

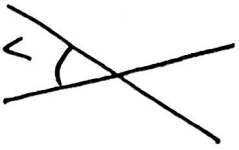
28 sep

10.1

Vinkler

2021

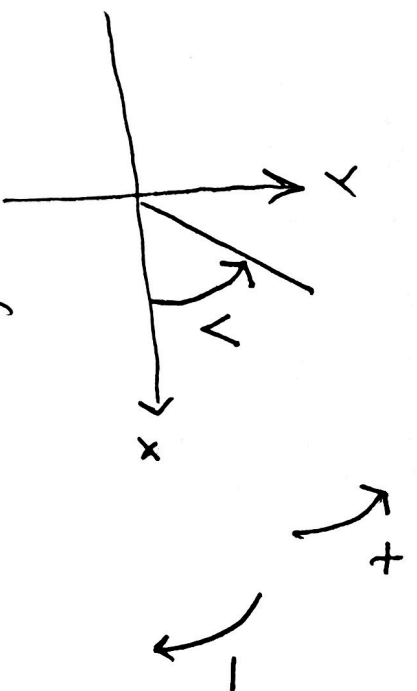
①



vinkel mellom linjer
 $0 \leq v \leq 90^\circ$



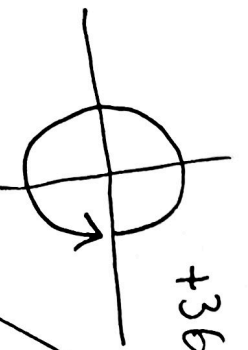
$0 \leq v \leq 180^\circ$



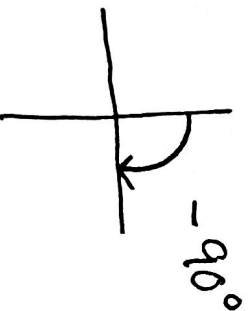
positiv retning
er motsatt rotationsretning

til urviseren.

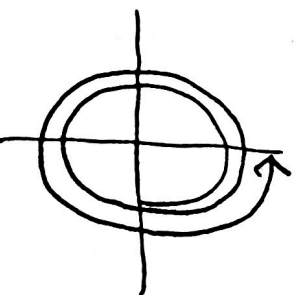
Vinkler kan være vilkårlige
vilkårlige vinkel



$+360^\circ$

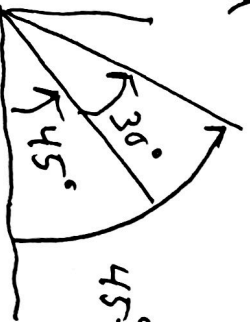


-90°



to hele omkøp
 $360^\circ + 360^\circ + 90^\circ$
 $\underbrace{720^\circ}_{+90^\circ}$

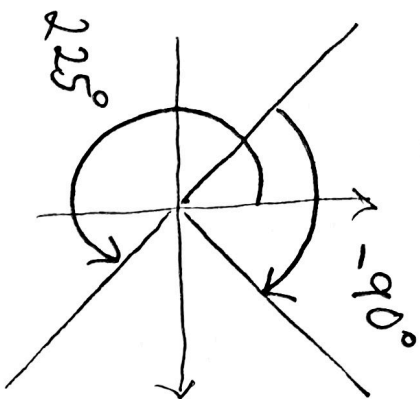
$= 810^\circ$



$45^\circ + 30^\circ = 75^\circ$



$90^\circ - 60^\circ = 30^\circ$



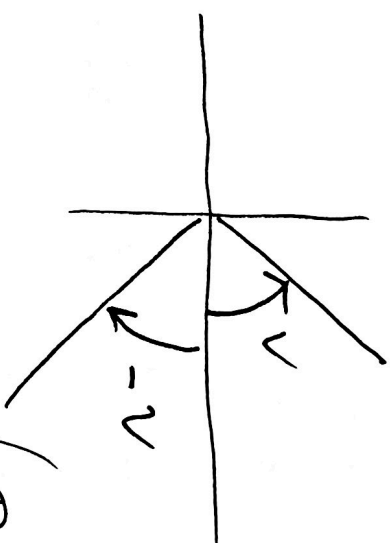
②

Absolutt vinkel mål (Radianer)

$$0^\circ + v = v$$

$$v + (-v) = 0$$

identitets element
invers element



(Radianer forholder)
rad

$$b = \text{konstant} \cdot v \cdot r$$

$$\frac{b}{r} = \text{konstant} \cdot v$$

velger en enhet for vinkel
slik at konstanten blir 1.



