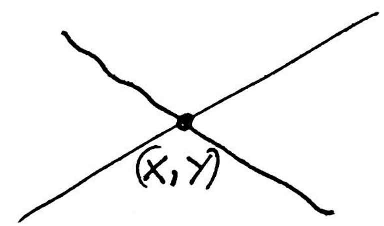
30 august   Likningssystem   3.1 & 2 i boka

Lineært likningssystem

$$\begin{cases} x + 2y = 1 \\ 2x - y = -3 \end{cases}$$



(1)

Løsninger er x, y som gjør begge likningene sanne (påstandene)

Innsettingsmetoden: Likning 1 gir $x = 1 - 2y$ (x bestemt av y)

setter dette inn for x i Likning 2

$$2(1-2y) - y = -3$$
$$2 - 4y - y = -3$$
$$-5y = -3 - 2 = -5 \quad \Big| \cdot \frac{1}{(-5)}$$
$$\underline{y = 1}$$
$$x = 1 - 2y = 1 - 2 \cdot 1 = \underline{\underline{-1}}$$

setter inn og sjekker svaret ✓

Løsningen til likningssystemet er $\underline{x = -1 \text{ og } y = 1}$

$x + y = 5 \quad \Leftrightarrow \quad y = 5 - x$

$2x - y = 1$

setter inn i den 2. likningen

$2x - (\overbrace{5 - x}^{y}) = 1$

$2x - 5 + x = 1$

$3x = 1 + 5 = 6 \quad | \cdot \frac{1}{3}$

$x = \frac{6}{3} = \underline{2}$

Så $y = 5 - 2 = 3$

Løsningen er $\underline{x = 2 \text{ og } y = 3}$

$(x, y) = (2, 3)$

(Sjekker:
$x + y = 2 + 3 = 5$ ✓
$2x - y = 2 \cdot 2 - 3 = 1$ ✓)

②

$a = b$
$c = d$ $\Leftrightarrow$ $ka + c = kb + d$

$a = b$

k konstant.

addisjonsmetoden.

$L1 + L2$ (likning 1 + likning 2)

$x + (2x) + y + (-y) = 5 + 1$

$3x = 6$

$x = \underline{2}$

L1 gir $y = 5 - x = \underline{3}$

Oppg

$$2x + y = 4$$
$$x + 2y = 5$$

$(-2) \cdot L2 + L1$:

addisjonsmetoden

$$\left(-2 \cdot L2: \quad -2x - 4y = -10\right)$$
$$(2x + y) - (2x + 4y) = 4 - 10$$
$$-3y = -6$$
$$\underline{y = 2}$$
$$x = 5 - 2 \cdot y = 5 - 4 = 1$$

Løsningen e $\underline{(x, y) = (1, 2)}$

3

Oppg

$$2x + y = 3$$
$$x - 3y = 4$$

L2 gir $x = 4 + 3y$

setter inn i L1 : $2(4 + 3y) + y = 3$

$$8 + 6y + y = 3$$
$$7 \cdot y = 3 - 8 = -5 \quad | \tfrac{1}{7}$$

$\underline{y = \frac{-5}{7}}$ og $x = 4 + 3 \cdot y = \frac{4 \cdot 7}{7} + \frac{-3 \cdot 5}{7} = \frac{28 - 15}{7} = \underline{\frac{13}{7}}$

Løsningen e $\underline{\left(\frac{13}{7}, -\frac{5}{7}\right)}$

Oppg

$$12x + 4y = 3$$
$$6x + 2y = 2$$

$2 \cdot L2 : 12x + 4y = 4$

$L1 - (2L2) = L1 + (-2L2)$

$$0 = 3 - 4 = -1$$

aldri sant

Finner ingen løsning.

