20.10
2021

13.7-8 **Determinanter** Faush

(1) Determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \cdot d - bc$

$$\begin{bmatrix} a & d \\ 0 & 0 \end{bmatrix} = a \cdot 0 - d \cdot 0 = 0$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \cdot 1 - 0 \cdot 0 = \underline{1}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = 0 \cdot 0 - 1 \cdot 1 = -1$$

generelt: $\begin{vmatrix} c & d \\ a & b \end{vmatrix} = b \cdot c - ad = -\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

bytter vi radene så snus fortegnet til determinanten.

$$\begin{vmatrix} 15 & 20 \\ c & d \end{vmatrix} = 15 \cdot d - 20 \cdot c = 5(3 \cdot d - 4c)$$
$$= 5\begin{vmatrix} 3 & 4 \\ c & d \end{vmatrix}$$

$$\begin{vmatrix} ra & rb \\ c & d \end{vmatrix} = rad - rbc = r(ad - bc) = r\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

(2)

Determinanter tar inn 2 radvektorer og gir en skalar $\begin{vmatrix} \vec{r}_1 \\ \vec{r}_2 \end{vmatrix}$. Antisymmetrisk under ombytte av vektorene.

rader

$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$ matrise av størrelse 2×2

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$$

SØYLER

3

Hvis $\vec{r}_1$ og $\vec{r}_2$ er parallelle, da er

$$\begin{vmatrix} \vec{r}_1 \\ \vec{r}_2 \end{vmatrix} = 0$$

$\vec{r}_1 = t\vec{r}_2$ (eller $\vec{r}_2 = s\vec{r}_1$)

$$\begin{vmatrix} \vec{r}_1 \\ \vec{r}_2 \end{vmatrix} = \begin{vmatrix} t\vec{r}_2 \\ \vec{r}_2 \end{vmatrix} = t \begin{vmatrix} \vec{r}_2 \\ \vec{r}_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 15, & 20 \\ 21 & 28 \end{vmatrix} = 15 \cdot 28 - 20 \cdot 21 = 0$$

oppg.

$$\begin{vmatrix} 5[3,4] \\ 7[3,4] \end{vmatrix} = 5 \cdot 7 \begin{vmatrix} 3 & 4 \\ 3 & 4 \end{vmatrix} = 0$$

oppg

$$\begin{vmatrix} 15 & 20 \\ 25 & 35 \end{vmatrix} = \begin{vmatrix} 5[3,4] \\ 5[5,7] \end{vmatrix} = 5^2 \begin{vmatrix} 3 & 4 \\ 5 & 7 \end{vmatrix}$$

$$= 25(\underbrace{3 \cdot 7 - 4 \cdot 5}_{21-20}) = \underline{25}$$

$$\begin{vmatrix} \vec{r}_1 + \vec{r}_2 \\ \vec{s} \end{vmatrix} = \begin{vmatrix} \vec{r}_1 \\ \vec{s} \end{vmatrix} + \begin{vmatrix} \vec{r}_2 \\ \vec{s} \end{vmatrix}$$

$\vec{r}_1 = [x_1, y_1]$ $\quad \vec{r}_2 = [x_2, y_2]$

$\vec{s} = [c, d]$

4

$$\begin{vmatrix} x_1 + x_2 , & y_1 + y_2 \\ c & d \end{vmatrix} = (x_1 + x_2) d - (y_1 + y_2) \cdot c$$

$$= x_1 d - y_1 c + x_2 d - y_2 c$$

$$= \begin{vmatrix} x_1 & y_1 \\ c & d \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ c & d \end{vmatrix}$$

$$\begin{vmatrix} 13 , & 17 \\ 28 & 41 \end{vmatrix} = \begin{vmatrix} 13 & 17 \\ 2 + 2 \cdot 13 , & 7 + 2 \cdot 17 \end{vmatrix}$$

