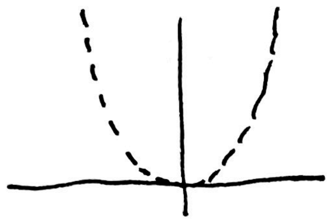
Øving Fredag 10 sep 2021

$y = x^2$

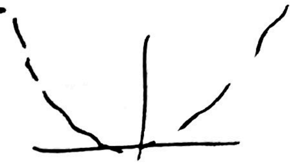
≥ 0

bare lik 0

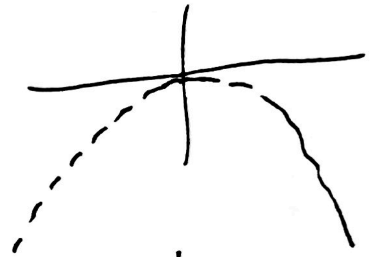
for $x = 0$



$y = ax^2$

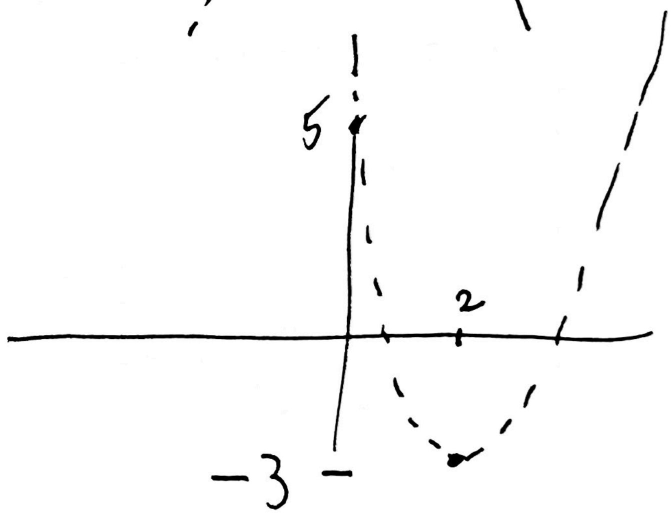


$a > 0$



$a < 0$

$y = +2(x-2)^2 - 3$



$$y = ax^2 + bx + c$$
$$= a\left(x^2 + \frac{b}{a}x\right) + c$$
$$a\left(x + \frac{b}{2a}\right)^2 - a \cdot \left(\frac{b}{2a}\right)^2 + c$$

$$y = ax^2 + bx + c$$
$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$$

oblig 1, ~~opg~~ #6 variasjoner

parabel $y = ax^2 + bx + c$

$\underset{\downarrow}{a}(x-k)^2 + \underset{\downarrow}{c}$ $a < 0$

toppunkt i k

2 variabler.

opg. Toppunkt i $(2, 3)$

går gjennom $(0, 1)$.

$y = a(x-2)^2 + c$

setter inn koordinatene til punktene som skal være på parabelen

$\underline{3} = y(2) = a(0)^2 + \underline{c}$ og $1 = y(0) = a(-2)^2 + c$

$c = 3$

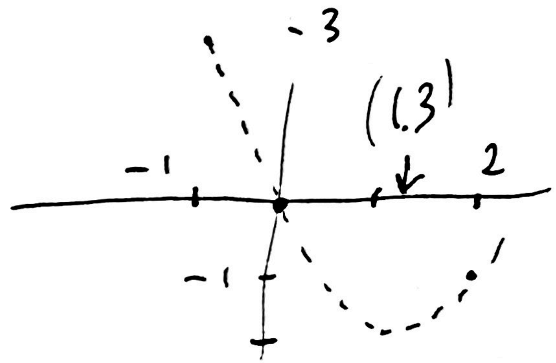
$1 = 4a + c$

$4a = 1 - c = 1 - 3$

$= -2$

$a = \frac{-2}{4} = \frac{-1}{2}.$

$$y(x) = \underline{\underline{-\frac{1}{2}(x-2)^2 + 3}}$$



<u>oppg</u>: Finn parabelen som går gjennom

$(-1, 3)$, $(0, 0)$, $(2, -1)$

origo

$y = ax^2 + bx + c$

$(-1,3)$: $3 = a - b + c = a - b$

$\underline{0 = c}$

$-1 = 4a + 2b + c = 4a + 2b$

L1 $a - b = 3$

L2 $4a + 2b = -1$

2L1 + L2

$$2a + 4a + (\overbrace{-2b + 2b}^{0}) = 6 - 1$$

$$6a = 5$$

$$\underline{a = 5/6}$$

$$b = a - 3 = \frac{5}{6} - 3 = \frac{5 - 18}{6} = \frac{-13}{6}$$

Parabelen er $y = \frac{5}{6}x^2 - \frac{13}{6}x$

$$y = \frac{5}{6}\left(x^2 - \frac{13}{5}x\right)$$

$$= \frac{5}{6}\left(\left(x - \frac{13}{10}\right)^2 - \left(\frac{13}{10}\right)^2\right)$$