$$v = \frac{b}{r}$$

omkrets

$$b = 2\pi \cdot r$$

$$\frac{b}{r} = 2\pi$$

③

definier
der: 3

Et helt omkøp.

$$360^\circ = 2\pi \text{ radianer}$$

$$\boxed{180^\circ = \pi \text{ rad}}$$

$$90^\circ = \frac{\pi}{2} \text{ rad}$$

$$45^\circ = \frac{\pi}{4} \text{ rad}$$

$$30^\circ = \frac{\pi}{6} \text{ rad}$$

$$60^\circ = \frac{\pi}{3} \text{ rad}$$

$$= 1.04719... \text{ rad.}$$

$$\frac{\text{vinkel i grader}}{180^\circ} = \frac{\text{vinkel i rad}}{\pi}$$

$$V \text{ i grader} = \frac{180^\circ}{\pi} \cdot \text{vinkel i rad.} = 57.2957... \cdot \text{vinkel i rad.}$$

10.2 Sin og cos for vilkårlige vinkler.

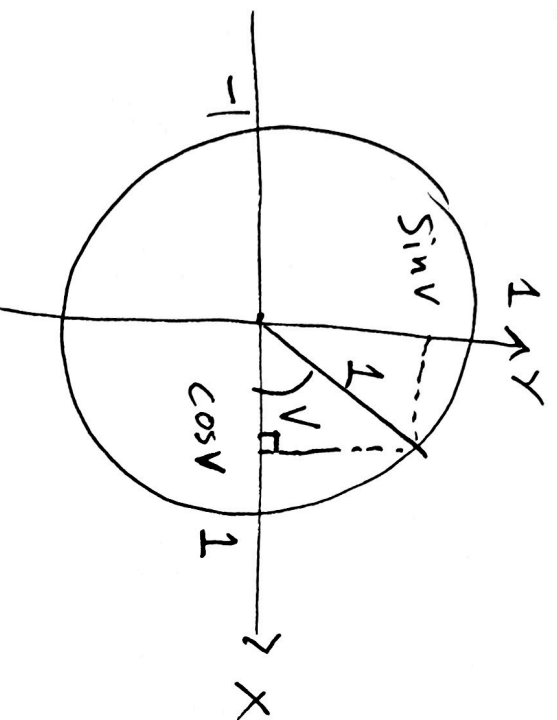
$$V = \frac{\text{buelængde}}{\text{radius}} = \frac{b}{r}$$

$$r=1 \text{ giver}$$

$$V = b$$

Vinkel = buelængde.

(5)



$$\sin(V + 2\pi) = \sin(V)$$

$$\cos(V + 2\pi) = \cos(V)$$

periodiske funktioner med
periode 2π ($\approx 360^\circ$)

$$\sin(V), \cos(V)$$

kan realiseres i

intervallet $[-1, 1]$

tangens $\tan V = \frac{\sin V}{\cos V}$ når $\cos V \neq 0$

$$V \neq 90^\circ \text{ og } 270^\circ$$

$\tan(V)$ er skningskallet til tangens.

og til hele omkøbet.

