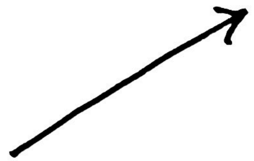
11.10.2021
H Fauske

# 12 Vektorer

1

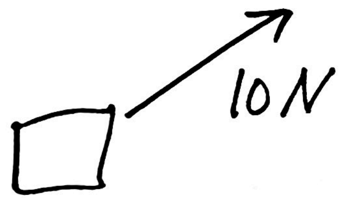


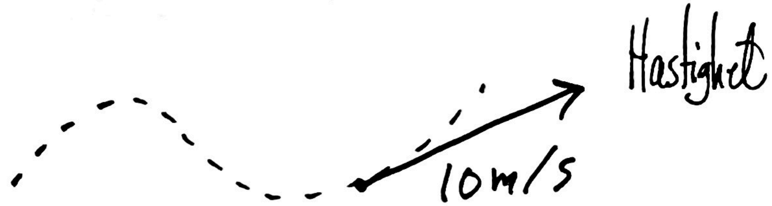
størrelse

retning

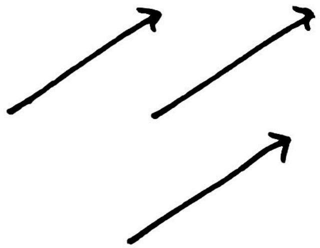
eksempler fra fysikk

Krefter



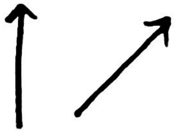

Like vektorer:



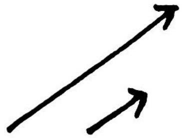
beskriver samme vektor. Er like

"vi kan parallellforskyve vektorer"

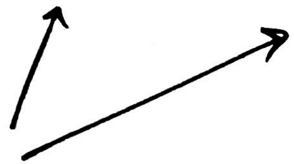
Forskjellige vektorer:



lik størrelse
ulik retning



samme retning
ulik størrelse



To vektorer er like hvis de har samme størrelse og samme retning.

null vektoren $\cdot$ ingen utstrekkning lengden er 0

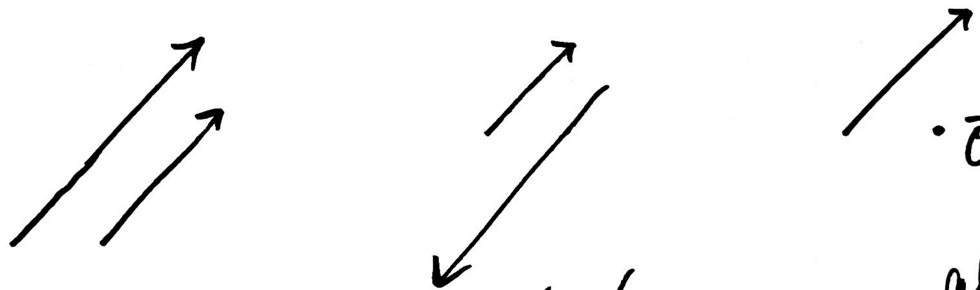
ingen retning – eller alle retninger.

(2) $\vec{a}$

$\vec{a}$ skriver heller $\vec{a}$

i tekst brukes ofte "fete" bokstaver for vektorer.

Parallellitet.



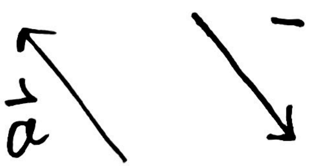
$\cdot \vec{0}$

samme retning

motsatt retning

alle vektorer er parallelle til $\vec{0}$, nullvektoren.

