqrt{b^2-4\cdot c}}{2}=\frac{-b\pm\sqrt{b^2-4c}}{2}$$

konjugatsetningen

$$\left(x+\frac{b}{2}\right)^2-\left(\left(\frac{b}{2}\right)^2-c\right)=0$$

$$x^2+bx+c=\left(x+\frac{b}{2}+\sqrt{\frac{b^2}{4}-c}\right)\left(x+\frac{b}{2}-\sqrt{\frac{b^2}{4}-c}\right)=0$$

$$2x^2 - 3x - 5 = 0$$

$a = 2 \quad b = -3 \quad c = -5$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 2(-5)}}{2 \cdot 2} = \frac{3 \pm \sqrt{9 + 40}}{4}$$

$$= \frac{3 \pm \sqrt{49}}{4} = \frac{3 \pm 7}{4}$$

④

$$x = \frac{3+7}{4} = \frac{10}{4} = \frac{5}{2} = 2.5 \quad \text{og} \quad \frac{3-7}{4} = \frac{-4}{4} = -1$$

Løsningene er $x = -1$ og 2.5.

$$2x^2 - 3x - 5 = 2\left(x - (-1)\right)\left(x - \frac{5}{2}\right)$$

$$= 2(x+1)\left(x - \frac{5}{2}\right)$$

$$= \underline{(x+1)(2x-5)}$$

Generelle tilfellet

$$ax^2+bx+c = a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$$

$$= a\left(x+\frac{b}{2a}+\sqrt{\frac{(b/a)^2-4c/a}{4}}\right)\left(x+\frac{b}{2a}-\sqrt{\frac{(b/a)^2-4c/a}{4}}\right)$$

Forenkler $\sqrt{\frac{(b/a)^2-4c/a}{4}} = \sqrt{\frac{b^2-4a\cdot c}{4a^2}} = \frac{\sqrt{b^2-4ac}}{2|a|}$

↑ "utvider med a^2"

(5)

siden $\sqrt{a^2} = |a|$ | absoluttverdien til a

$$= \begin{cases} -a & a<0 \\ a & a\geq 0 \end{cases}$$

$\sqrt{(-2)^2} = \sqrt{4} = 2$

$\sqrt{2^2} = \sqrt{4} = 2$

$$ax^2+bx+c = a\left(x+\frac{b}{2a}+\frac{\sqrt{b^2-4ac}}{2a}\right)\left(x+\frac{b}{2a}-\frac{\sqrt{b^2-4ac}}{2a}\right)$$

Løsningene til $ax^2+bx+c=0$ "abc-formel"

er $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ 2.gradsformelen

1) $3x^2 - x - 5 = 0$

2) $3x^2 - 13 = 0$

3) $2x^2 + 7x = 0$

4) $x^2 - 2x + 3 = 0$

1) $a = 3$, $b = -1$, $c = -5$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 3 \cdot (-5)}}{2 \cdot 3}$$

$$= \frac{1 \pm \sqrt{1 + 60}}{6} = \frac{1 \pm \sqrt{61}}{6}$$

$$= (1 \pm \sqrt{61})/6$$

$$= (1 \pm \sqrt{(61)})/6$$

(6)

Løsningen er

$x = \frac{1}{6}(1 + \sqrt{61})$ og $x = \frac{1}{6}(1 - \sqrt{61})$

2) $3x^2 + 0 \cdot x - 13 = 0$

$b = 0$

$3x^2 = 13$

$x^2 = 13/3$

$x = \pm\sqrt{\frac{13}{3}}$

$$x = \frac{-0 \pm \sqrt{0^2 - 4ac}}{2a}$$

$$= \pm \frac{2\sqrt{-ac}}{2a} = \sqrt{\frac{-c}{a}}$$

$$= \pm \frac{\sqrt{-ac}}{\sqrt{a^2}} = \pm\sqrt{\frac{-ac}{a^2}}$$

$$= \pm\sqrt{\frac{-c}{a}}$$

3) $2x^2 + 7x + 0 = 0$

$a=2 \quad b=7 \quad c=0$

$x(2x+7) = 0$

$x=0$ eller $2x+7=0$

$x = \frac{-7}{2}$

$x = \frac{-b \pm \sqrt{b^2-0}}{2a}$

$= \frac{-b \pm |b|}{2a}$

gir $x=0$ og $x = \frac{-b}{a}$

4) $x^2 - 2x + 3 = 0$

$a=1 \quad b=-2 \quad c=3$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$

$= \frac{2 \pm \sqrt{4-12}}{2}$

$= \frac{2 \pm \sqrt{-8}}{2}$ ingen reelle løsninger.

(7)

($1 \pm \sqrt{2}i$)

Løsninger som komplekse tall

oppg Faktoriser

(8)

$$P(x) = 3x^2 - 12x + 12.$$

$$= 3(x^2 - 4x + 4)$$

$$= 3\left((x-2)^2 \underbrace{-(-2)^2 + 4}_{0}\right) = 3(x-2)^2$$

$$P(x) = 3(x-x_1)(x-x_2)$$

x_1, x_2 røtter
(løsninger til $P(x) = 0$)

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 3 \cdot 12}}{2 \cdot 3} \quad \ldots \text{ tungvint}$$

Deler først med 3 : $x^2 - 4x + 4 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 4}}{2} = \frac{4 \pm 0}{2}$$

$$= \frac{4}{2} = \underline{2}$$

Tall på standard form. $r = \pm a \cdot 10^n$ n heltall

$1 \leq a < 10$

$1234 = 1{,}234 \cdot 10^3$

$0.00003478 = 3{,}478 \cdot 10^{-5}$

(9) $4578932 = 4{,}578932 \cdot 10^6$

Med 4 gyldige siffer: $4{,}578 \cdot 10^6$

(rundet opp $4{,}579 \cdot 10^6$)

$r(1+a) + s(1+b) = r+s + (a \cdot r + b \cdot s)$

$r+s\left(1 + \frac{a \cdot r + b \cdot s}{r+s}\right)$

$(1,1)(1,1) = 1,21$

10% 10% ~20%

avvik avvik.

$a \cdot b \cdot c$ relativ usøyaktighet

er ~summen av relativ usøyaktighet til a, b og c.

(10)

$1^{10} = 1$

$(1,1)^{10} = 2.59...$

10% 160% avvik.

avvik

Formler

radius

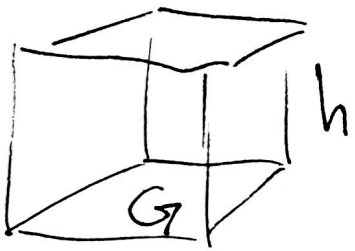
Areal $A = \pi \cdot r^2$

$\pi = 3.14....$
irrasjonalt tall.

← omkrets

$O = 2\pi \cdot r$

(11)



Volum

$V = G \cdot h$ ← høyde.

↑ areal grunnflate

sylinder

$V = \pi r^2 \cdot h$

$$h = \frac{V}{\pi r^2}$$

$$r^2 = \frac{V}{\pi \cdot h}$$

$$r = \sqrt{\frac{V}{\pi \cdot h}} \qquad (r > 0)$$