$$= \frac{5}{6}\left(x - \frac{13}{10}\right)\Big)^2 - \frac{5}{6}\left(\frac{169}{100}\right)$$

$$\underline{= \frac{5}{6}\left(x - \frac{13}{10}\right)^2 - \frac{169}{120}}$$

13.sep
2021

Kap 5 Polynomdivisjon.

Polynom uttrykk : eks x^2-3x+2 (grad 2), $x-5$ (grad 1), 10 (grad 0)

$-3x^{12}+10x^5-7$ grad 12

1. x^7-3x^5+12x grad 7,

ledende koeffisient

generelt $p(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$

konstant-leddet.

a_i koeffisienter. $a_n \neq 0$

Graden til $p(x)$ er n $(a_n \neq 0)$

høyeste eksponent til x som forekommer

$p(x)$ gir en funksjon med naturlig definisjonsmengde $\mathbb{R}$

oppg. Hva er graden til

1) $(x+1)(x-1)+3 = x^2+2$ grad 2

2) $x^2-7x-x^2 = -7x$ grad 1

3) $0\cdot x^5+3x^4-12x$ grad 4

$p(x) = 0$ n-te grads likning.

$\deg(p(x)) = \text{grad}(p(x)) = n$

② En n-te grads likning har n eller færre løsning. $n \geq 0$

Grafen til et n-te gradspolynom har $(n-1)$ eller færre topp/bunnpunkt. $n \geq 1$

$x^2 + \frac{1}{x}$ er ikkje et polynom.

$x^2 \cdot \frac{1}{x} = x$ er et polynom. $(x \neq 0)$

Polynomer med flere variabler

$x^2 + y^2$ 2 variable

$xy - y^2 + z^2$ 3 variabler

$3x^4 - 7xyz$ —"—

5.2 Polynomdivisjon

$$\frac{p}{q} = \underset{\text{kvotient}}{s} + \underbrace{\frac{r}{q}}_{\text{rest}} \qquad 0 \leq \frac{r}{q} < 1 \qquad q > 0$$

p, q, s, r heltall.

3

$$\frac{10}{3} = \frac{1+3\cdot 3}{3} = 3 + \frac{1}{3}$$

Alternativt:

$$p = s \cdot q + r$$

$$10 = 3 \cdot 3 + 1$$

$$\frac{13}{3} = \frac{4\cdot 3+1}{3} = 4 + \frac{1}{3} \iff 13 = 4\cdot 3 + 1$$

$$-\frac{13}{3} = -4 - \frac{1}{3}, \quad \frac{-13}{3} = \frac{-5\cdot 3+2}{3} = \underline{-5 + \frac{2}{3}}$$

(4) $\dfrac{p(x)}{q(x)} = S(x) + \dfrac{r(x)}{q(x)}$ $\qquad \deg(r(x)) < \deg(q(x))$

deg = grad (engelsk)