Motsattvektoren $\vec{a}$



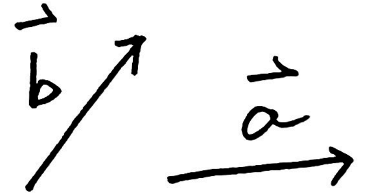
$-\vec{a}$ motsattvektoren til $\vec{a}$

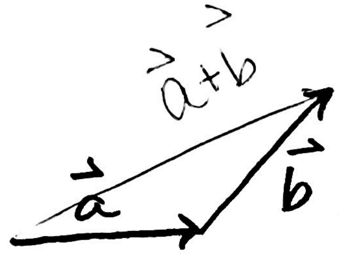
samme lengde som $\vec{a}$,

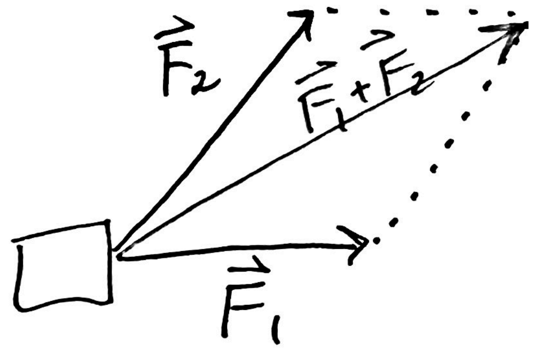
men motsatt retning.

$\vec{0} = -\vec{0}$

## 12.2 Addisjon av vektorer.



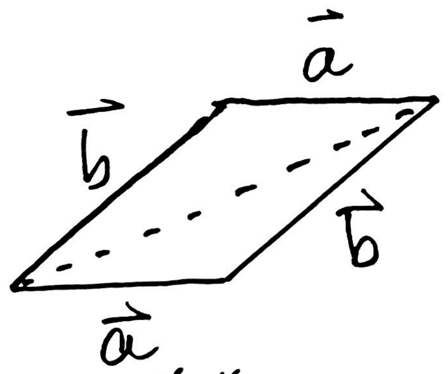




③

$\vec{a}+\vec{b} = \vec{b}+\vec{a}$ kommutativ

$\vec{0}+\vec{a} = \vec{a} = \vec{a}+\vec{0}$

$\vec{a}+(-\vec{a}) = \vec{0}$



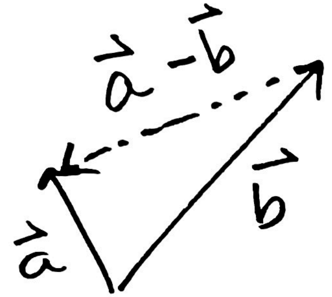

parallellogram

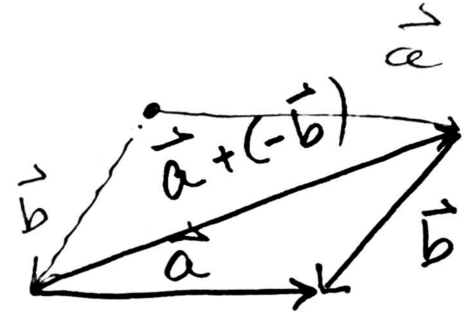

$(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$ assosiativ

tegn...

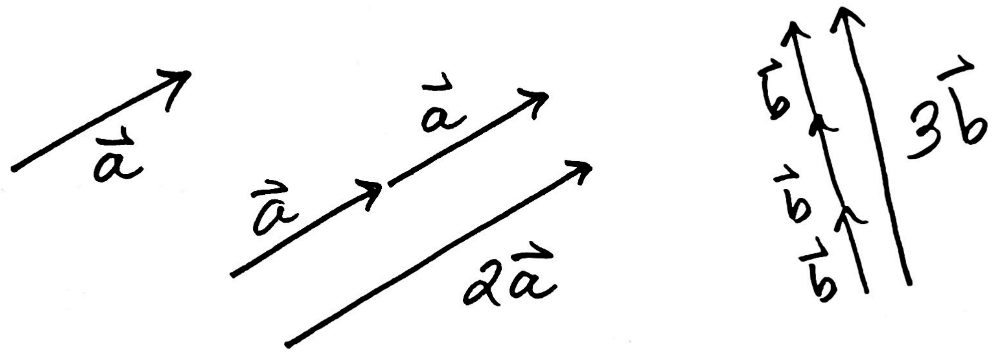
$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

(4)



