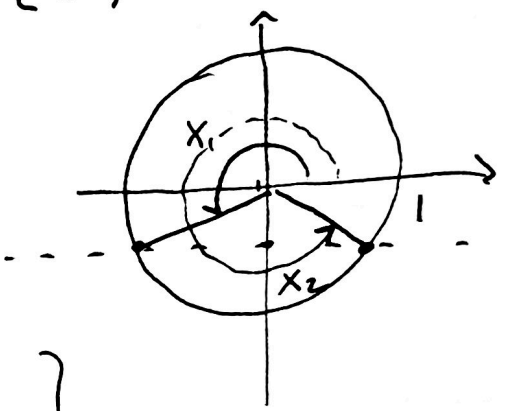
10 Trigonometriske likninger

29.09
2021

① 10.3 $\sin x = -0.4$ $\quad x \in [0, 360°]$



$x = \arcsin(-0.4)$

$= \sin^{-1}(-0.4) \sim -23{,}58°$

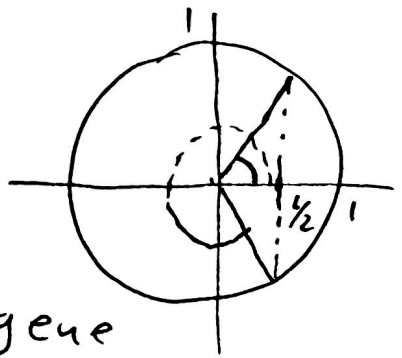
$x_2 = 360° + (-23.58°) = \underline{336.42°}$

$x_1 = 180° - (-23.58°) = \underline{203.58°}$

} er løsningene.

10.4 cosinus likning.

$\cos x = \frac{1}{2}$ $\quad x \in [0, 2\pi]$

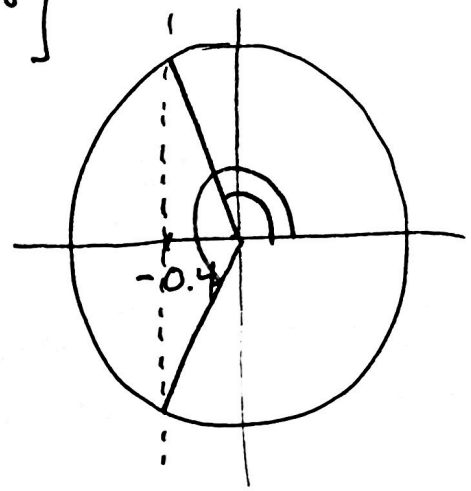


$x_1 = \arccos\left(\frac{1}{2}\right) = 60° = \underline{\pi/3}$

$x_2 = 2\pi + (-\pi/3) = \frac{6\pi}{3} - \frac{\pi}{3} = \underline{\underline{\frac{5\pi}{3}}}$

} løsningene

Oppg. $\cos v = -0.4$ $\quad v \in [0, 500°]$



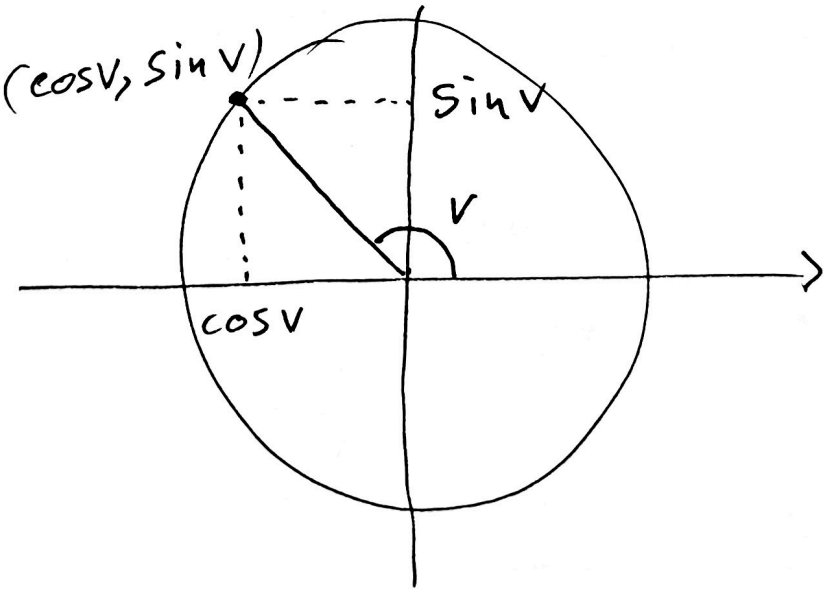
$\cos^{-1}(-0.4) = \arccos(-0.4) \sim \underline{113.58°}$

