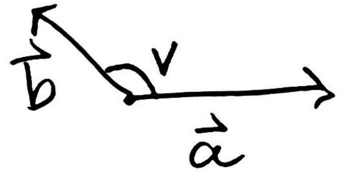
26.10
2021

14.4 Skalarprodukt i $\mathbb{R}^3$.

(1)

$$\vec{a} \bullet \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos(v)$$

v vinkelen mellom $\vec{a}$ og $\vec{b}$.



$0 \leq v \leq 180°$

$$-|\vec{a}||\vec{b}| \leq \vec{a} \bullet \vec{b} \leq |\vec{a}| \cdot |\vec{b}|$$

$\vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a}$ symmetrisk

$\vec{a} \bullet \vec{b}$ lineært i begge vektorene:

$(t\vec{a}) \bullet \vec{b} = t(\vec{a} \bullet \vec{b})$ og $(\vec{a}_1 + \vec{a}_2) \bullet \vec{b} = \vec{a}_1 \bullet \vec{b} + \vec{a}_2 \bullet \vec{b}$

Skalarprodukt på koordinatform:

$$[x_1, x_2, x_3] \bullet [y_1, y_2, y_3] = x_1 y_1 + x_2 \cdot y_2 + x_3 y_3$$

$$[x_1, x_2, x_3] = x_1 \vec{e}_1 + x_2 \vec{e}_2 + x_3 \vec{e}_3$$

og $\vec{e}_i \cdot \vec{e}_j = \begin{cases} 1 & i=j \\ 0 & \end{cases}$

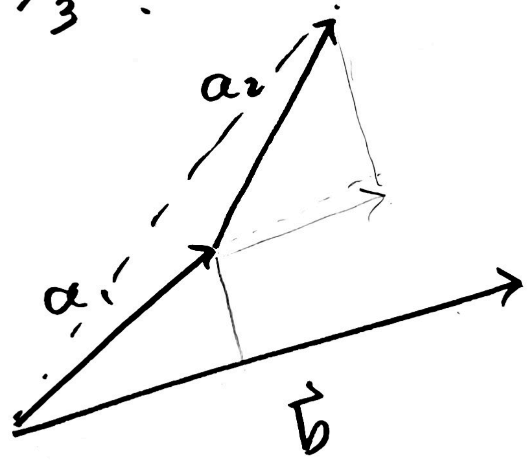
ved linearitet

$$[x_1, x_2, x_3] \bullet [y_1, y_2, y_3] = \sum_{\substack{1 \le i \le 3 \\ 1 \le j \le 3}} \underbrace{x_i \vec{e}_i \bullet y_j \vec{e}_j}_{x_i y_j \underbrace{\vec{e}_i \bullet \vec{e}_j}_{\substack{1 \text{ for } i=j \\ 0 \quad i \neq j}}}$$

sum over

2

$$= x_1 y_1 + x_2 y_2 + x_3 y_3 .$$



(illustrere linearitet)

$$[1, 1, -1] \bullet [2, 3, 2] = 1 \cdot 2 + 1 \cdot 3 + (-1) \cdot 2 \underline{= 3}$$

$$[2, 4, 1] \bullet [1, -1, 1] = 2 - 4 + 1 = -1$$

$\vec{a}, \vec{b}$ ortogonale hvis $\vec{a} \cdot \vec{b} = 0$

$\vec{a}, \vec{b}$ er $\vec{0}$ eller $\vec{a} \perp \vec{b}$.

vinkelrette.

3

Bestem t slik at $\vec{u}_t = [2t, 3t-1, -t]$ er ortogonal til $\vec{v} = [-2, 3, 3]$

$\vec{u}_t \cdot \vec{v} = 0$

$[2t, 3t-1, -t] \cdot [-2, 3, 3]$

$= -4t + 3(3t-1) - t \cdot 3 \quad = -4t + 9t - 3 - 3t = 0$

$= 2t - 3 = 0$

så $\underline{t = \frac{2}{3}}$. [Dette er galt!]

