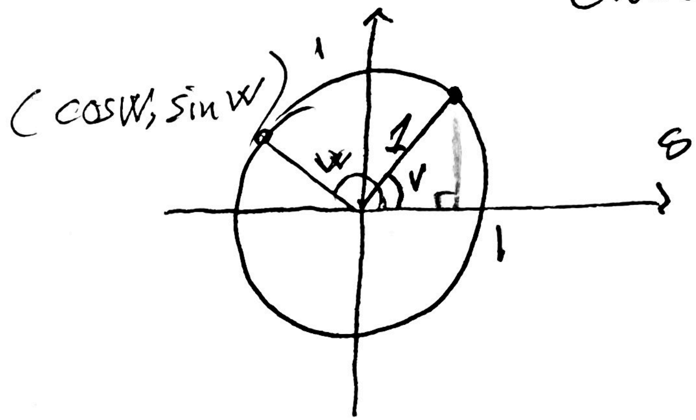
27sep 2021

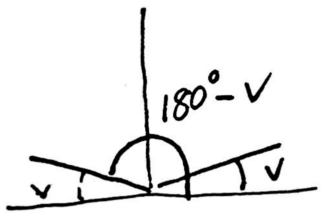
9.7–9.9

10.2 Sinus og cosinus

enhetssirkel
radius 1
senter i origo



①



$$-1 \leq \cos(v), \sin(v) \leq 1$$

$$\sin(v) = \sin(180° - v)$$

$$\cos(v) = -\cos(180° - v)$$

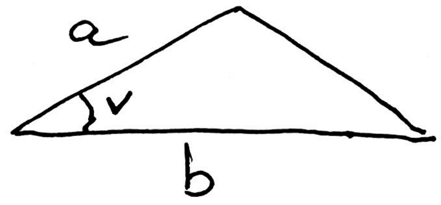
$$\sin(135°) = \sin(90° + 45°) = \sin(180° - 45°)$$

$$= \sin(45°) = \frac{1}{\sqrt{2}}$$

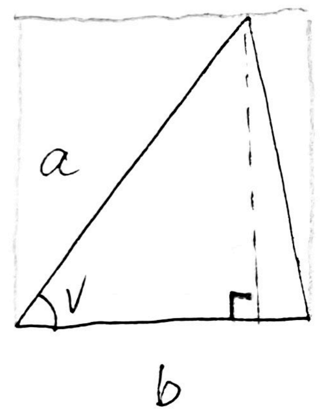
$$\cos(135°) = -\cos(45°) = \frac{-1}{\sqrt{2}}.$$

9.7 Arealsetningen

Arealet $A = \frac{1}{2} a \cdot b \cdot \sin(v)$

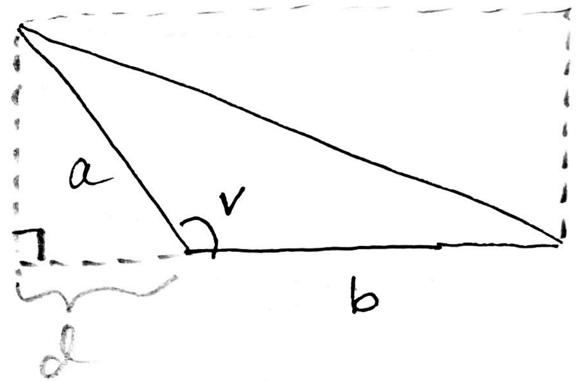


②



høgden $a \sin v$

$h = a \sin v$



areal $\frac{h \cdot (d+b)}{2} - \frac{h \cdot d}{2}$

$= \frac{h \cdot b}{2} = \frac{ab \cdot \sin(v)}{2}$

Finn arealet til Δ hvor to av sidene har lengde 3, og 5 og vinkelen mellom de to sidene er $45°$

$$A = \frac{1}{2} 3 \cdot 5 \overbrace{\sin(45°)}^{1/\sqrt{2}} = \frac{3 \cdot 5}{2\sqrt{2}}$$

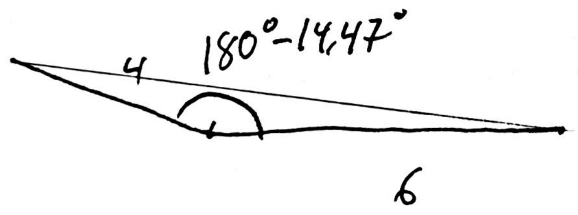
oppgave 3

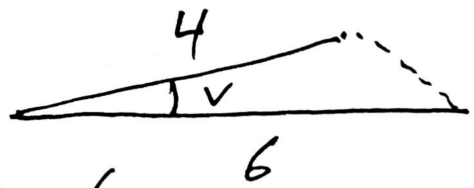
En trekant har areal $A=3$ og lengden på to av sidene er $a=4$ og $b=6$.
Hva er vinkelen V mellom de to sidene?