$$1 = 180^\circ = \pi$$

$$\sin(180^\circ) = 0,$$

$$\cos(180^\circ) = -1$$

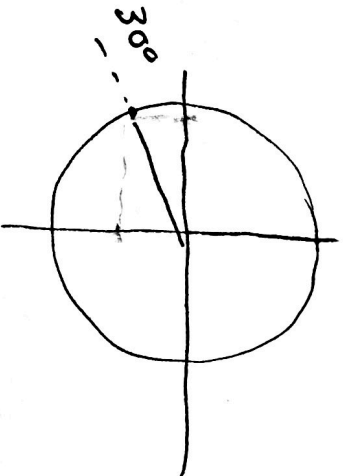
$$V = 270^\circ = \frac{3\pi}{2}$$

$$\sin(270^\circ) = -1,$$

$$\cos(270^\circ) = 0$$

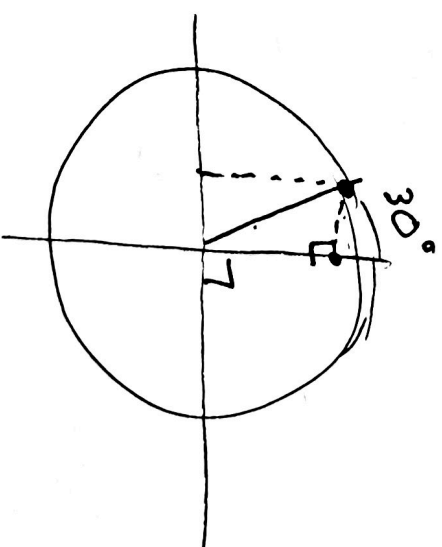
$$V = 210^\circ = 180^\circ + 30^\circ$$

$$= \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$



$$\sin(210^\circ) = -\frac{1}{2}$$

$$\cos(210^\circ) = -\frac{\sqrt{3}}{2}.$$



⑥

opp.

Finan $\sin$ og $\cos$

$$\text{Hil } V = 480^\circ = 360^\circ + 120^\circ$$

$$\sin(480^\circ) = \sin(360^\circ + 120^\circ) = \sin(120^\circ)$$

$$= \cos(30^\circ) = \underline{\underline{\frac{\sqrt{3}}{2}}}$$

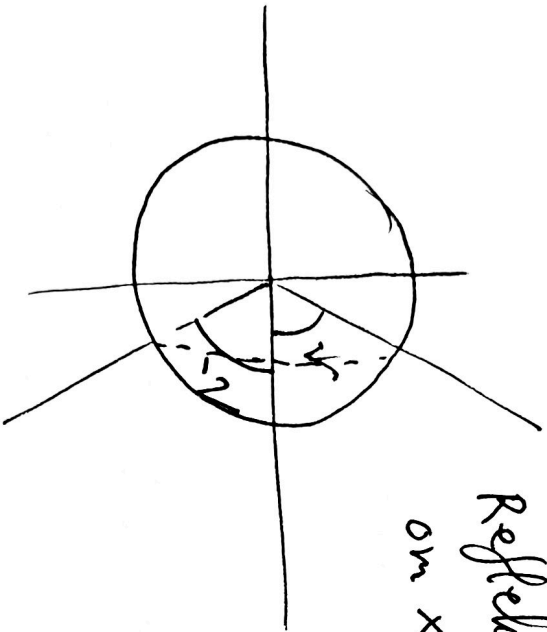
$$\cos(480^\circ) = \cos(120^\circ) = -\sin(30^\circ) = \underline{\underline{-\frac{1}{2}}}$$

Ques. Find

$$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} = \frac{-1}{\sqrt{2}}$$

$$\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

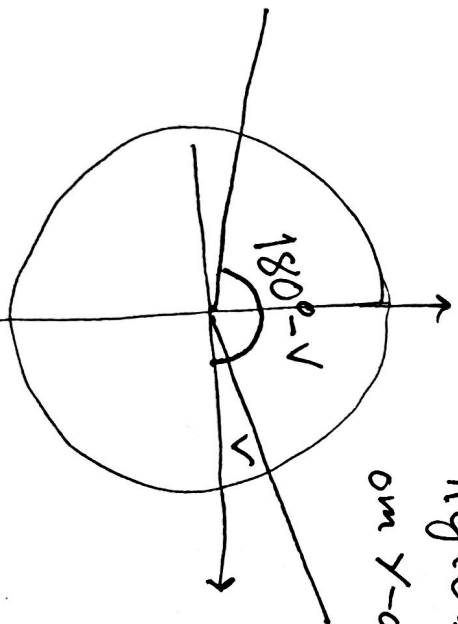
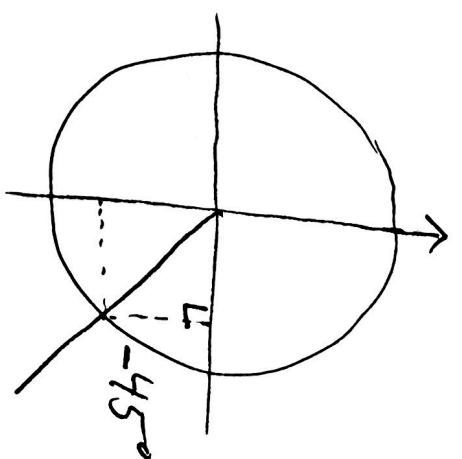
(7)