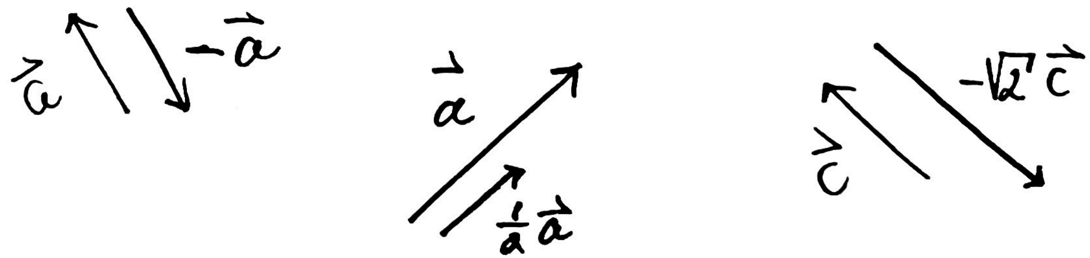




12.3 skalering.




$-1 \cdot \vec{a} = -\vec{a}$

snur retningen



$r \cdot \vec{v}$ har lengde $r \cdot$ lengden til $v$

$r > 0$ samme retning

$r < 0$ motsatt retning.

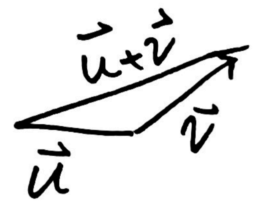
$r = 0$ $\vec{0}$

5

$1 \cdot \vec{v} = \vec{v}$

$(r+s)\vec{v} = r \cdot \vec{v} + s \cdot \vec{v}$

$r(\vec{u}+\vec{v}) = r\vec{u} + r\vec{v}$

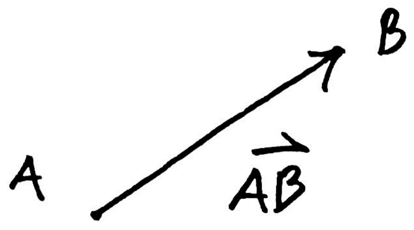


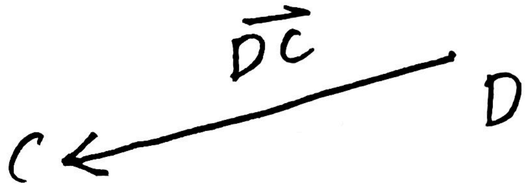

$(r \cdot s)\vec{v} = r \cdot (s \cdot \vec{v})$

$r, s$ skalarer, reelle tall

$\vec{u}, \vec{v}$ vektorer

# 12.1 Punkt og vektorer



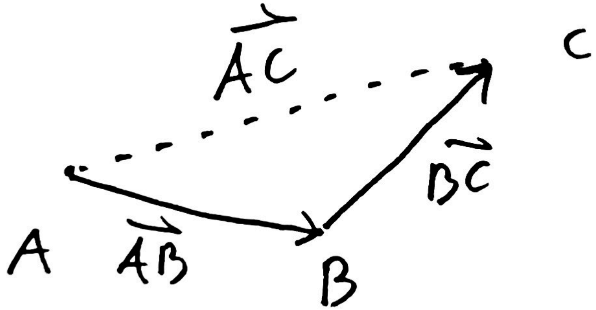




(6)

$$\boxed{\vec{AB} = -\vec{BA}}$$

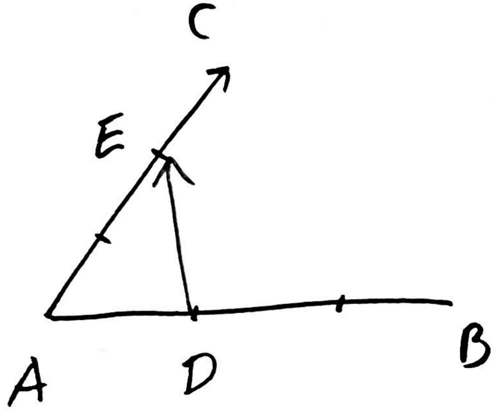
"husker ikkje A og B"