$$A = \frac{1}{2} \cdot 4 \cdot 6 \cdot \sin V = 3$$

$$\sin(V) = \frac{3}{3 \cdot 4} = \frac{1}{4} = 0.25$$

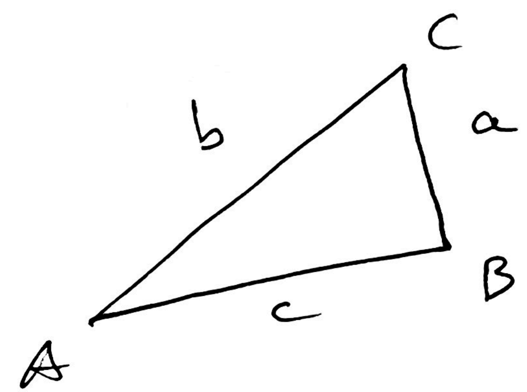
$$V_1 = \arcsin\left(\frac{1}{4}\right) = 14.47^\circ$$ og $$V_2 = 180^\circ - 14.47^\circ = \underline{165.53^\circ}$$





To muligheter!

9.8 Sinussetningen



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

(4) for alle trekanter.

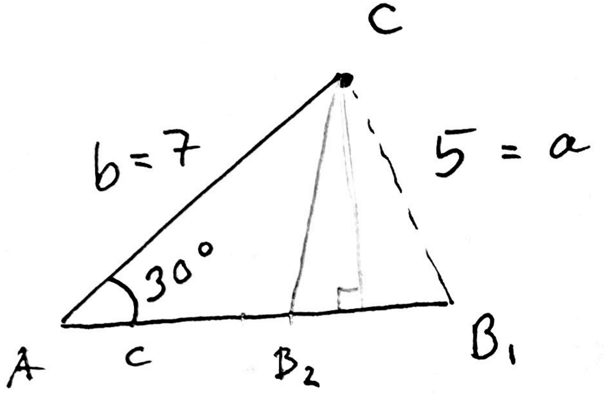
Hvorfor er det slik?

$$A = \frac{1}{2} b \cdot c \sin(A) = \frac{1}{2} a \cdot c \sin(B) = \frac{1}{2} a \cdot b \sin(C)$$

deler med $\frac{1}{2} abc$ (ganger med $\frac{2}{abc}$)

$$\frac{2A}{a \cdot bc} = \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

(5)



Bestem $\angle C$

sidelengden c

Benytter sinussetningen

$$\frac{\sin A}{a} = \frac{\sin(30°)}{5} = \frac{1/2}{5} = \frac{1}{10}$$

$$= \frac{\sin B}{b} = \frac{\sin B}{7}$$

$$= \frac{\sin C}{c} = \frac{1}{10}$$

så $\sin(B) = \frac{7}{10} = 0.7$

Vi får $B_1 = \arcsin(0.7) = 44.427°$

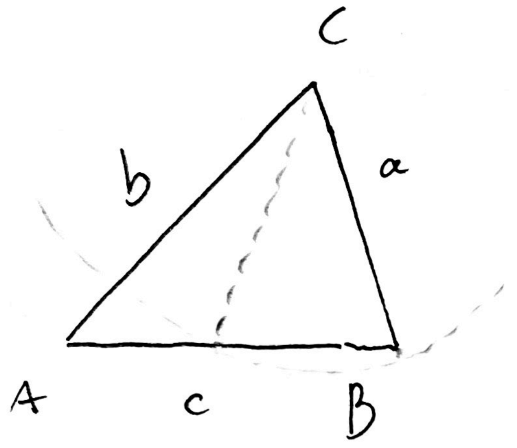
$B_2 = 180° - B_1 = 135.573°$

$C_1 = 180° - 30° - B_1 = 105.573°$

$C_2 = 180° - 30° - B_2 = 14.427°$

$c_1 = 10 \sin C_1 = 9.63$

$c_2 = 10 \sin C_2 = 2.49$



6

a, b og $\angle A$ er kjent.

