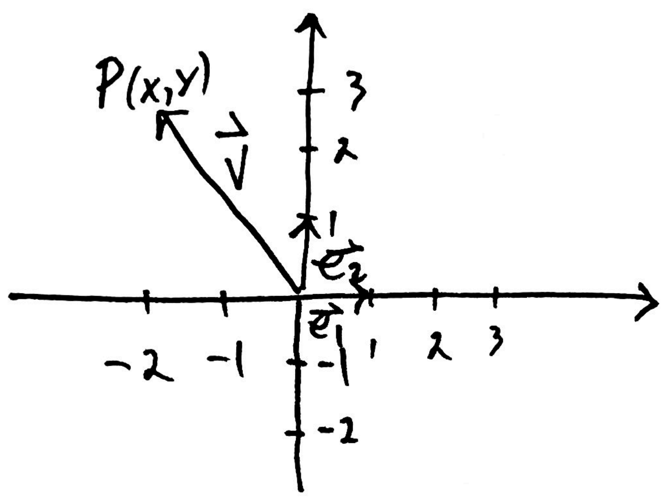
13.10
2021

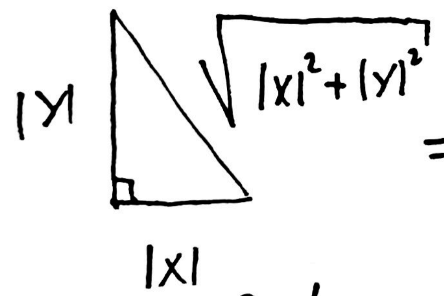
12.7-9 Lengde, parallellitet

(1)



$$\vec{v} = \vec{OP} = [x,y] = x\vec{e_1} + y\vec{e_2}.$$

$$\vec{e_1} = [1,0]$$
$$\vec{e_2} = [0,1]$$



$= \sqrt{x^2+y^2}$

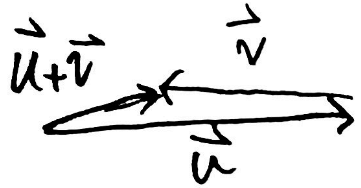
Pytagoras sin sats

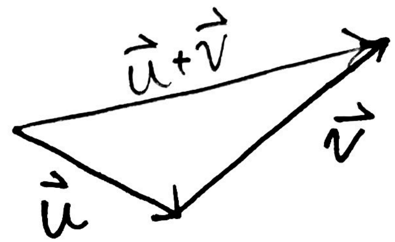
(Euklidske) Normen (lengden) til $[x,y]$ er

$$|[x,y]| = \sqrt{x^2+y^2}$$

Egenskaper:
* $|\vec{v}| \geq 0$
* $|\vec{v}| = 0 \Leftrightarrow \vec{v} = \vec{0}$
* $|t\vec{v}| = |t| \cdot |\vec{v}|$

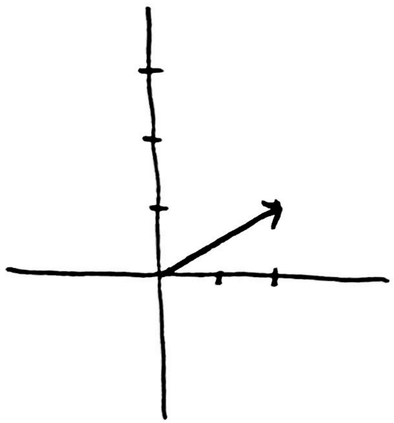
* Trekantulikheten $\quad |\vec{u}+\vec{v}| \leq |\vec{u}| + |\vec{v}|$





(Det følger at $\big|\,|\vec{u}|-|\vec{v}|\,\big| \leq |\vec{u}+\vec{v}|$)

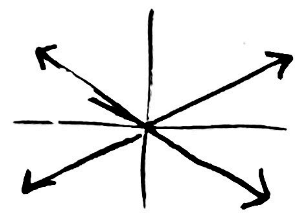
2



$$|[2,1]| = \sqrt{2^2+1^2}$$
$$= \sqrt{4+1} = \underline{\sqrt{5}} \approx 2.236\ldots$$

