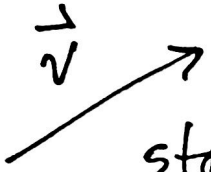
Vektorer på koordinatform 12.4-6.

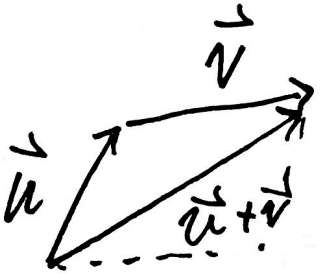
12.10
2021

Repetisjon

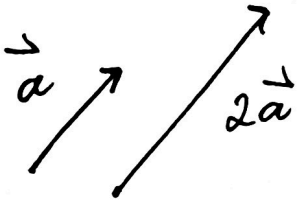


retning

størrelse $|\vec{v}|$

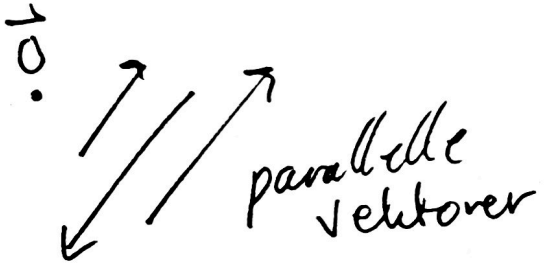


sum



motsatt vektor

$r\vec{a}$ skalerer $\vec{a}$ med r

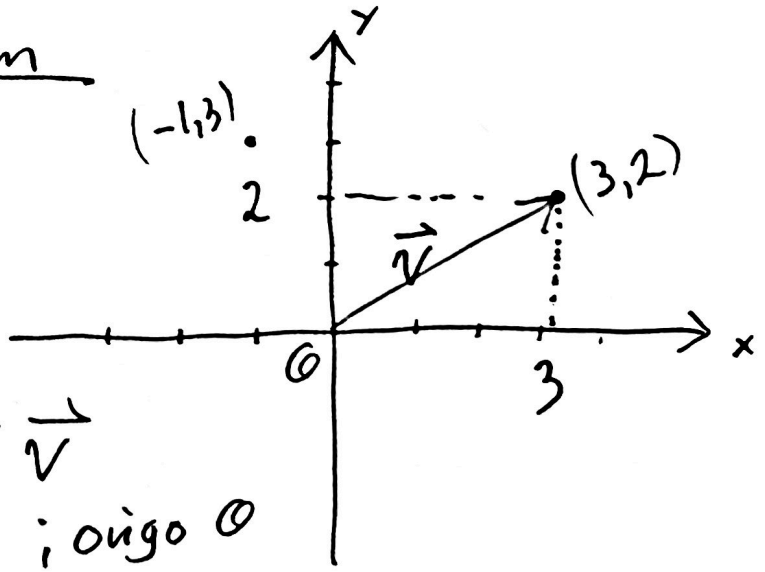


parallelle vektorer

①

Koordinatform

koordinatsystem



Vektorkoordinatene

til $\vec{v} = [x, y]$

hvor (x,y) er endepunktet til $\vec{v}$

når den starter i origo O

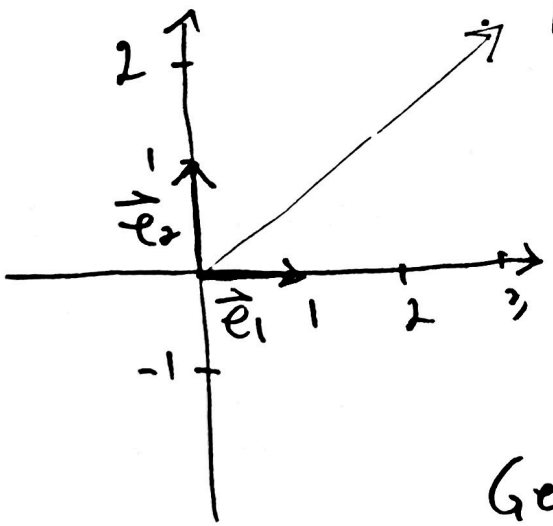
Hvis $P(x,y)$ er et punkt. Da er $\overrightarrow{OP} = [x,y]$