Antall løsninger (mulige trekanter)

1) $a < b\sin(A)$: ingen løsning

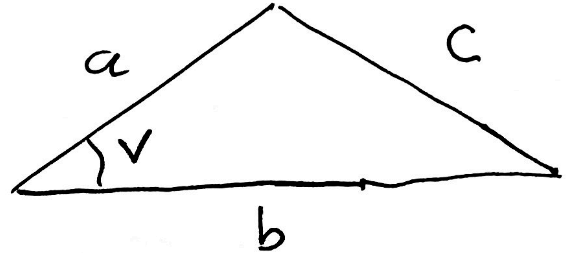
2) $a = b\sin(A)$: én løsning (rettvinkla Δ)

3) $b\sin(A) < a < b$: to løsninger

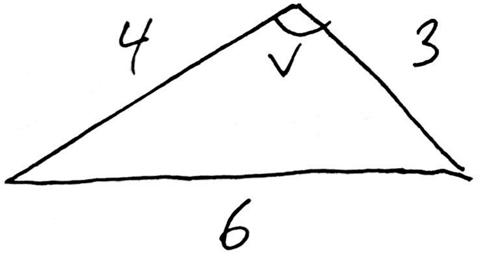
4) $a \geq b$: én løsning.

9.9 cosinussetningen

(7)



$$\boxed{a^2 + b^2 - 2ab\cos(V) = c^2}$$

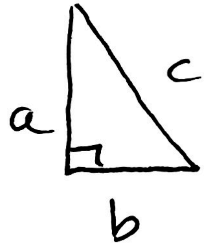


Hva er vinkel v?

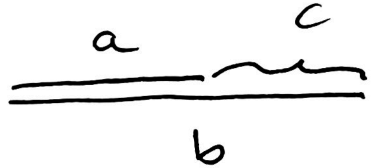
cosinussetningen gir

$6^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cos v$

$V = 90°$ Pythagoras



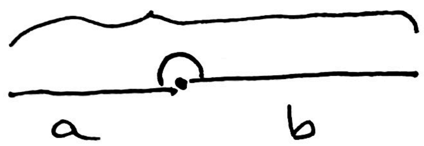
$V = 0°$



$c = |b - a|$

$V = 180°$

$c = a + b$



8

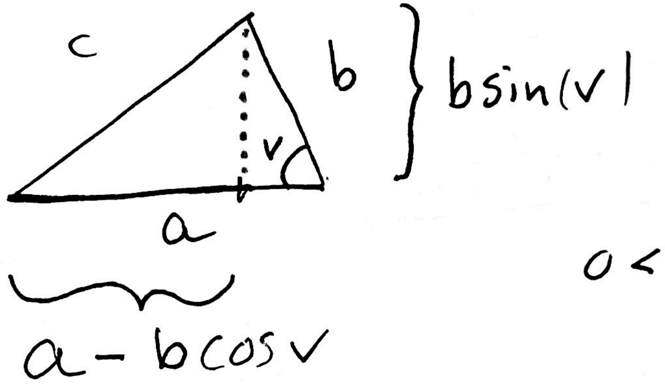
$$6^2 - 3^2 - 4^2 = -2 \cdot 3 \cdot 4 \cos v$$

$$36 - 9 - 16 = -2 \cdot 3 \cdot 4 \cos v$$

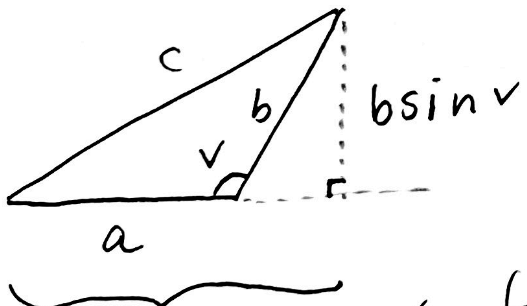
$$\cos v = \frac{-11}{2 \cdot 3 \cdot 4} \approx -0.4583..$$

$$\arccos(-0{,}4583) = v = \underline{117.28^\circ}$$

bevis for cosinussetningen



$0 < v \leq 90^\circ$



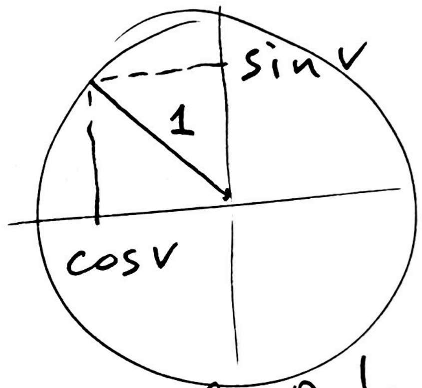
a - bcosv for $90^\circ \leq v < 180^\circ$