$\vec{u}_{2/3} = [\frac{4}{3}, 1, -\frac{2}{3}] = \underline{\frac{1}{3}[4, 3, -2]}$

sjekk: $\vec{u}_{2/3} \cdot \vec{v} \neq 0$! Feil $\rightarrow$

Det skal være $2t = 3$

så $t = \underline{3/2}$

(4)

$\vec{u}_{3/2} = [3, \frac{9}{2} - 1, -\frac{3}{2}] = \underline{\frac{1}{2}[6, 7, -3]}$

sjekker: $\vec{u}_{3/2} \cdot \vec{v} = \frac{1}{2}(-12 + 21 - 9) = 0$

V vinkel mellom $\vec{a}$ og $\vec{b}$

$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos V$

$V = \arccos\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}\right)$

eks $\vec{a} = [1, 1, -1]$ $\vec{b} = [4, 2, 3]$

Hva vinkelen mellom $\vec{a}$ og $\vec{b}$?

$\vec{a} \cdot \vec{b} = 4 + 2 - 3 = 3.$

$|\vec{a}| = \sqrt{3}$ $\quad |\vec{b}| = \sqrt{16+4+9} = \sqrt{29}$

5

$$V = \arccos\left(\frac{3}{\sqrt{3}\cdot\sqrt{29}}\right)$$

$$= \arccos\left(\sqrt{\frac{3}{29}}\right) \sim \underline{71.238^\circ}$$

oppg

Finn vinkelen mellom $\vec{a} = [1, 2, 3]$ og $\vec{b} = [3, 2, 1]$

$$\vec{a}\cdot\vec{b} = 3 + 4 + 3 = 10$$

$$|\vec{a}| = |\vec{b}| = \sqrt{1 + 2^2 + 3^2} = \sqrt{14}$$

$$V = \arccos\left(\frac{10}{\sqrt{14}\sqrt{14}}\right) = \arccos\left(\frac{5}{7}\right)$$

$$\arccos(0.7142..)$$

$$= \underline{44.415^\circ}$$

(6) Gitt to punkt $A(5, 11, -3)$ og $B(1, 11, -1)$

Finn alle punkt C på linjen mellom A og B slik at $|\vec{AC}| = 3\,|\vec{BC}|$

$$\vec{AB} = \vec{OB} - \vec{OA} = [1, 11, -1] - [5, 11, -3]$$
$$= \underline{[-4, 0, 2]} = 2[-2, 0, 1]$$

$$\vec{AC} = t\vec{AB}.$$

$$|\vec{AC}| = |t| \cdot |\vec{AB}|$$
$$= |t|\, 2\sqrt{5}$$

$$\left(|\vec{AB}| = 2\sqrt{(-2)^2 + 0^2 + 1^2} = 2\sqrt{5}\right)$$

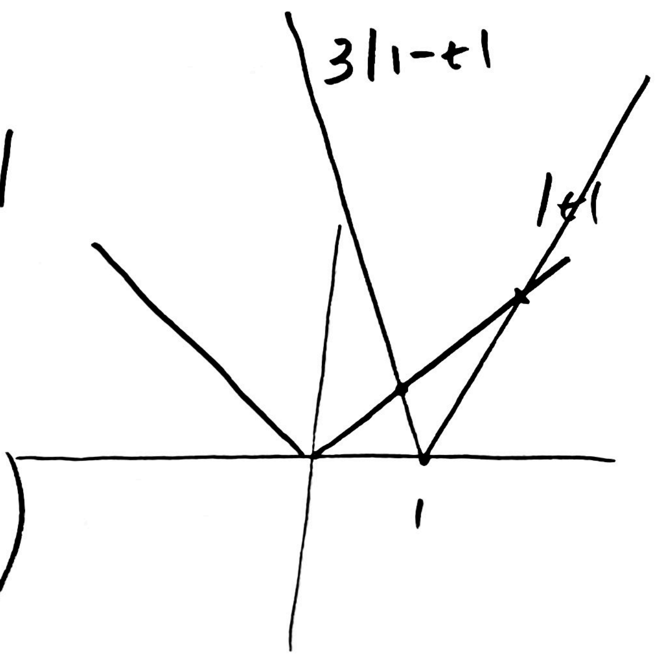
$$\vec{CB} = \vec{CA} + \vec{AB} = \vec{AB} - \vec{AC} = \vec{AB} - t\vec{AB}$$
$$\vec{CB} = (1-t)\,\vec{AB}.$$
$$|\vec{CB}| = |(1-t)|\,|\vec{AB}| = \underline{|1-t| \cdot 2\sqrt{5}}$$

(7)

$$|\vec{AC}| = 3|\vec{BC}|$$

$$|t|\,|\vec{AB}| = 3|1-t|\cdot|\vec{AB}|$$

$$\underline{|t| = 3|1-t|}$$



$t<0$:

$$-t = 3(1-t)$$
$$2t = 3 \quad \text{gir} \quad t = 3/2$$
$$\text{ingen} \quad t<0$$