$$\vec{CD} = -\vec{DC}$$




$$\boxed{\vec{AB} + \vec{BC} = \vec{AC}}$$

$$\begin{aligned} \vec{AB} - \vec{BA} &= \vec{AB} + (-\vec{BA}) \\ &= \vec{AB} + \vec{AB} \\ &= 2\vec{AB}. \end{aligned}$$




$$\vec{AE} = \frac{2}{3}\vec{AC}$$

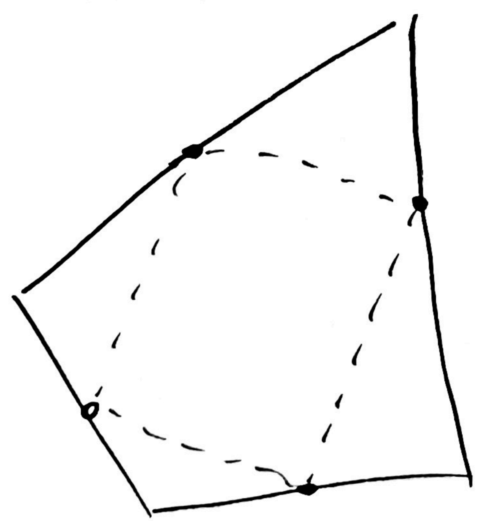
$$\vec{AD} = \frac{1}{3}\vec{AB}$$

Uttrykk $\vec{DE}$ ved hjelp av $\vec{AC}$ og $\vec{AB}$.

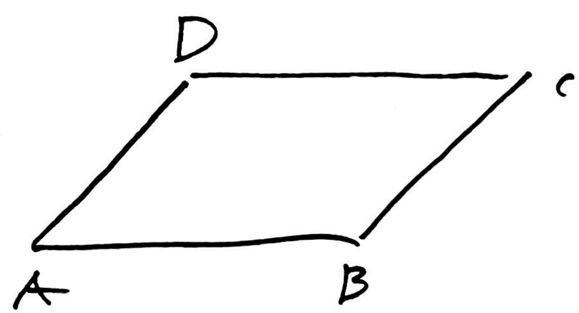
(7)

$$\vec{DE} = \vec{DA} + \vec{AE}$$

$$\vec{DE} = -\frac{1}{3}\vec{AB} + \frac{2}{3}\vec{AC}$$



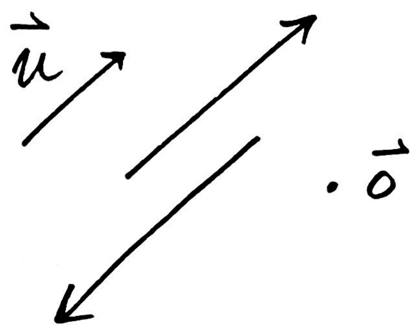
parallellogram




$$\vec{AB} = \vec{DC}$$

$$\vec{AD} = \vec{BC}$$

Skaleing og parallellitet




(12.8 og 12.9)

$\vec{v}$ er parallell til $\vec{u} \neq \vec{0}$ hvis og bare hvis det finnes skalar $s$ slik at $\vec{v} = s \cdot \vec{u}$

Hvis $\vec{a}$ og $\vec{b}$ ikkje er parallelle (så $\vec{a}, \vec{b} \neq \vec{0}$)

og $\vec{u} = x\vec{a} + y\vec{b}$. Da er $x$ og $y$ entydige.

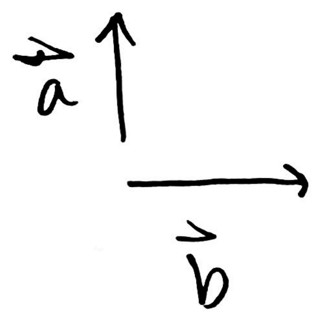
Anta $x_1\vec{a} + y_1\vec{b} = \vec{u} = x_2\vec{a} + y_2\vec{b}$

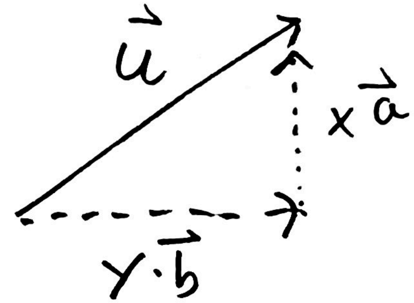
$(x_1 - x_2)\vec{a} = (y_2 - y_1)\vec{b}$

8

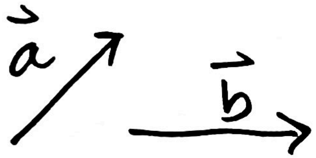
hvis $x_1 \neq x_2$ så må $\vec{a}$ og $\vec{b}$ være parallelle, ikkje mulig.