Benytter Pytagoras sin sats på de rettvinkla trekantene : $c^2 = (b \sin v)^2 + (a - b \cos v)^2$

$$c^2 = b^2\sin^2 v \quad + \quad a^2 + b^2\cos^2 v \underbrace{- 2ab\cos v}_{\text{kryssledd}}$$

$$\left(\sin^2 v \underset{\text{betyr}}{=} (\sin v)^2 \right)$$

(9)



ved Pytagoras
$$\underline{\cos^2 v + \sin^2 v = 1}$$

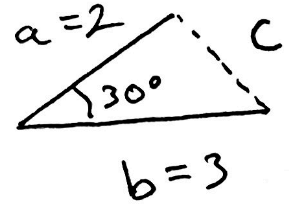
$$c^2 = b^2(\overbrace{\sin^2 v + \cos^2 v}^{1}) + a^2$$
$$-2ab\cos v$$

dette gir cosinussetningen

$$\underline{c^2 = a^2 + b^2 - 2ab\cos v}.$$

Oppg.

(10)



Hva er lengden til side c ?

Benytter cosinussetningen

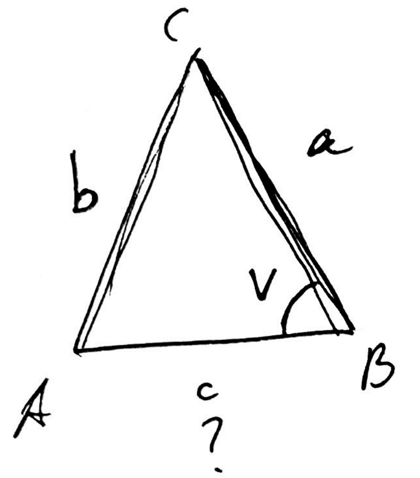
$$c^2 = a^2 + b^2 - 2ab\underbrace{\cos(30^\circ)}_{\frac{\sqrt{3}}{2}}$$

$$c^2 = 2^2 + 3^2 - 2\cdot 2\cdot 3\,\frac{\sqrt{3}}{2}$$

$$\cong \underline{1.6148}$$

Øving

(11)



a, b kjent
vinkel v kjent.

Sinussetningen

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

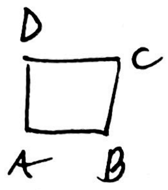
regner så ut $\sin A$. (muligens to løsninger) for $\angle A$.

Regner ut $\angle C = 180° - \angle A - \angle B$.

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\underline{c = b \cdot \frac{\sin(C)}{\sin(B)}}$$

oppg i boka 9.72



AB og DC paralle

$\angle A = 60°$

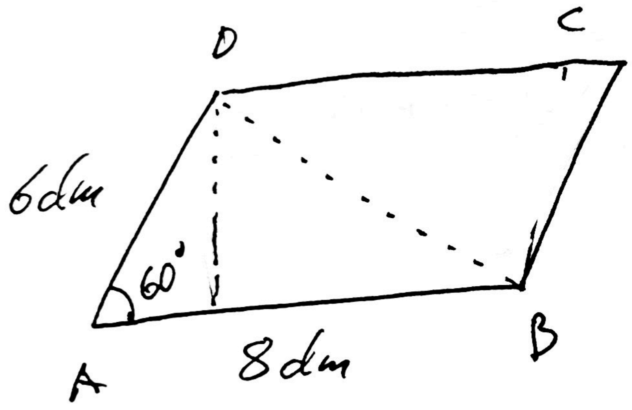
$AB = 8\,dm$

$AD = 6\,dm$

Finn arealet

parallellogram

(12)



Areal A = bredde × høyde

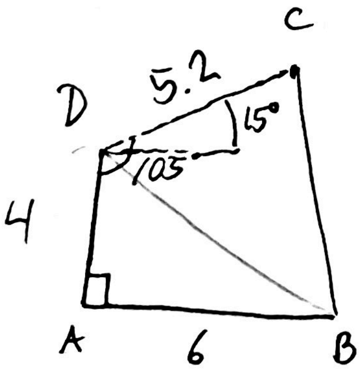
$A = 8dm \cdot (6dm \cdot \sin 60°)$

$= 6 \cdot 8\,dm^2 \cdot \frac{\sqrt{3}}{2}$

$= 3 \cdot \sqrt{3} \cdot 8\,dm^2$

$= \underline{41.57\,dm^2}$

9.73



1) Finner BD.