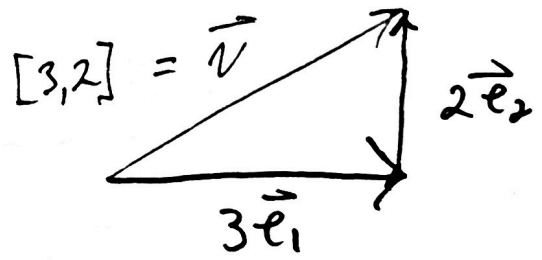
Korrespondanse: Punkt i $\mathbb{R}^2$ $\leftrightarrow$ vektorer i planet. $\mathbb{R}^2$.

(2)

$[x_1,y_1] = [x_2,y_2] \iff x_1 = x_2$ og $y_1 = y_2$.

$[3,2] = 3\vec{e}_1 + 2\vec{e}_2$





Generelt $[x,y] = x\vec{e}_1 + y\vec{e}_2$

$$t[x,y] = t(x\vec{e}_1 + y\vec{e}_2)$$
$$= t\cdot x\,\vec{e}_1 + t\cdot y\,\vec{e}_2$$
$$= [tx, ty]$$

③ $\underline{t[x,y] = [tx, ty]}$

$$[x_1,y_1] + [x_2,y_2] = (x_1\vec{e}_1 + y_1\vec{e}_2) + (x_2\vec{e}_1 + y_2\vec{e}_2)$$
$$= (x_1+x_2)\vec{e}_1 + (y_1+y_2)\vec{e}_2$$

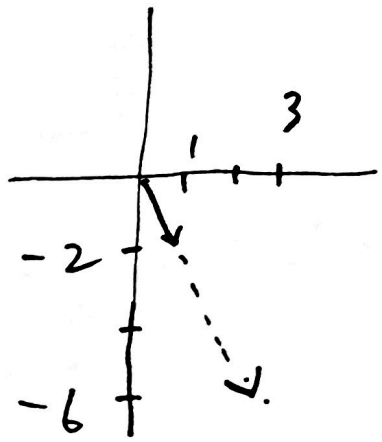
$\underline{[x_1,y_1]+[x_2,y_2] = [x_1+x_2, y_1+y_2]}$

$\vec{0} = [0,0]$ $\quad -[x,y] = [-x,-y]$

Ex $\quad [2,3] + [7,-5] = [2+7, 3-5]$
$$= [9,-2]$$

$$3[1,-2] = [3\cdot 1, 3(-2)] = [3,-6]$$

(4)