$$= \begin{vmatrix} 13 & 17 \\ 2 & 7 \end{vmatrix} + 2 \underbrace{\begin{vmatrix} 13 & 17 \\ 13 & 17 \end{vmatrix}}_{0}$$

$$= \begin{vmatrix} 13 & 17 \\ 2 & 7 \end{vmatrix} = 13 \cdot 7 - 2 \cdot 17$$

$$(10+3)(10-3) - 34$$

$$10^2 - 3^2$$

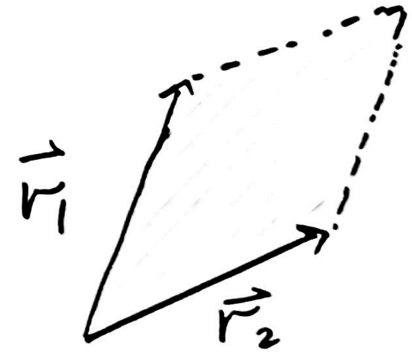
$$= 91 - 34 = \underline{57}$$

(konjugatsetningen
$(a-b)(a+b) = a^2 - b^2$)

5

$$\left| \det \begin{pmatrix} \vec{r}_1 \\ \vec{r}_2 \end{pmatrix} \right| = \text{arealet til parallellogrammet utspent av } \vec{r}_1 \text{ og } \vec{r}_2 .$$

$$\left| \begin{vmatrix} \vec{r}_1 \\ \vec{r}_0 \end{vmatrix} \right|$$

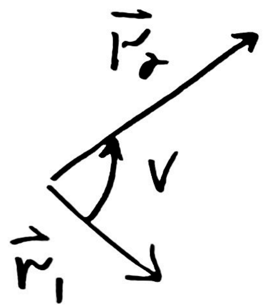


$\vec{r}_1$ og $\vec{r}_2$ er parallelle $\iff \begin{vmatrix} \vec{r}_1 \\ \vec{r}_2 \end{vmatrix} = 0$

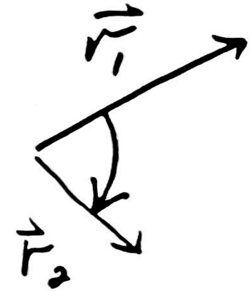
(6) $-|\vec{r}_1||\vec{r}_2| \leq \det\left(\begin{bmatrix} \vec{r}_1 \\ \vec{r}_2 \end{bmatrix}\right) \leq |\vec{r}_1| \cdot |\vec{r}_2|$

$$\left|\det\left(\begin{bmatrix} \vec{r}_1 \\ \vec{r}_2 \end{bmatrix}\right)\right| = |\vec{r}_1| \cdot |\vec{r}_2|.$$

ortogonale $\left\{ \begin{array}{l} \vec{r}_1 \perp \vec{r}_2 \\ \text{eller } 0 \end{array} \right.$



$0^\circ \leq v \leq 180^\circ$



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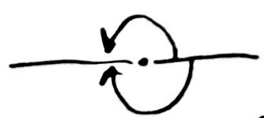
÷

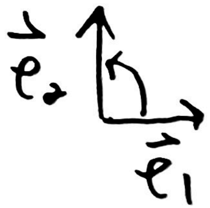
$\begin{vmatrix} \vec{r}_1 \\ \vec{r}_2 \end{vmatrix} \geq 0$ hvis (korteste) rotasjonsretningen fra $\vec{r}_1$ til $\vec{r}_2$ er positiv



$v = 0^\circ, 180^\circ$ så er $\begin{vmatrix} \vec{r}_1 \\ \vec{r}_2 \end{vmatrix} = 0$

$\begin{vmatrix} \vec{r}_1 \\ \vec{r}_2 \end{vmatrix} \leq 0$ hvis (korteste) rotasjonsretning fra $\vec{r}_1$ til $\vec{r}_2$ er negativ.