$$3x + y = 5$$
$$4x - 3y = 7$$

$\frac{4}{3} \cdot L1: \quad 4x + \frac{4}{3}y = \frac{4}{3} \cdot 5$

$L2 - \frac{4}{3}L_1: \quad -3y - \frac{4}{3}y = 7 - \frac{20}{3}$

ganger opp med 3

$$-9y - 4y = 21 - 20$$
$$-13y = 1$$

4

$$\underline{\underline{y = \frac{-1}{13}}} \quad \left(= \frac{1}{-13}\right)$$

$$3x = 5 - y$$
$$x = \frac{1}{3}(5 - y) = \frac{1}{3}\left(5 + \frac{1}{13}\right)$$
$$= \frac{1}{3} \cdot \frac{65+1}{13} = \frac{66}{3} \cdot \frac{1}{13}$$
$$\underline{\underline{x = \frac{22}{13}}}$$

Oppg $\begin{aligned} 2x + y &= 3 \\ x - 3y &= 4 \end{aligned}$ med addisjonsmetoden

$3L1 + L2: \quad 3(2x + y) + (x - 3y) = 3 \cdot 3 + 4$

$$6x + 3y - 3y + x = 9 + 4 = 13$$
$$7x = 13$$
$$\underline{x = 13/7}$$

$L1$ gir $y = 3 - 2x = \frac{3 \cdot 7 - 2 \cdot 13}{7} = \underline{\underline{\frac{-5}{7}}}$

$L1: \quad ax + cy = e \qquad a \neq 0$

$L2: \quad bx + dy = f$

$\frac{b}{a}L1: \quad b \cdot x + \frac{b \cdot c}{a}y = \frac{b \cdot e}{a}$

$L2 - \frac{b}{a}L1: \quad dy - \frac{bc}{a}y = f - \frac{b \cdot e}{a}$

$(bx - bx = 0)$

Alternativt: $bL1 - aL2 \ldots$

$$\cdot 2 \begin{cases} x+y+z=6 \\ x-y+z=2 \\ 2x-2y+z=1 \end{cases} \iff \begin{aligned} x+y+z&=6 \\ 2x+0+2z&=6+2=8 \quad \Big| \tfrac{1}{2} \\ 2x+2x+0+2z+z&=12+1=13 \end{aligned}$$

$$\iff \begin{aligned} x+y+z&=6 \\ x+z&=4 \\ 4x+3z&=13 \end{aligned} \;\Big)(-3) \iff \begin{aligned} x+y+z&=6 \\ x+z&=4 \\ \underbrace{(4x-3x)+(3z-3z)}_{x\;+\;0}&=13-3\cdot 4 \\ &=1 \end{aligned}$$

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Så $x=1$, $\quad z=4-x=4-1=3$

$y=6-x-z=6-1-3=2$

Løsningen er $(x,y,z)=\underline{(1,2,3)}$

Per og Kari er 50 år tilsammen.
Per er 10 år eldre enn Kari.

$$P + K = 50$$
$$\underbrace{P = K + 10}$$
$$P - K = 10$$

Likningssystem med 2 variabler P, K
2 likninger.

Alder til Per : P
Kari : K

(6)

$P = 30$ år
$K = 20$ år

3.2
$$x^2 + y = 4$$
$$2x + y = 1$$

L2 gir $y = 1 - 2x$ setter inn i L1

$$x^2 + (1 - 2x) = 4$$
$$x^2 - 2x + 1 - 4 = 0$$
$$(x - 1)^2 - 1 - 3 = 0$$
$$(x - 1)^2 = 4 \quad , \quad x = 1 \pm 2$$

2-grads likning i x

$\underline{x = -1}$, $y = 1 - 2(-1) = \underline{3}$
$x = 3$, $y = 1 - 2(3) = \underline{\underline{-5}}$

Løsningene er
$(-1, 3)$
$(3, -5)$

$x^2 + y^2 = 4$

$x - 2y = 0 \iff x = 2y \implies x^2 = (2y)^2 = 4y^2$

Setter inn i L1: $x^2 + y^2 = (2y)^2 + y^2 = 5y^2 = 4$

$y^2 = \frac{4}{5}$ så $y = \pm\sqrt{\frac{4}{5}} = \pm \frac{\sqrt{4}}{\sqrt{5}} = \pm \frac{2}{\sqrt{5}}$