2) $\angle ADB$, $\angle BDC = 105° - \angle ADB$.

3) $A = \frac{AD \cdot AB}{2} + \frac{BD \cdot DC}{2} \sin(\angle BDC)$

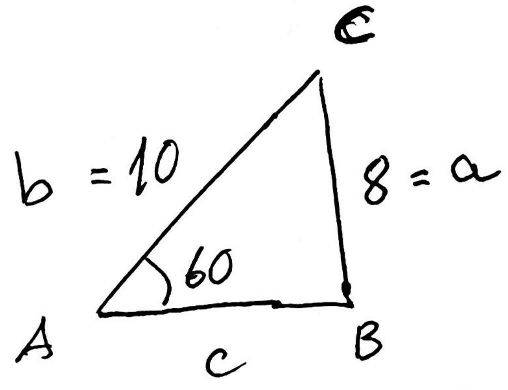
opg. (sinussetning) ΔABC

(13) $\angle A = 60°$

$a = 8$

$b = 10$

Finn c (om mulig)



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

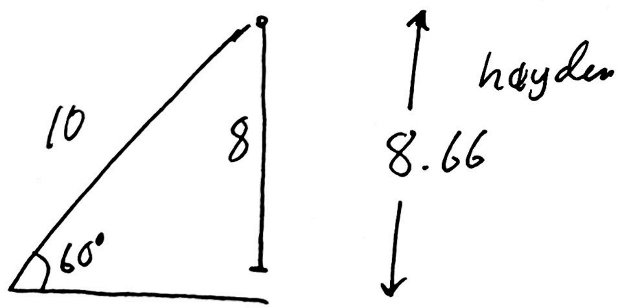
$$\sin B = \frac{b}{a} \cdot \sin A$$

$$= \frac{10}{8} \frac{\sqrt{3}}{2} \sim 1.0825\ldots$$

$$\cong \frac{0.866}{0.8} > 1$$

ingen løsning!

Hva er galt?



"rekker ikkje ned"

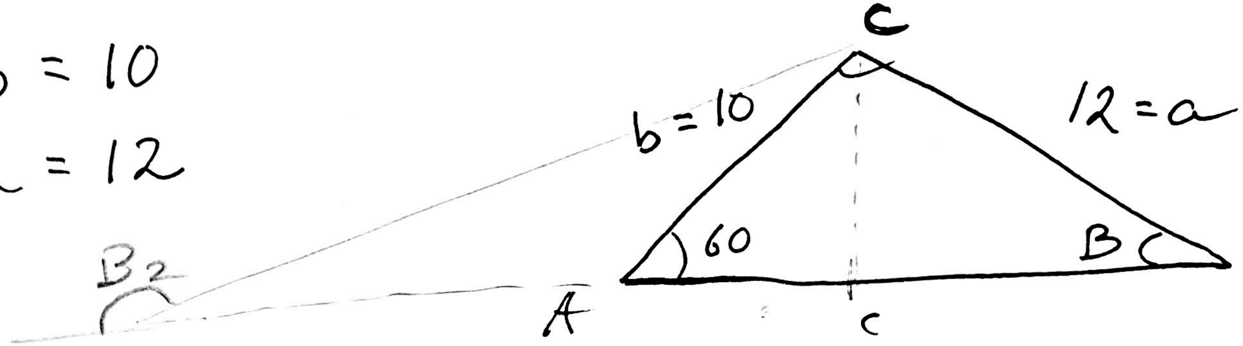
Det finnes ikkje en slik trekant.

14

$\angle A = 60°$ Finn $\angle B$ og $\angle C$ (og c)

$b = 10$

$a = 12$



Sinussetning

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\sin B = \frac{b}{a} \sin A = \frac{10}{12} \cdot \frac{\sqrt{3}}{2} \sim 0.72168..$$

$B_1 = 46.194°$ og ~~$180° - 46.194°$~~

~~$B_2 = 133.806°$~~

~~$B_2 + A = 133.8 + 60°$~~

~~$= 193.8°$~~ $> 180°$?

$C = 180° - B - A = 180° - 106.2°$

$\underline{C = 73.8°}$

$c = a \cdot \frac{\sin C}{\sin A} \sim \cancel{11.52}$ $\underline{13.3}$