Reflection
on x-axis

$$\cos(\theta) = \cos(-\theta)$$

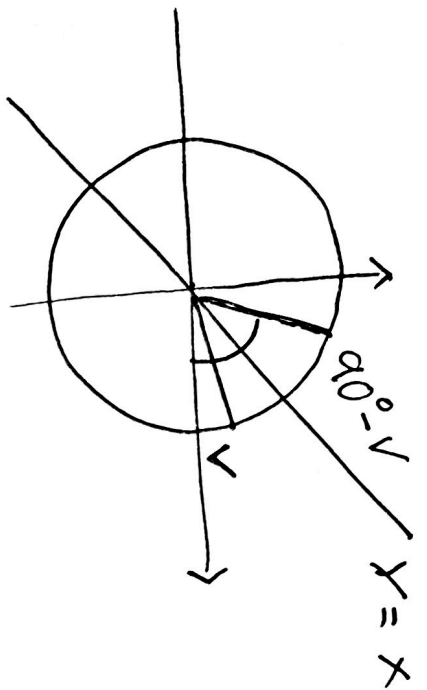
$$\sin(\theta) = -\sin(-\theta)$$



Reflection
on y-axis

$$\sin(\theta) = \sin(180^\circ - \theta)$$

$$\cos(\theta) = -\cos(180^\circ - \theta)$$



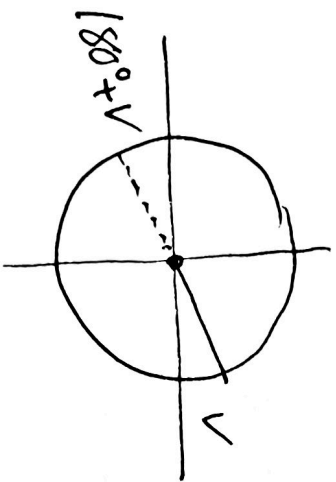
Refleksion
om linjen $x=y$.

x og y-koordinatene byttes om.

$$\cos(90^\circ - V) = \sin V$$

$$\sin(90^\circ - V) = \cos V$$

⑧



Refleksion
om origo

$$\cos(180^\circ + V) = -\cos(V)$$

$$\sin(180^\circ + V) = -\sin(V)$$

$$\begin{aligned} \frac{\sin(V)}{\cos(V)} &= \tan(V) \\ &= \frac{-\sin(180^\circ + V)}{-\cos(180^\circ + V)} = \tan(V + 180^\circ) \end{aligned}$$

$\tan(v + \pi) = \tan v$
 $\tan$ er periodisk med periode π .
 (180°)

②

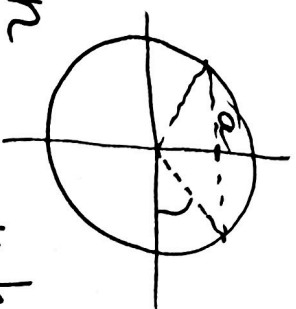
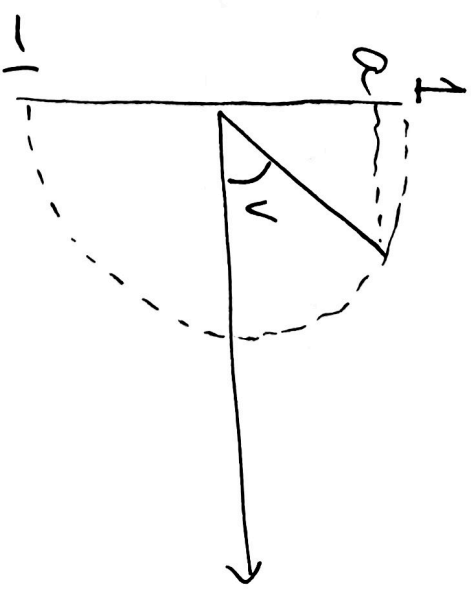
$$\sin(v) = a$$