$\Leftrightarrow \quad p(x) = S(x) \cdot q(x) + r(x)$

$x \neq 2$

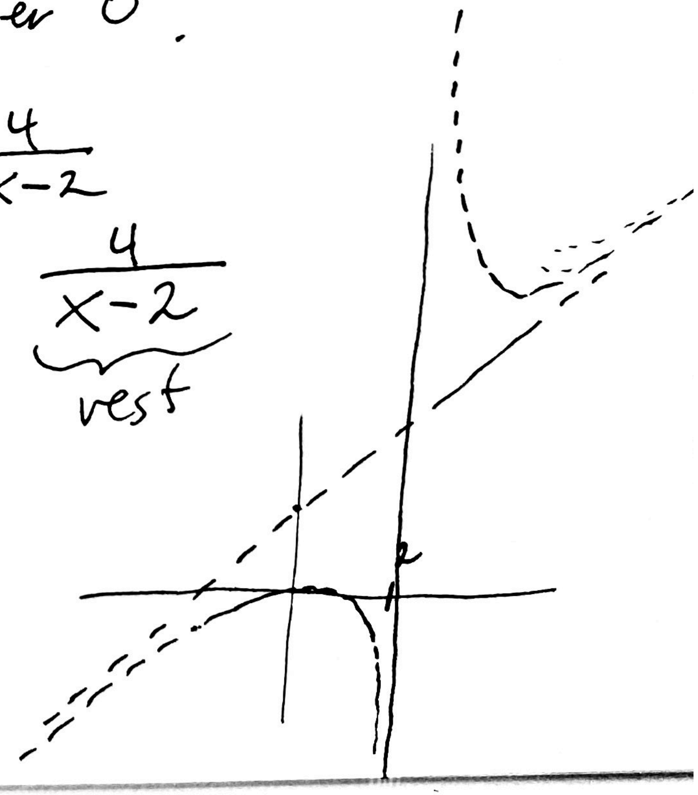
$$\frac{x^2-4}{x-2} = \frac{(x+2)(x-2)}{x-2} = \underbrace{x+2}_{\text{kvotient}}$$

restleddet er 0.

$$\frac{x^2}{x-2} = \frac{x^2 \overbrace{-4+4}^{0}}{x-2} = \frac{x^2-4}{x-2} + \frac{4}{x-2}$$

$$\frac{x^2}{x-2} = \underbrace{x+2}_{\text{kvotient}} + \underbrace{\frac{4}{x-2}}_{\text{rest}}$$

Dette kan for eksempel hjelpe oss å tegne grafen



5

$$\frac{x^2}{x-2} = \frac{x(x-2+2)}{x-2} = \frac{x(x-2)+2x}{x-2}$$

$$= x + \frac{2x}{x-2}$$

$$= x + \frac{2(x-2+2)}{x-2} = \underline{x+2+\frac{4}{x-2}}$$

$$x^2+0\cdot x+0 : x-2 = x+2+\frac{4}{x-2}$$

$$\begin{array}{l} x^2-2x+0 \\ \hline 0 \quad 2x+0 \\ -\;(2x-4) \\ \hline \quad 0+4_{\text{rest}} \end{array}$$

$$\frac{x^3 - 2x}{x+1}$$

6

$$x^3 + 0 - 2x + 0 : x+1 = x^2 - x - 1 + \frac{1}{x+1}$$

$$\begin{array}{l}
\underline{x^3 + x^2} \\
-x^2 - 2x + 0 \\
\underline{-x^2 - x} \\
-x \\
\underline{-x - 1} \\
1 \text{ rest}
\end{array}$$

$$\Updownarrow$$

$$x^3 - 2x = (x^2 - x - 1)(x+1) + 1.$$

ganger ut ogsjekka

$$\begin{array}{llll}
x^3 - x^2 - x & & & \\
\quad x^2 - x & - 1 & & +1 \\
= x^3 \quad - 2x & & & ✓
\end{array}$$

a tall $\quad p(x) = a = a \cdot x^0$

$\text{grad}(p(x)) = 0$ for $a \neq 0$.

7 $\quad \text{grad}(10) = 0, \quad \text{grad}(-17) = 0$

$\text{grad}(0) = -1$

Hva er graden til

oppg. 4) $(x-2)(x+3) - x^2 = x^2 - x^2 - 2x + 3x - 6 = x - 6 = x^1 - 6 \cdot x^0$ grad lik: 1

5) $x^7 + 5x^5 - 3x - \underbrace{x^2(x^5+2)}_{(x^7+2x^2)} = 5x^5 + \dots$ grad lik 5

$5x^5 - 2x^2 - 3x$

6) $(x^7+3x+2)(x^9-8x^7+3)(x^{17}+9x+7)$

$= x^7 x^9 \cdot x^{17}$ + ledd av lavere grad

$= x^{33} + \dots$

graden er lik 33

$\frac{1}{x} = x^{-1}, \quad \sqrt{x} = x^{1/2}$ ikkje polynomer.