oppg Finn koordinatene til vektoren

$$2[3,1] - 5[2,-1]$$

$$= [6,2] - [10,-5]$$

$$= [6-10, 2+5] = \underline{[-4,7]}$$

oppg Finn koordinatene til vektoren $\vec{v}$ slikt at

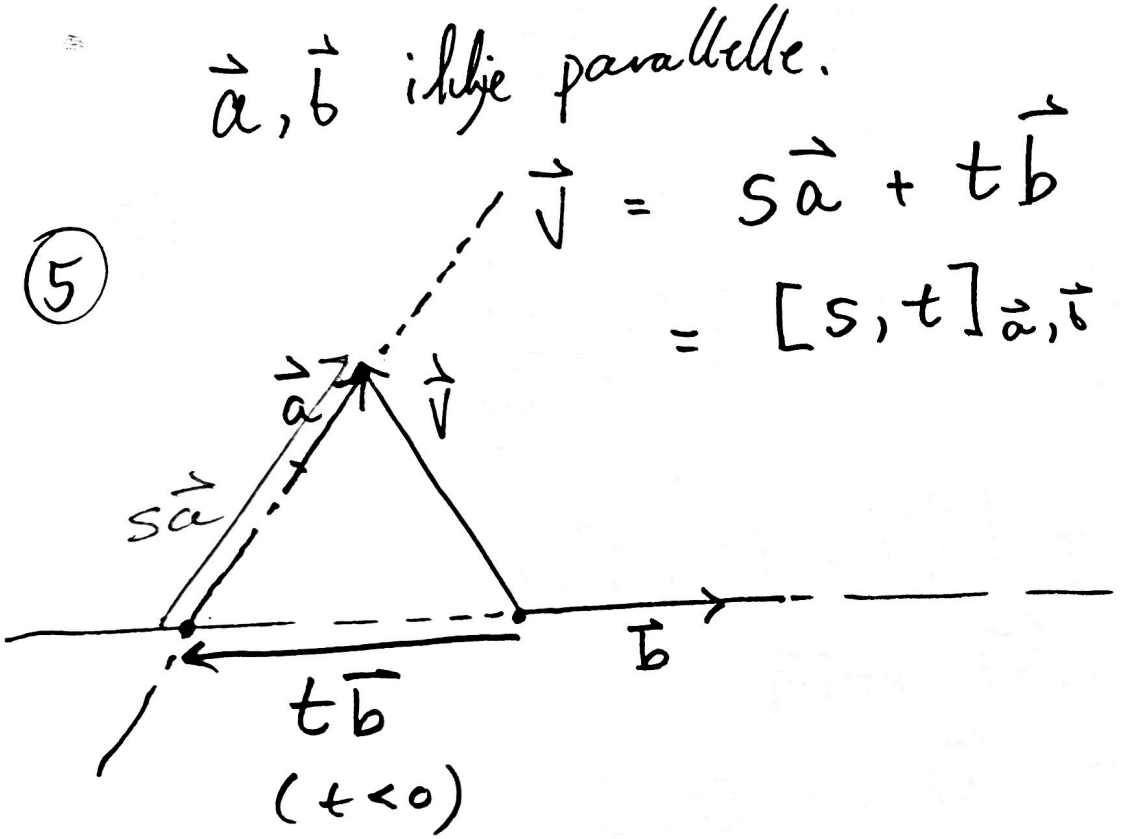
$$[1,2] + 2\vec{v} = [4,4]$$

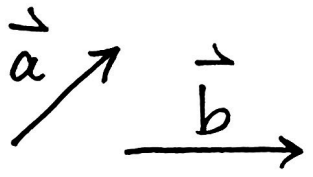
legger til $-[1,2]$ på begge sider av =-tegnet

$$2\vec{v} = [4,4] - [1,2] = [4-1, 4-2] = [3,2]$$

Skalerer med $\frac{1}{2}$: $\vec{v} = (\frac{1}{2}\cdot 2)\vec{v} = 1\cdot\vec{v} = \frac{1}{2}[3,2] = \underline{[3/2, 1]}$

$\vec{a}, \vec{b}$ ikke parallelle.





koordinatene i basisen $\vec{a}$ og $\vec{b}$.

⑤

oppg. Finn vektoren $\vec{v}$ slik at $3(\vec{v} + [1,-1]) = -2\vec{v} + 3[2,4]$

$3\vec{v} + 3[1,-1] = -2\vec{v} + [3\cdot 2, 3\cdot 4]$

$3\vec{v} + 2\vec{v} = [6,12] - [3,-3]$

$5\vec{v} = [6-3, 12-(-3)] = [3,15]$

skalerer med $\frac{1}{5}$ $\quad \vec{v} = \frac{1}{5}5\vec{v} = \frac{1}{5}[3,15] = [\frac{3}{5}, \frac{15}{5}] = \underline{[\frac{3}{5}, 3]}$

$s\vec{v} = [1,2]$ $\quad s \neq 0$

⑥ $\frac{1}{s} \cdot (s\vec{v}) = \frac{1}{s}[1,2]$

$\left(\frac{1}{s} \cdot s\right)\vec{v} = 1 \cdot \vec{v} = \underline{\vec{v} = \frac{1}{s}[1,2]}$

oppg $\quad 2\left([1,3] - \vec{w}\right) + [2,2] = 2\vec{w} - [3,4]$

Finn $\vec{w}$.