En løsning:

$$\sin^{-1}(a) = \arcsin(a)$$

giver vinkelen v mellem -90° og 90°

$$\text{Sik at } \sin(v) = a$$



$$\sin(v) = a$$

Løsningene er

$$\arcsin(a) + 360^\circ \cdot n$$

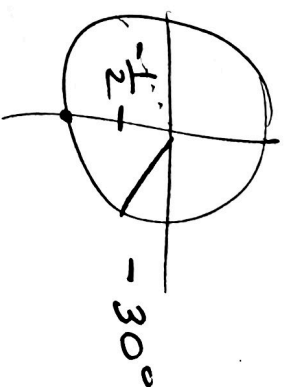
$$180^\circ - \arcsin(a) + 360^\circ \cdot n$$

n helheder

$$\sin v = \frac{-1}{2}$$

$$v_0 = \arcsin\left(\frac{-1}{2}\right) = -30^\circ$$

$$180^\circ - v_0 = 180^\circ - (-30^\circ) = 210^\circ.$$



⑩

Løsningene er

$$v = -30^\circ + 360^\circ \cdot n$$

$$\text{og } v = 210^\circ + 360^\circ \cdot n$$

$$\sin v = -1$$

$$v = \arcsin(-1) = -90^\circ$$

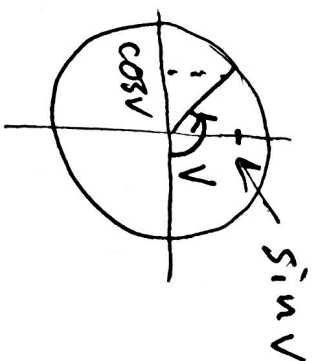
Den andre løsningen

$$180^\circ - (-90^\circ) = 270^\circ = -90^\circ + 360^\circ$$

Løsningene

$$v = \underline{-90^\circ + 360^\circ \cdot n} \quad n \in \mathbb{Z}$$

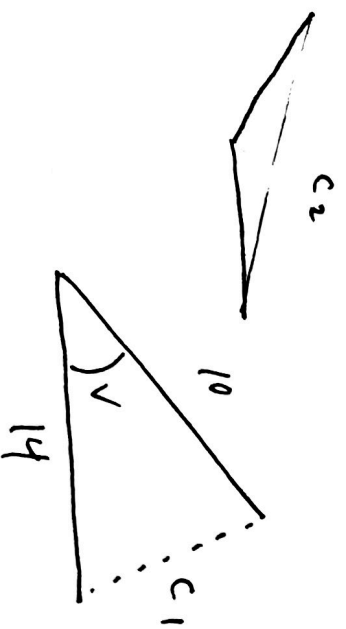
$$\pi_{rad} = 180^\circ$$



(11)

Bestem trekanens hvor to sider har
 længde 10 og 14, og areal $A = 35$.
 Her er vinklen V
 mellem siderne a og b .

$$a = 10, \quad b = 14.$$



$$A = \frac{1}{2} \cdot b \cdot a \cdot \sin V$$

$$35 = \frac{1}{2} \cdot 14 \cdot 10 \cdot \sin V$$

$$\sin V = \frac{35 \cdot 2}{14 \cdot 10} = \frac{70}{140} = \frac{1}{2}$$

$$V_1 = \arcsin\left(\frac{1}{2}\right) = 30^\circ \quad (= \pi/6)$$

$$V_2 = 180^\circ - V_1 = 150^\circ$$

Det er to mulige
 trekanter:
 $V = 30^\circ$
 $V = 150^\circ$

Finne længdene C_1 og C_2

cosinussetningen

$$C^2 = a^2 + b^2 - 2ab \cos(\gamma).$$

⑫

$$\begin{aligned} C_1^2 &= 10^2 + 14^2 - 2 \cdot 10 \cdot 14 \cdot \left(\frac{\sqrt{3}}{2}\right) \\ &= 100 + 196 - 140\sqrt{3} \end{aligned}$$

$$C_1 = \sqrt{296 - 140\sqrt{3}} \sim 7.315$$

C_2 tilsvarende

$$C_2 = \sqrt{296 + 140\sqrt{3}} \sim 23.205.$$

$$\begin{aligned} \cos(\gamma_2) &= \cos(150^\circ) \\ &= -\sqrt{3}/2 \end{aligned}$$

Utryk vinklene

315°	,	240°	og	255°	i radianer.
$360^\circ - 45^\circ$		$180^\circ + 60^\circ$		$270^\circ - 15^\circ$	
$2\pi - \frac{\pi}{4}$		$\pi + \pi/3$		$\frac{3\pi}{2} - \frac{\pi}{12}$	$= \frac{18\pi - \pi}{12}$
$= \frac{8\pi - \pi}{4} = \frac{7\pi}{4}$		$= \frac{3\pi}{3} + \frac{\pi}{3} = \frac{4\pi}{3}$		$= \frac{17\pi}{12}$	