(7)



$$\begin{vmatrix} \vec{e}_1 \\ \vec{e}_2 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 > 0$$

$$\begin{vmatrix} \vec{e}_2 \\ \vec{e}_1 \end{vmatrix} = -1 < 0$$

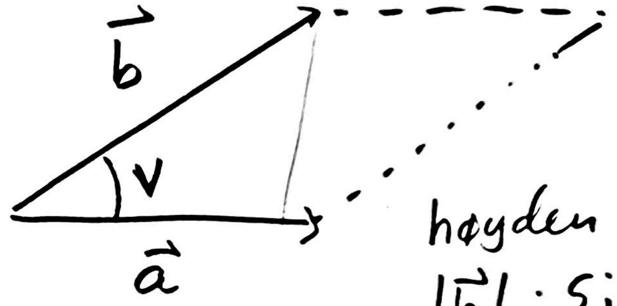
$$\begin{vmatrix} 3 & 9 \\ 24 & 32 \end{vmatrix} = \begin{vmatrix} 3\,[1,3] \\ 8\,[3,4] \end{vmatrix} = 3\cdot 8 \begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} = 3\cdot 8\,\overbrace{(4-9)}^{-5} = \underline{-120}$$

$$\begin{vmatrix} -2 & -4 \\ 5 & 6 \end{vmatrix} = -2 \begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix} = -2(6-10) = \underline{8}$$

$$\begin{vmatrix} 91 & 105 \\ -180 & -200 \end{vmatrix} = \begin{vmatrix} 91 & 105 \\ \underbrace{-180+2\cdot 91}_{2}, & \underbrace{-200+2\cdot 105}_{10} \end{vmatrix} = \begin{vmatrix} 91 & 105 \\ 2 & 10 \end{vmatrix}$$

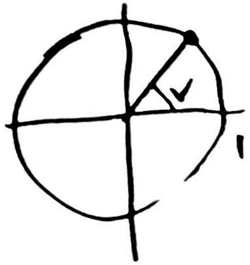
$$= 910 - 210 = \underline{700}$$

$$\left|\det\begin{pmatrix}\vec{a}\\ \vec{b}\end{pmatrix}\right| = |\vec{a}|\cdot|\vec{b}|\cdot\sin(v)$$



høyden
$|\vec{b}|\cdot\sin v$
$(0 \le v \le 180°)$

8



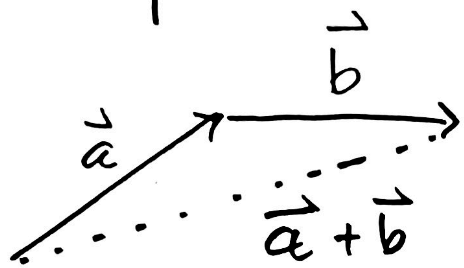
$$\cos^2 v + \sin^2 v = 1$$

$$\left|\det\begin{pmatrix}\vec{a}\\ \vec{b}\end{pmatrix}\right|^2 + \left(\vec{a}\bullet\vec{b}\right)^2 = |\vec{a}|^2\cdot|\vec{b}|^2$$

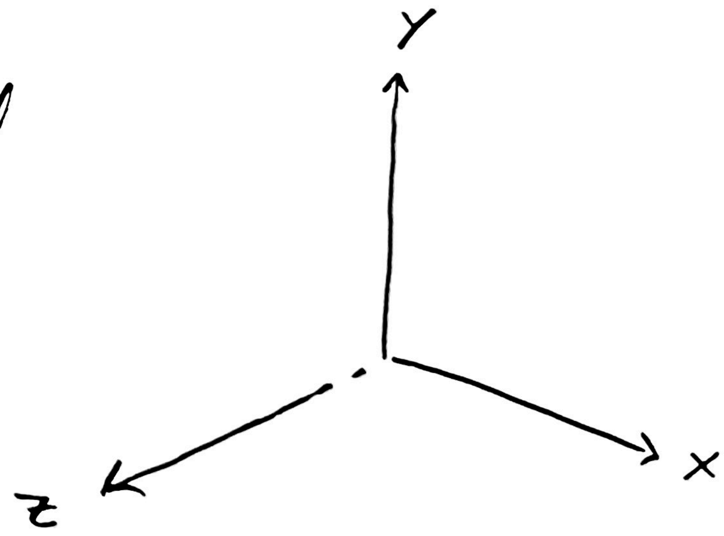
13.8 Vektorer i rommet $\mathbb{R}^3$

vektorer : piler i rommet

(9)