$2[1,3] - 2\vec{w} + [2,2] = 2\vec{w} - [3,4]$

$-2\vec{w} - 2\vec{w} = -[3,4] - [2,2] - [2,6]$

$= -\left([3,4] + [2,2] + [2,6]\right)$

$= -\left([3+2+2, 4+2+6]\right)$

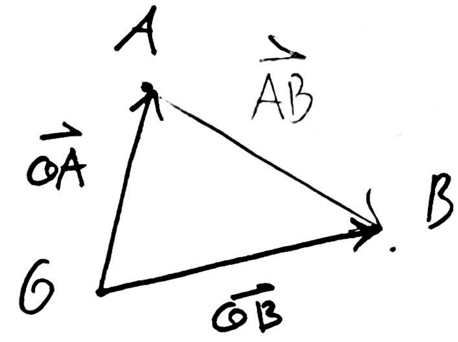
$-4\vec{w} = -[7,12]$

$\vec{w} = \frac{-1}{4}(-4w) = \left(\frac{-1}{4}\right)(-[7,12]) = \frac{1}{4}[7,12] = \underline{\left[\frac{7}{4}, 3\right]}$

(7)

$A(x_1, y_1)$ $B(x_2, y_2)$

$\vec{OA} = [x_1, y_1]$ $\vec{OB} = [x_2, y_2]$.



$\vec{AB} = \vec{OB} - \vec{OA}$

$\left(= \vec{OB} + \vec{AO} \right.$

$\left. = \vec{AO} + \vec{OB} \right)$

$= [x_2, y_2] - [x_1, y_1]$

$= [x_2 - x_1, y_2 - y_1]$

ikkje skriv

~~$\vec{AB} = B - A$~~ ...

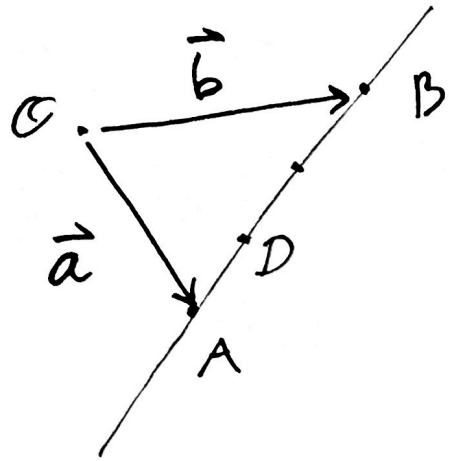
$A(1, 2)$

$B(5, -3)$

$\vec{AB} = [5, -3] - [1, 2]$

$= [5 - 1, -3 - 2]$

$= \underline{[4, -5]}$



D ligger mellom A og B på linjen og avstanden AD er halvparten av avstanden DB.

⑧ Beskriv $\vec{OD}$ ved hjelp $\vec{a}$ og $\vec{b}$

$$\vec{AD} = \frac{1}{3}\vec{AB} = \frac{1}{3}(\vec{b}-\vec{a})$$

$$\vec{OD} = \vec{OA} + \vec{AD} = \vec{a} + \frac{1}{3}(\vec{b}-\vec{a})$$

$$= \vec{a} + \frac{1}{3}\vec{b} - \frac{1}{3}\vec{a}$$

$$= (1-\frac{1}{3})\vec{a} + \frac{1}{3}\vec{b}$$

$$\vec{OD} = \underline{\frac{2}{3}\vec{a} + \frac{1}{3}\vec{b}}$$

$\vec{a} = [1,2]$

$\vec{b} = [3,1]$ ikkje parallelle

(9) Finn s og t slik at $s\vec{a} + t\vec{b} = [-7,1]$ } uttrykk $[-7,1]$ i basise. $\vec{a}$ og $\vec{b}$