$x = 2 \cdot y$ Løsningene er $\underline{\left(\frac{4}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)}$

$\underline{\left(\frac{-4}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\right)}$

7

Øvingstime

oppg. 8

$$2x - y = -3$$
$$4x + 3y = 5$$

$-2 \cdot L1 + L2$:

$$-2(2x - y) + 4x + 3y = (-2)(-3) + 5$$
$$2y + 3y = 11$$
$$5y = 11 \quad \Big| \cdot \frac{1}{5}$$
$$y = \frac{11 \cdot 2}{5 \cdot 2} = \frac{22}{10} = \underline{2{,}2}$$

$$2x = -3 + y$$
$$x = \frac{1}{2}(y - 3)$$
$$= \frac{1}{2}\left(\frac{11}{5} - 3\right) = \frac{1}{2}\left(\frac{11 - 15}{5}\right)$$
$$= \frac{1}{2} \cdot \frac{-4}{5} = \frac{2}{2} \cdot \left(\frac{-2}{5}\right) = \frac{-2}{5} = \frac{-4}{10} = \underline{-0.4}$$

Løsningen er $\underline{(-0.4, 2.2)}$

Alternativt $3 \cdot L1 + L2$ da kansellers y-ledden

$$3 \cdot 2x \underbrace{- 3y + 4x + 3y}_{} = 3(-3) + 5$$
$$10x = -9 + 5 = -4 \text{ så } x = -0.4$$

etc.

$$x + 3y = 4$$
$$-x + 2y = 1$$

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L1+L2 $\quad \underbrace{x-x}_{0} + 3y + 2y = 4+1$

$$5y = 5 \quad \Big| \tfrac{1}{5}$$

$$y = 1$$

L1 gir $\quad x = 4 - 3y = 4 - 3\cdot 1 = 1$

Løsningen er $(x,y) = \underline{(1,1)}$

3.20 a) $x - y = 1$
$-x^2 + 4x + y = 3$

innsettingsmetoden $\quad$ L1: $y = x - 1$

setter inn i L2:

$$-x^2 + 4x + (x-1) = 3$$
$$-x^2 + 5x - 1 - 3 = 0 \quad \Big| \cdot(-1)$$
$$x^2 - 5x + 4 = 0$$
$$\left(x - \tfrac{5}{2}\right)^2 - \left(\tfrac{5}{2}\right)^2 + 4 = 0$$
$$\left(x - \tfrac{5}{2}\right)^2 + \frac{-25+16}{4} = 0$$

$$\left(x - \tfrac{5}{2}\right)^2 = \frac{9}{4} = \left(\frac{3}{2}\right)^2$$

så $x - \frac{5}{2} = \pm \frac{3}{2}$, $\quad x = \frac{5}{2} \pm \frac{3}{2}$

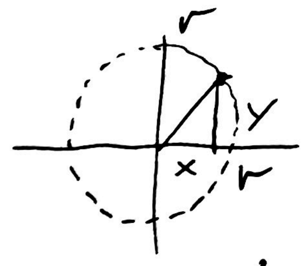
$$x = 4, \quad x = 1.$$

$(x-4)(x-1) = x^2 - 5x + 4$

Løsningene er $\underline{(1,0)}$ og $\underline{(4,3)}$

3.21 b)

$$x+y=2$$
$$x^2-4x+y^2-6y=-4$$
$$(x-2)^2-4+(y-3)^2-9=-4$$
$$(x-2)^2+(y-3)^2=9=3^2$$



$x^2+y^2=r^2$

Sirkel med radius 3 og senter i (2,3).

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$y=2-x$ setter inn i $L2$

$$x^2-4x+(2-x)^2-6(2-x)=-4$$
$$x^2-4x+4+x^2-4x-12+6x=-4$$
$$2x^2-2x-8=-4 \quad | \cdot \tfrac{1}{2}$$
$$x^2-x-4=-2$$
$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$

$x=-1 \qquad y=2-(-1)=3$

$x=2 \qquad y=2-2=0$

Løsningene er $(-1,3)$ og $(2,0)$