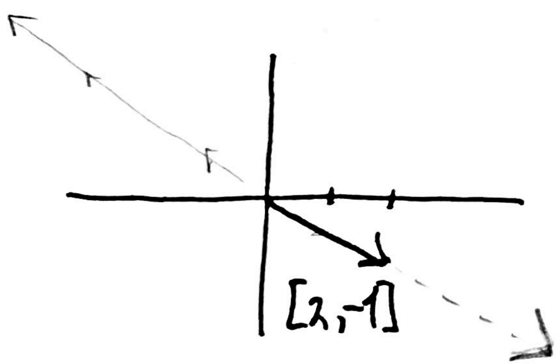
$$|[-3,4]| = \sqrt{(-3)^2+4^2} = \sqrt{9+16} = \underline{5}$$

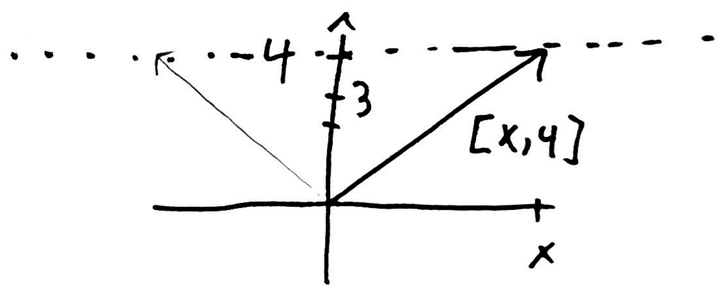
$$|[\pm x, \pm y]| = \sqrt{x^2+y^2}$$



oppg. 1) Bestem t slik at $t[2,-1] = [2t,-t]$ har lengde 10

③ 2) Finn x slik at $[x,4]$ har lengde 5
har lengde 3.





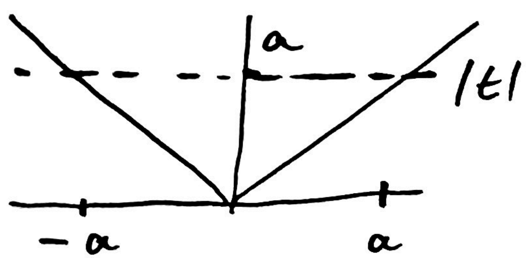
$[2,-1] = \sqrt{2^2+(-1)^2} = \sqrt{5}$.

$|[2t,-t]| = \sqrt{(2t)^2+(-t)^2} = |t|\sqrt{4+1} = |t|\sqrt{5}$

$|t|\sqrt{5} = 10$ så $|t| = \frac{10}{\sqrt{5}} = \frac{2\cdot\sqrt{5}\cdot\sqrt{5}}{\sqrt{5}} = 2\sqrt{5}$

så $t = 2\sqrt{5}$ og $t = -2\sqrt{5}$.

$\sqrt{t^2} = |t|$



(4)

$|[x,4]| = 5$

$\sqrt{x^2+4^2} = 5$

$x^2+4^2 = 25$

$x^2 = 25-16 = 9$

$x = \pm\sqrt{9} = \pm 3$

$\underline{x = 3 \text{ og } x = -3}$.

$|[x,4]| \geq 4$ så det finnes ikkje en x slik at $|[x,4]| = 3$.