$$s[1,2] + t[3,1] = [-7,1]$$

x-koord $\quad s + 3t = -7$,

y-koord $\quad 2s + t = 1$ så $t = 1-2s$

setter inn i første likning

$$s + 3(1-2s) = -7$$

$$s + (-6s) + 3 = -7$$

$$-5s = -7 - 3 = -10$$

$$s = -10/(-5) = \underline{2}.$$

$$t = 1 - 2s = 1 - 2 \cdot 2 = \underline{-3}$$

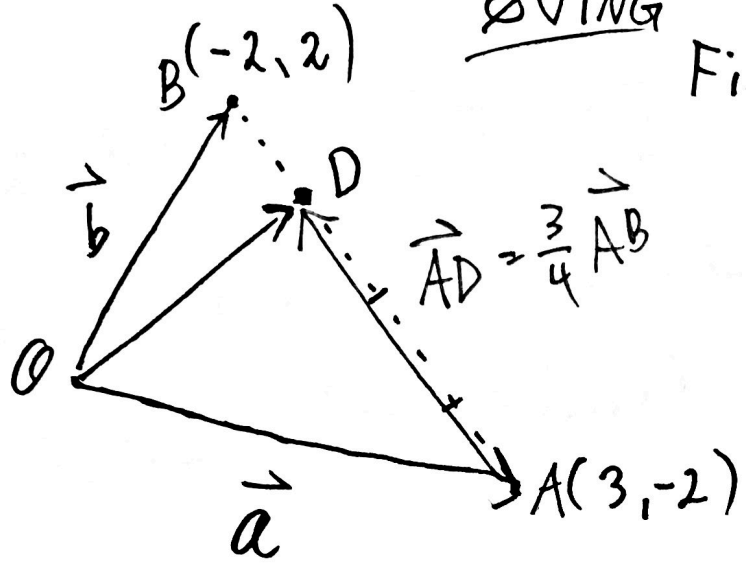
sjekker $2[1,2] - 3[3,1] = [-7,1]$

(10) $[2,-3]_{\vec{a},\vec{b}} = [-7,1]_{\vec{e}_1,\vec{e}_2}$

oppgave

ØVING

Finn koordinatene til D.



(11)

$\vec{a} = [3,-2]$

$\vec{b} = [-2,2]$.

$\vec{AB} = \vec{OB} - \vec{OA}$

$= \vec{b} - \vec{a}$

$\vec{OD} = \vec{OA} + \frac{3}{4}\vec{AB}$

$= \vec{a} + \frac{3}{4}(\vec{b} - \vec{a})$

$= 1\cdot\vec{a} - \frac{3}{4}\vec{a} + \frac{3}{4}\vec{b}$

$\vec{OD} = \frac{1}{4}\vec{a} + \frac{3}{4}\vec{b} = \frac{1}{4}[3,-2] + \frac{3}{4}[-2,2]$

$= [\frac{3}{4}, \frac{-1}{2}] + [\frac{-3}{2}, \frac{3}{2}] = [\frac{3}{4} - \frac{3}{2}, \frac{-1}{2} + \frac{3}{2}]$

$= \underline{[\frac{-3}{4}, 1]}$ så $\underline{D(\frac{-3}{4}, 1)}$

$$-5[2,3] + [7,12]$$

$$= [-10,-15] + [7,12] = [-10+7, -15+12]$$

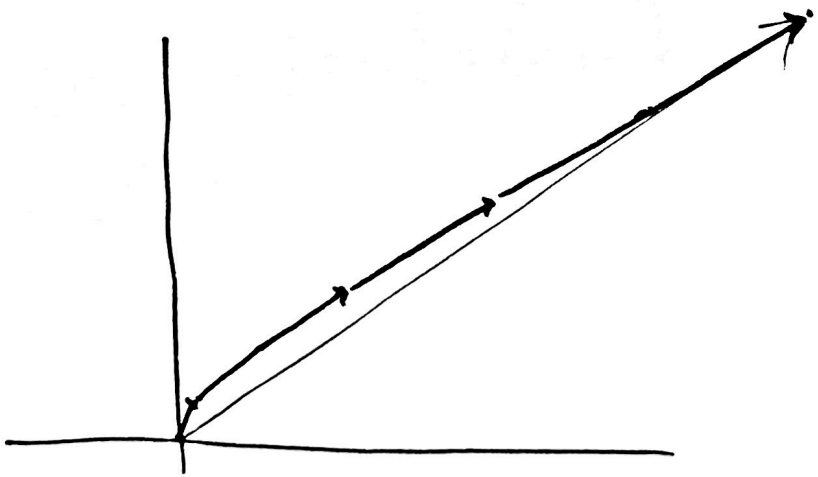
$$= [-3,-3] = (-1)\cdot 3\,[1,1] = \underline{-3[1,1]}$$