1) $113.58° + 360° = \underline{473.58°}$ $\quad (< 500°)$

2) $-113.58° + 360° \cdot n$

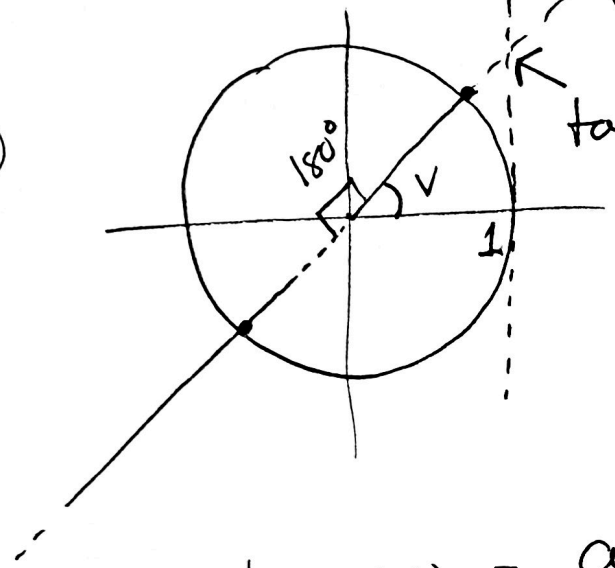
$v = \underline{246.42°}$

② Løsningene er $\underline{113.58°,\ 246.42° \text{ og } 473.58°}$



10.5 Tangens likninger

③



← stigningstallet er lik $\tan v = \frac{\sin v}{\cos v}$

$v \neq 90° + 360°$
$\neq 270° + 360°$
$\Big\} v \neq 90° + 180° \cdot n$

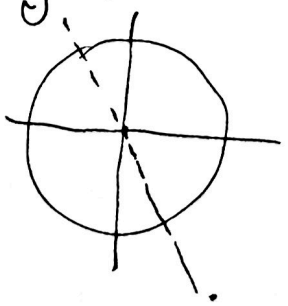
$\tan(v) = a \quad a \in \mathbb{R}$

$v = \arctan(a) + 180° \cdot n \quad , \quad n \in \mathbb{Z}$

Løs likningen $\tan v = -\sqrt{3} \quad v \in [0, 4\pi]$.

oppgave

$\tan 0° = 0$
$\tan 30° = 1/\sqrt{3}$
$\tan 45° = 1$
$\tan(60°) = \sqrt{3}$

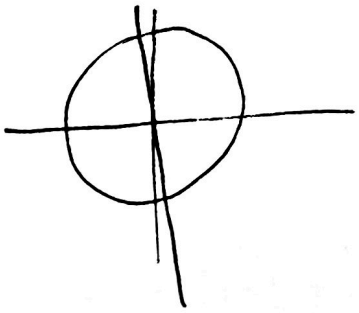


$v = \arctan(-\sqrt{3}) = -\frac{\pi}{3} + \pi \cdot n$

$(-60° + 180° \cdot n)$

$v = \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}, \frac{11\pi}{3}$

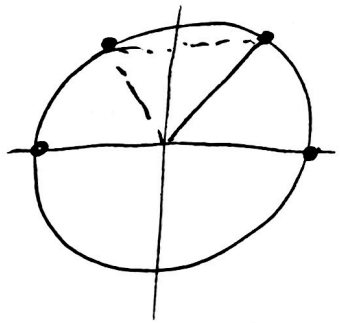
oppg. $\tan v = -80$ $\quad v \in [0, 180°]$.



$v = \arctan(-80) \sim -89.284 + 180° \cdot n$

$v = \underline{90.72°}$

④



10.8 og 10.9

$2\sin^2 x = \sqrt{3}\sin x$

$2\sin^2 x - \sqrt{3}\sin x = 0$

$\sin x (2\sin x - \sqrt{3}) = 0$

$\sin x = 0$ $\quad$ eller $\quad 2\sin x - \sqrt{3} = 0$

$\Leftrightarrow \sin x = \frac{\sqrt{3}}{2} \sim 0.866$

$x = \left.\begin{matrix} 0° + 360° \cdot n \\ 180° + 360° \cdot n \end{matrix}\right\} 180° \cdot n$, $n \in \mathbb{Z}$

$x = \underline{60° + 360° n}$

$x = \underline{120° + 360° \cdot n}$

Løsningene er $x = \underline{0°, 60°, 120° \text{ eller } 180°}$ opp til hele omløp