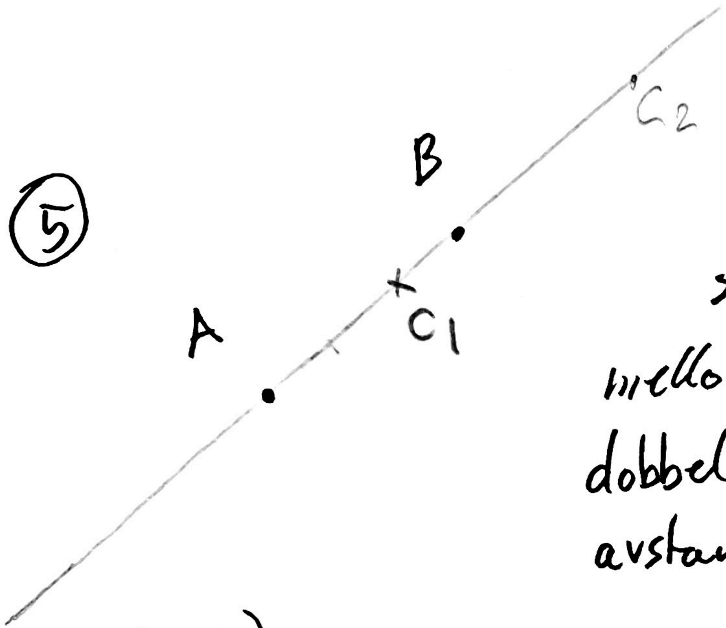
$\overrightarrow{AB} = \underbrace{[b_1, b_2]}_{\overrightarrow{OB}} - \underbrace{[a_1, a_2]}_{\overrightarrow{OA}}$

$= [b_1-a_1, b_2-a_2]$

Avstand mellom punkt A og B er $|\overrightarrow{AB}|$

$|\overrightarrow{AB}| = \sqrt{(b_1-a_1)^2+(b_2-a_2)^2}$

(5)



Finn alle punkt C på linjen slik at avstanden mellom A og C er dobbelt så stor som avstanden mellom B og C

$A = (1,2)$

$B = (3,4)$

$\vec{v} = \vec{AB}$

$\vec{OC} = \vec{OA} + t\vec{v}$

$t \cdot \vec{v} = \vec{OC} - \vec{OA} = \vec{AC}$

$|\vec{AC}| = |t\vec{v}|$

$\vec{CB} + \vec{BA} = \vec{CA}$

$\vec{CB} = \vec{CA} - \vec{BA}$

$= \vec{CA} + \vec{AB}$

$= \vec{AB} - \vec{AC}$

$= \vec{v} - t\vec{v} = \underline{(1-t)\vec{v}}$

Finn lengden til

$\vec{a} = [1, 3]$

$\vec{b} = [2, -7]$

$\vec{a} + \vec{b} = [3, -4]$

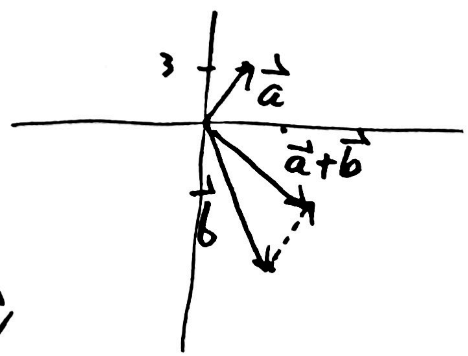
$|\vec{a}| = \sqrt{1^2 + 3^2} = \sqrt{10} \sim 3.16$

$|\vec{b}| = \sqrt{2^2 + (-7)^2} = \sqrt{53} \sim 7.28$

$|\vec{a}+\vec{b}| = \sqrt{3^2 + 4^2} = 5$

$||\vec{a}| - |\vec{b}|| \leq |\vec{a}+\vec{b}| \leq |\vec{a}| + |\vec{b}|$

$|3.16 - 7.28| \sim 4.12.$



6

$$|\vec{AC}| = 2|\vec{CB}|$$

$$|t\vec{v}| = 2|(1-t)\vec{v}|$$

$$|t|\cdot|\vec{v}| = 2|1-t|\,|\vec{v}| \qquad |\vec{v}| \neq 0$$

$$|t| = 2|1-t|$$

$t<0$

$$-t = 2(1-t) = 2-2t$$

$$2t-t = t = 2$$

ingen løsning.

$0 \leq t \leq 1$

$$t = 2(1-t) = 2-2t$$

$$3t = 2$$

så $\underline{t = \frac{2}{3}}$

$t>1$

$$|t| = t = 2|1-t| = -2(1-t)$$

$$= -2+2t$$

$$\underline{2 = t}$$

Koordinatene til C.