Koordinatsystem i rommet

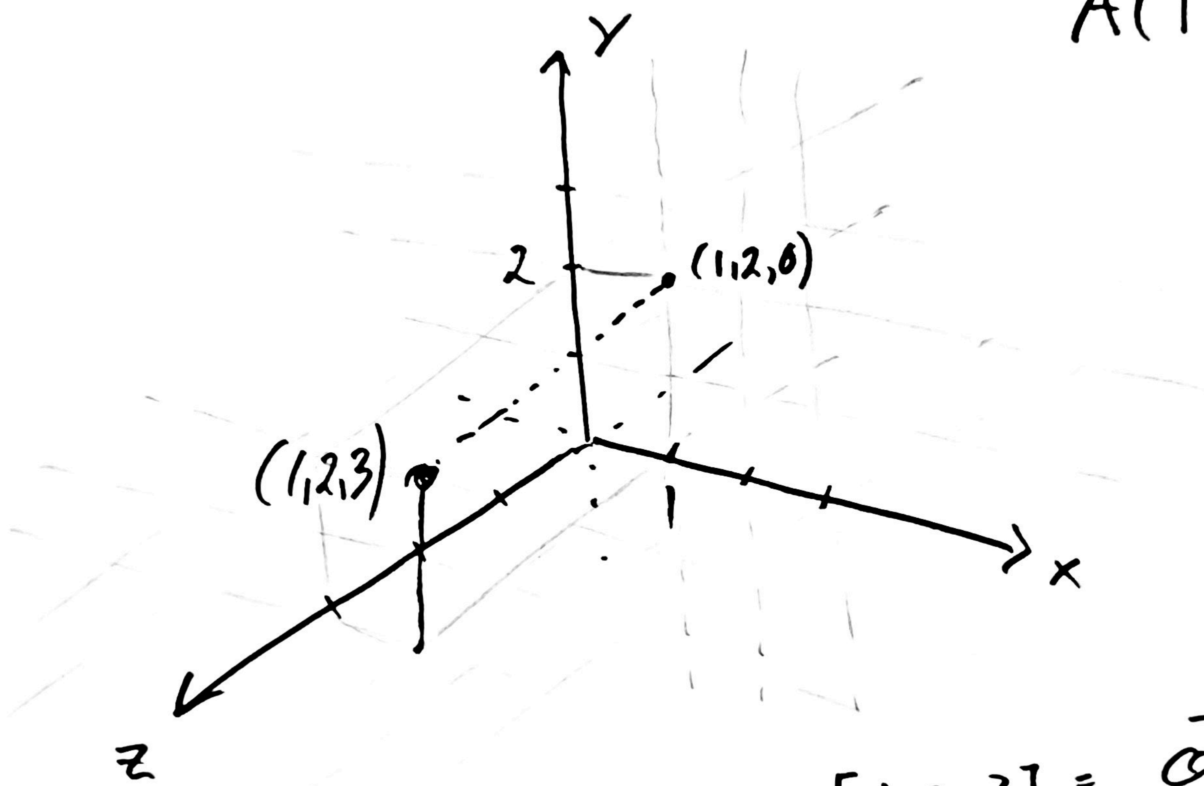


høyrehandsystem.

x, y, z
1. 2. 3. aksen

10

$A(1, 2, 3)$



$[1, 2, 3] = \overrightarrow{O(1,2,3)}$