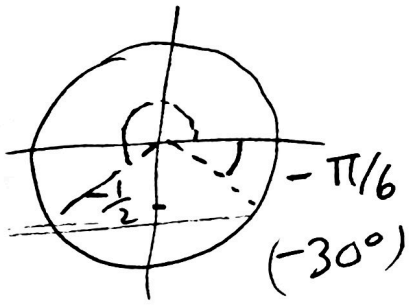
$$\sin^2 x + 2.5 \sin x + 1 = 0 \qquad U = \sin x$$

(5)

$$U^2 + 2.5U + 1 = 0$$

$$(U+2)(U+0.5) = 0$$

1) $U = -2$ og $U = -0.5$



2) $\sin x = -2$

ingen løsning.

$-2 \cdot$

$$\sin x = -0.5$$

$$x = \frac{-\pi}{6} + 2\pi \cdot n$$

$$= \pi - \left(\frac{-\pi}{6}\right) = \frac{7\pi}{6} + 2\pi \cdot n$$

Løsningene er $x = \frac{-\pi}{6} + 2\pi \cdot n$

og $x = \frac{7\pi}{6} + 2\pi \cdot n$

⑥ $\sin^2 V + \cos V \sin V - 6\cos^2 V = 0$

($\cos V = 0 \Rightarrow \sin V = \pm 1$ ingen løsning)

Kan anta $\cos V \neq 0$. Deler med $\cos^2 V$:

1) $\frac{\sin^2 V}{\cos^2 V} + \frac{\cos V}{\cos V} \cdot \frac{\sin V}{\cos V} - 6\frac{\cos^2 V}{\cos^2 V} = 0$

$\tan^2 V + \tan V - 6 = 0$

2) $(\tan V + 3)(\tan V - 2) = 0$

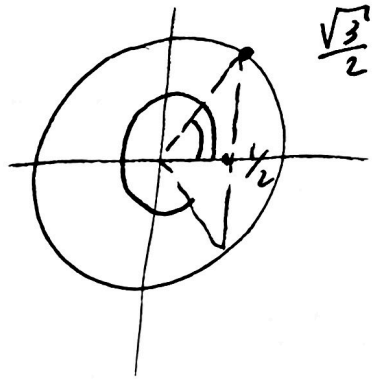
$\tan V = -3$ eller $\tan V = 2$

3) $V = \arctan(-3) + 180° \cdot n$, $V = \arctan(2) + 180° \cdot n$

$V = -71.565° + 180° \cdot n$ $V = 63.435° + 180° \cdot n$

(7)

Pytagoras $\sin^2 V + \cos^2 V = 1$ $\quad V \in [0, 2\pi]$



$\sin^2 V + \cos V - \frac{5}{4} = 0$

Utnytter $\sin^2 V$ som $1 - \cos^2 V$ v.h.a $\cos V$.

ved hjelp av

$1 - \cos^2 V + \cos V - \frac{5}{4} = 0 \quad \cdot(-1)$

$-\cos^2 V + \cos V - \frac{1}{4} = 0$

$\cos^2 V - \cos V + \frac{1}{4} = 0$

$(\cos V - \frac{1}{2})^2 = 0$

$\cos V = \frac{1}{2}$

$V_1 = 60° + 360° \cdot n$

$= \frac{\pi}{3} + 2\pi \cdot n$

$V_2 = -\frac{\pi}{3} + 2\pi \cdot n$

$V = \frac{\pi}{3}, \frac{5\pi}{3}$

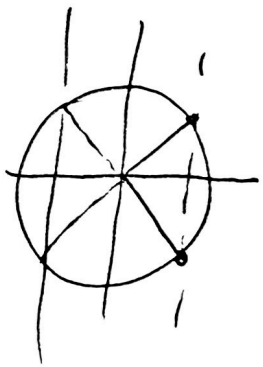
⑧

$$3\cos^2 v + 2\sin^2 v = 2.5$$

$\sin^2 v = 1 - \cos^2 v$ setter inn:

$$3\cos^2 v + 2(1 - \cos^2 v) = 2.5$$

$$\cos^2 v = 2.5 - 2 = 0.5 = \frac{1}{2}$$



$$\cos v = \frac{1}{\sqrt{2}} \qquad \cos v = \frac{-1}{\sqrt{2}}$$

$v = \pm 45° + 360° \cdot n$ $\qquad$ $v = \pm 135° + 360° \cdot n$

Løsningene er $45°, 135°, 225°, 315°$

opp til hele omløp.