Hva er $\frac{11\pi}{8}$ i grader?

$$\pi + \frac{3\pi}{8}$$

$$\frac{\pi}{4} = 45^\circ$$

$$\frac{\pi}{8} = 22.5^\circ$$

$$\frac{3\pi}{8} = 67.5^\circ$$

13

$$= 180^\circ + 67.5^\circ$$

$$= \underline{247.5^\circ}$$

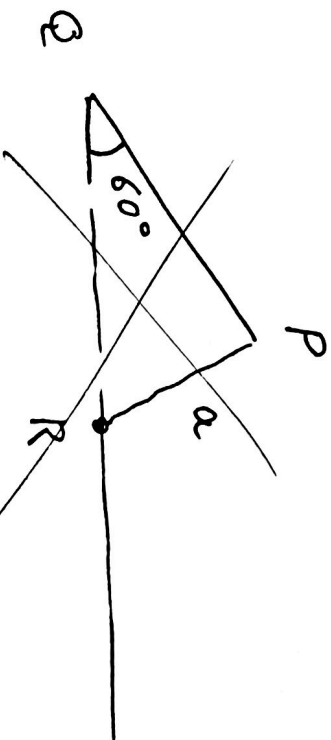
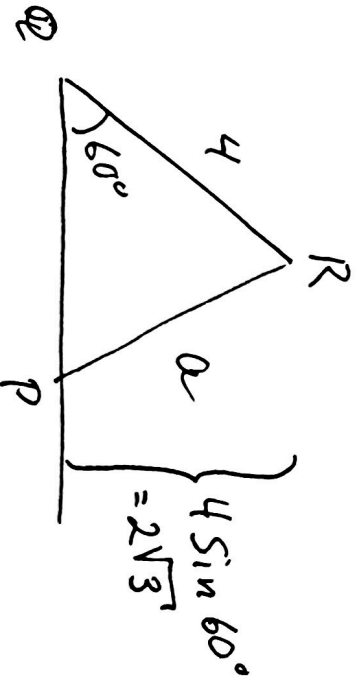
opp. $\frac{9.84}{\sin 60^\circ}$

ΔPQR

$$\angle Q = 60^\circ$$

$$QR = 4$$

$$PR = a$$



ingen Δ

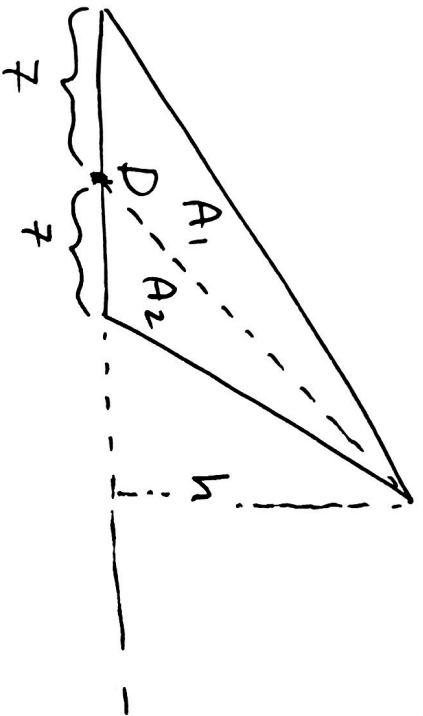
én Δ

$$\begin{cases} a = 2\sqrt{3} \\ a \geq 4 \end{cases}$$

to $2\sqrt{3} < a < 4$

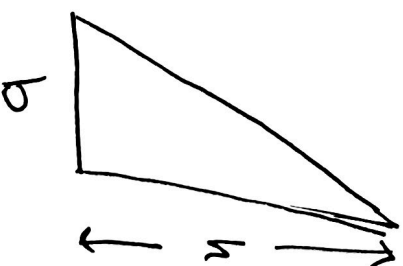
9,194 e)

(14)



Hvorfor er
 $A_1 = A_2$?

De to trekantene har
samme højde og samme bredde.
Derfor er arealerne deses lige.



areal
bredde $\times$
højde $/ 2$