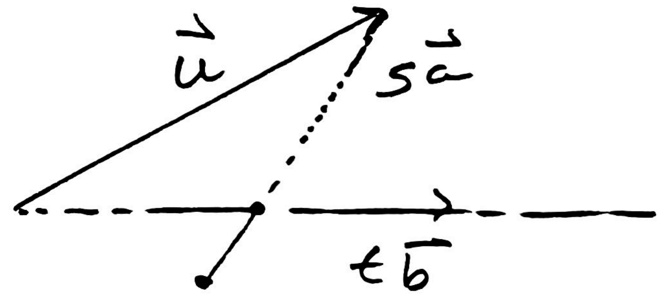
Derfor må $x_1 = x_2$ og $y_1 = y_2$.




9




Hvis $\vec{a}, \vec{b}$ ikkje er paralle, $\vec{u}$ en vilkårlig vektor i planet,
da finnes det (entydige) $x$ og $y$ slik at

$$\vec{u} = x\vec{a} + y\vec{b}$$

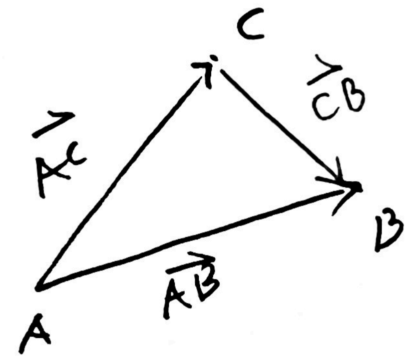
$\vec{a}, \vec{b}$ er en <u>basis</u> for vektorer i planet.

Skalering $4(2\vec{a} - 3\vec{b}) = 4(2\vec{a}) + 4(-3)\vec{b}$

$= 8\vec{a} - 12\vec{b}$

oppgave

Forenkl: $\vec{AB} + \vec{CB} - \vec{AC}$




$= \vec{AB} + \vec{CB} + \vec{CA}$

$= \underbrace{\vec{CA} + \vec{AB}}_{\vec{CB}} + \vec{CB}$

10

$= \underline{2\vec{CB}}$.

opg. La $\vec{u} = \vec{a} - 2\vec{b}$ og $\vec{v} = 2\vec{a} + 3\vec{b}$

Uttrykk $2\vec{u} - \vec{v}$ ved hjelp av $\vec{a}$ og $\vec{b}$

$$2(\vec{a}-2\vec{b}) - (2\vec{a}+3\vec{b})$$

$$2\vec{a}+2(-2\vec{b}) \quad (-1)2\vec{a} + (-1)(3\vec{b})$$

$$\underbrace{2\vec{a}-2\vec{a}}_{\vec{0}} \quad -4\vec{b}-3\vec{b} = \underline{-7\vec{b}}$$

(11)

$\vec{a}, \vec{b}$ ikkje parallelle

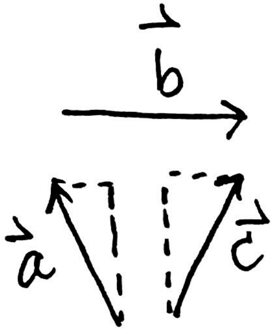
Bestem skalaren $s$ slik at $s\vec{a}+4\vec{b}$ og $2\vec{a}-\vec{b}$ er paralle

$$t(2\vec{a}-\vec{b}) = s\vec{a}+4\vec{b}$$

$$2t\vec{a}+(-t)\vec{b} = s\vec{a}+4\vec{b} \iff \begin{matrix} -t=4 \\ 2t=s \end{matrix}$$