(12)

oplg

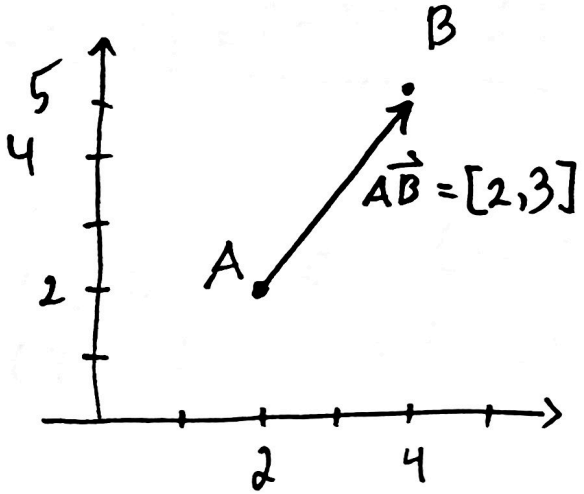
$$[1,2] + 2[3,4] + 3[5,6]$$

$$= [1+2\cdot 3+3\cdot 5,\ 2+2\cdot 4+3\cdot 6] = \underline{[22,28]}$$



Gitt $\vec{AB} = [2,3]$ og $B(4,5)$

Hva er koordinatene til A?



(13)

$\vec{OB} = [4,5]$

$\vec{AB} = \vec{OB} - \vec{OA}$

$\vec{OA} = \vec{OB} - \vec{AB}$

$(\vec{OB} + \vec{BA})$

$\vec{OA} = [4,5] - [2,3] = [2,2]$ så $A(2,2)$

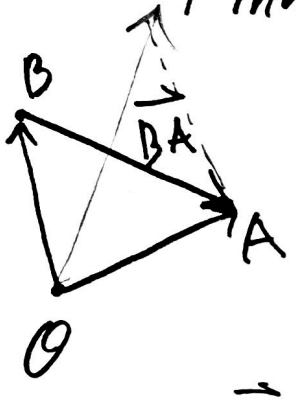
$= 2[1,1]$

$A(2,-3)$ $B(7,-1)$

Finn $-3\vec{AB}$

$\vec{BA} = \vec{OA} - \vec{OB} = [2,-3] - [7,-1] = [-5,-2]$

$-3\vec{AB} = 3(-\vec{AB}) = 3\vec{BA} = 3[-5,-2] = \underline{[-15,-6]}$



Den grønne pilen er $\vec{OA} + \vec{OB}$.

$A(1,-2)$ $C = (\sqrt{2},-37)$

$B(7,18)$ $D = (4,6)$

(14)

$\vec{AB} - \vec{CB} + \vec{CD}$

$\underbrace{\vec{AB} + \vec{BC}}_{\vec{AC}} + \vec{CD}$

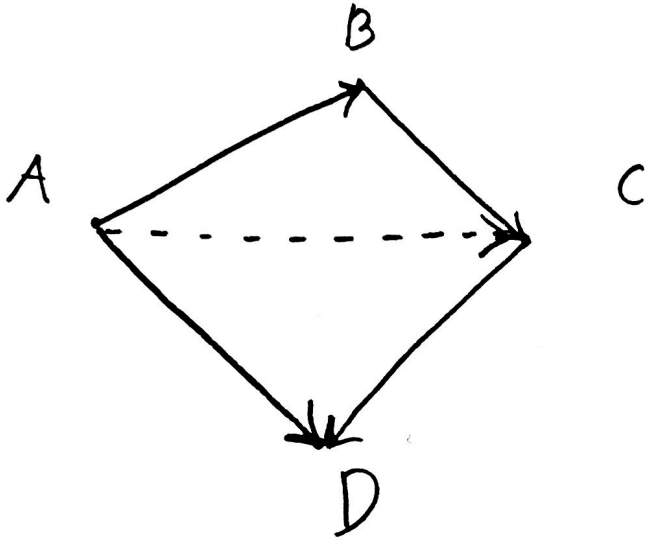
$+ \vec{CD} = \vec{AD} = \vec{OD} - \vec{OA}$

$= [4,6] - [1,-2]$

$= [4-1, 6-(-2)]$

$\vec{AD} = \underline{[3,8]}$

(15)



$\vec{AB} + \vec{BC}$

$= \vec{AC}$

$\vec{AC} + \vec{CD} = \vec{AD}$