(8)

$$x^3+0x^2+0x+0 : x-2 = x^2+2x+4+\frac{8}{x-2}$$

$$x^3-2x^2$$

$$2x^2+0+0$$

$$-\left(2x^2-4x\right)$$

$$0+4x+0$$

$$4x-8$$

$$8$$

$$(x^2+2x+4)(x-2)+8$$

$$x(x+2)(x-2)+4(x-2)+8$$

$$x(x^2-4)+4x-8+8 = x^3$$

oppg.

$$\frac{x^3+1}{x^2+1}$$

$$x^3+\quad+\quad+1 : x^2+1 = x-1$$

$$\cancel{x^3+x^2}$$

$$\cancel{-x^2\quad+1}$$

$$\cancel{-x^2-x}$$

$$\cancel{x+1}$$

rest

galt!

$$x(x^2+1)$$

$$=x^3+x$$

(ikke x^3+x^2)

9

$$x^3 + 0 + 0 + 1 : x^2 + 1 = x + \frac{-x+1}{x^2+1}.$$

$$\frac{-(x^3 + 0 + x)}{-x+1}$$

$$(x^2+1) \cdot x \quad - x + 1 \quad = x^3 + x - x + 1 = x^3 + 1$$

2 grads uttrykk ax^2+bx+c

Finn 2.grads funksjonen $p(x)$ som går gjennom $(0,1)$, $(1,-1)$ og $(-2,11)$.

(10)

$c=1$

$$\left.\begin{aligned} a+b+c &= -1 \\ 4a-2b+c &= 11 \end{aligned}\right\}$$

setter inn $c=1$

$a+b=-2$

$4a-2b=10$

$2L1+L2$ gir

$6a = 10-4=6$

$\underline{a=1}$

$\underline{b}=-2-a=\underline{-3}$

$\underline{p(x)=x^2-3x+1}$

går gjennom de 3 punktene.

Polynomdivisjon $\dfrac{3x^2+1}{2x-1}$

11

$$3x^2 + \quad + 1 : 2x - 1 = \frac{3}{2}x + \frac{3}{4} + \frac{7/4}{2x-1}$$

$$\underline{3x^2 - \frac{3}{2}x + 0}$$

$$\frac{3}{2}x + 1$$

$$\underline{\frac{3}{2}x - \frac{3}{4}}$$

$$0 \quad \underbrace{1+\frac{3}{4}}_{7/4}$$

$$\left(\frac{3}{2}x + \frac{3}{4}\right)(2x-1) + \frac{7}{4}$$

$$= 3x^2 + 1$$

oppg

$$\frac{7x^3}{3x^2+2x+1}$$

12

$$7x^3 + \quad + \quad + \quad : 3x^2+2x+1 = \frac{7}{3}x - \frac{14}{9} + \frac{(7/9)x + 14/9}{3x^2+2x+1}$$

$$= \frac{1}{9}\left(21x - 14 + \frac{7x+14}{3x^2+2x+1}\right)$$

$$7x^3 + \frac{14}{3}x^2 + \frac{7}{3}x$$

$$-\frac{14}{3}x^2 - \frac{7}{3}x + 0$$

$$-\frac{14}{3}x^2 - \frac{28}{9}x - \frac{14}{9}$$

$$\underbrace{-\frac{7}{3}x + \frac{28}{9}x}_{} + \frac{14}{9}$$

$$\frac{-21+28}{9}x + \frac{14}{9}$$

13

$$\frac{x^4}{x^2-3} = \frac{x^2(x^2-3)+3x^2}{x^2-3} = x^2 + \frac{3x^2}{x^2-3}$$

$$= x^2 + \frac{3(x^2-3)+9}{x^2-3} = \frac{x^2+3+\frac{9}{x^2-3}}{}$$

$$x^4 + \quad + \quad + \quad + \; : \; x^2 + \; -3 = x^2+3+\frac{9}{x^2-3}$$

$$\begin{array}{l} x^4 \quad -3x^2 \\ \hline \quad 3x^2 \\ \quad 3x^2 \quad -9 \\ \hline \qquad\quad 9 \end{array}$$

oppg.

$$\frac{x^2-13x+42}{x-6} = \frac{x(x-6)-7(x-6)}{x-6} = x-7$$

$$x^2-13x+42 : x-6 = \underline{x-7}$$

$$\begin{array}{l} x^2-6x \\ \hline \quad -7x+42 \\ \quad -7x+42 \\ \hline \qquad 0 \end{array}$$

divisjonen "går opp"

$x^2-13x+42$
$=(x-7)(x-6)$