7

$$\vec{OC} = \vec{OA} + t\vec{AB} = [1,2] + t[2,2]$$

$$\vec{OA} = [1,2]$$

$$\vec{AB} = \vec{OB} - \vec{OA} = [3,4] - [1,2] = [2,2].$$

$t = \frac{2}{3}$

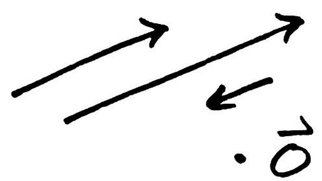
$$\vec{OC_1} = [1,2] + \frac{2}{3}[2,2] = [1,2] + \left[\frac{4}{3}, \frac{4}{3}\right] = \left[\frac{7}{3}, \frac{10}{3}\right]$$

$t = 2$

$$\vec{OC_2} = [1,2] + 2[2,2] = [1,2] + [4,4] = [5,6]$$

Koordinatene til de to punktene er $\underline{C_1\left(\frac{7}{3}, \frac{10}{3}\right)}$ og $\underline{C_2(5,6)}$

Parallellitet

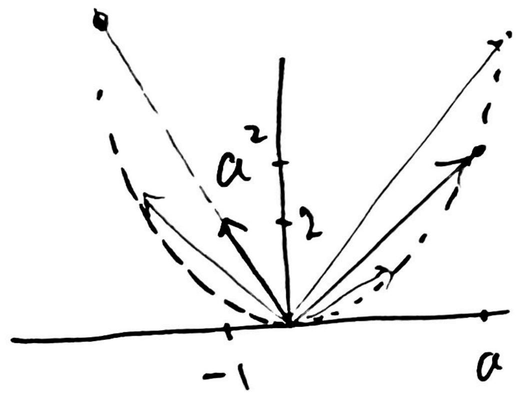


$\vec{u} \neq \vec{0}$

$\vec{u}$ og $\vec{v}$ er parallelle

$\iff \quad \vec{v} = t\,\vec{u} \qquad t$ skalar.

(8) Finn a slik at $[a, a^2]$ og $[-1, 2]$ blir parallelle.



Parallelle $\iff \quad t[-1,2] = [a, a^2]$

$-t = a$ x-koord.

$2t = a^2$ y-koord.

like
g.i.l

(−2) gangel likning 1

$2t = -2a$

så $\quad -2a = a^2$

$a^2 = -2a \iff a^2 + 2a = 0$

$a(a+2) = 0$

$\iff a = 0$ og $a = -2$.

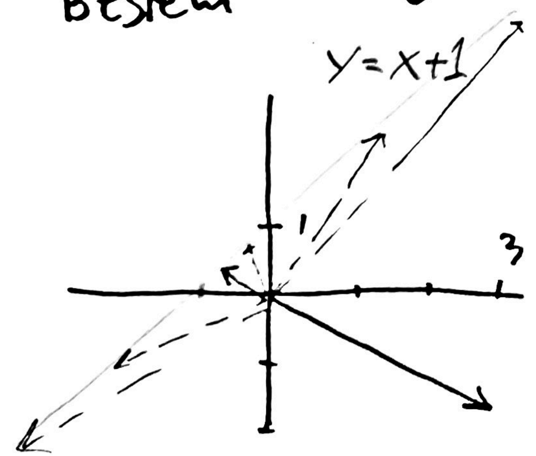
9

$a = 0 \quad [a, a^2] = \vec{0}$

$a = -2 \quad [a, a^2] = [-2, 4] = 2[-1, 2]$

Løsningene er $\underline{a = 0 \text{ og } a = -2}$

Oppg. Bestem t slik at $[t, t+1]$ og $[3, -2]$ blir parallelle.



$[t, t+1] = s[3, -2]$

$= [3s, -2s]$

$t = 3s$

$t+1 = -2s$

10

$t = 3s$

$t = -2s - 1$

$3s = -2s - 1$ så $5s = -1$

$s = -1/5$.

$t = 3s = \underline{-3/5}$

Løsningen er $\underline{t = -3/5}$