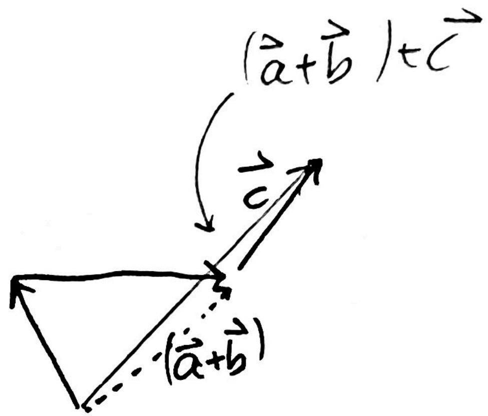
så $t=-4$ og da

$$s = 2t = \underline{-8}$$




## ØVING

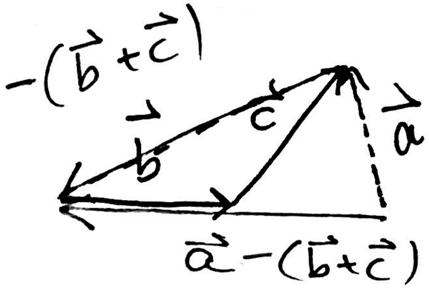
a) $(\vec{a}+\vec{b})+\vec{c}$



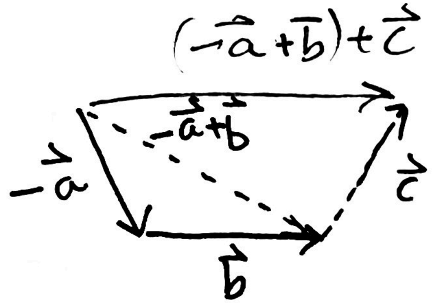

12

b) $\vec{a}+(-\vec{b}-\vec{c})$

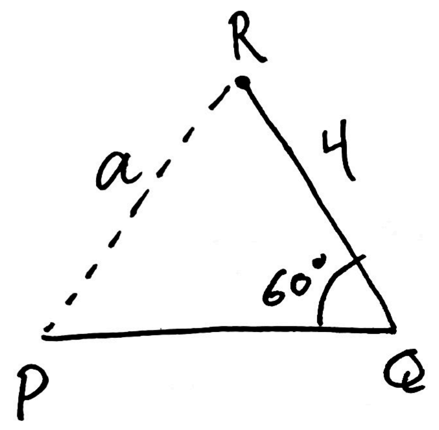
$= \vec{a}-(\vec{b}+\vec{c})$




c) $(-\vec{a}+\vec{b})+\vec{c}$




$$\vec{a}-(\vec{b}+\vec{c}) = \underbrace{\vec{a}-\vec{b}-\vec{c}}_{b)} = -\underbrace{(-\vec{a}+\vec{b}+\vec{c})}_{c)}$$




$a < 4 \cdot \sin(60°) = \sqrt{3} \cdot 2$ ingen løsning

$\left.\begin{array}{l} a = 2\sqrt{3} \\ \text{og } a \geq 4 \end{array}\right\}$ én løsning.

$2\sqrt{3} < a < 4$ to løsninger.

13

5.74

$$x^4 - 10x^2 + 9 = 0$$

$x^2 = u$

$x^4 = (x^2)^2 = u^2$

$$u^2 - 10u + 9 = 0$$

$$(u-9)(u-1) = 0 \iff u = 1 \text{ eller } u = 9$$

$$x^2 = 1 \text{ eller } x^2 = 9$$

Løsningene er $x = \underline{-1, 1, -3, 3}$.

$$x^4 - 10x^2 + 9 = (x^2 - 9)(x^2 - 1)$$

oppg.

$$x^5 - 13x^3 + 36x$$

$$= x(x^4 - 13x^2 + 36)$$

$$= x(x^2 - 9)(x^2 - 4)$$

$$= x(x-3)(x+3)(x-2)(x+2)$$

$13 = 4 + 9$

$36 = 4 \cdot 9$

14