$0 \leq t \leq 1$

$$t = 3(1-t) = 3 - 3t$$
$$4t = 3 \quad \text{så} \quad \underline{t = 3/4}$$

$t > 1$

$$t = 3|1-t| = 3(t-1)$$
$$t = 3t - 3$$
$$3 = 3t - t = 2t$$
$$\text{så} \quad \underline{t = 3/2}$$

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$\vec{AC} = t\vec{AB}$ $\qquad t = 3/4$ or $3/2$

$\vec{OC} = \vec{OA} + \vec{AC}$

$= [5, 11, -3] + t[-4, 0, 2]$

$t = \frac{3}{4}$:

$OC_1 = [5, 11, -3] + \underbrace{\frac{3}{4}[-4, 0, 2]}_{[-3, 0, 3/2]}$

$= [2, 11, -3/2]$

$C_1(2, 11, -1.5)$

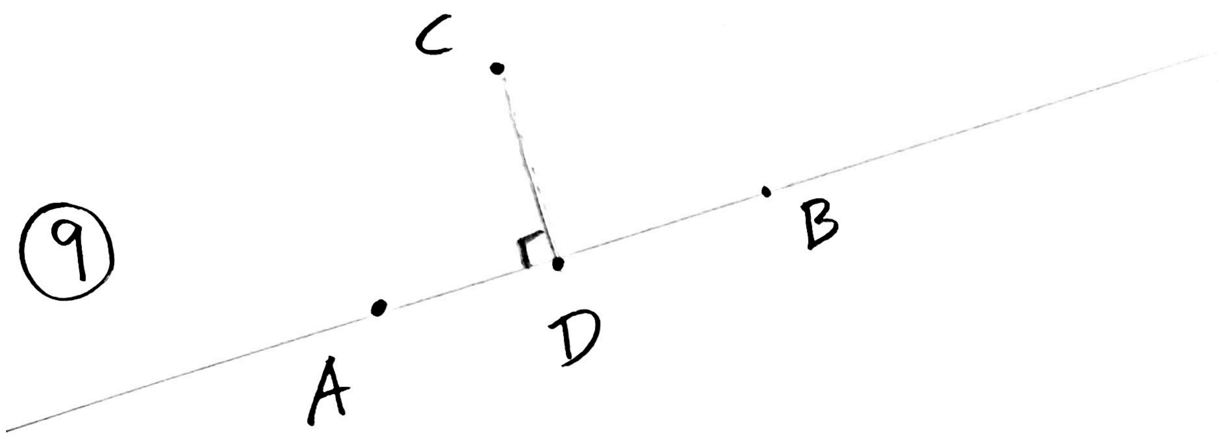
$t = \frac{3}{2}$

$OC_2 = [5, 11, -3] + \underbrace{\frac{3}{2}[-4, 0, 2]}_{[-6, 0, 3]}$

$= [-1, 11, 0]$

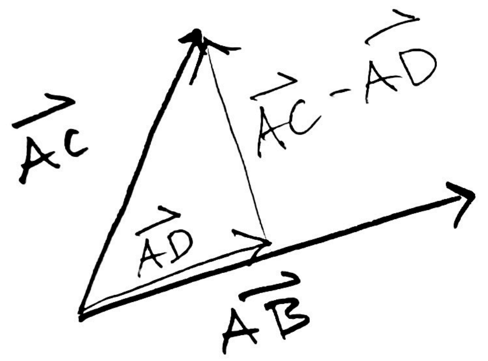
$C_2(-1, 11, 0)$

(9)



Finn D på linjen gjennom AB slik at $\overrightarrow{DC}$ er ortogonalt til $\overrightarrow{AB}$

(linjen fra D til C står vinkelrett på linjen gjennom A og B)



$\overrightarrow{AD} = t\overrightarrow{AB}$

$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$ så $\overrightarrow{DC} = \overrightarrow{AC} - \overrightarrow{AD} = \overrightarrow{AC} - t\overrightarrow{AB}$

Krever $\overrightarrow{DC} \bullet \overrightarrow{AB} = 0$

$(\overrightarrow{AC} - t\overrightarrow{AB}) \bullet \overrightarrow{AB} = 0$

$$\vec{AC}\cdot\vec{AB} - t\,\underbrace{\vec{AB}\cdot\vec{AB}}_{|\vec{AB}|^2} = 0$$

10

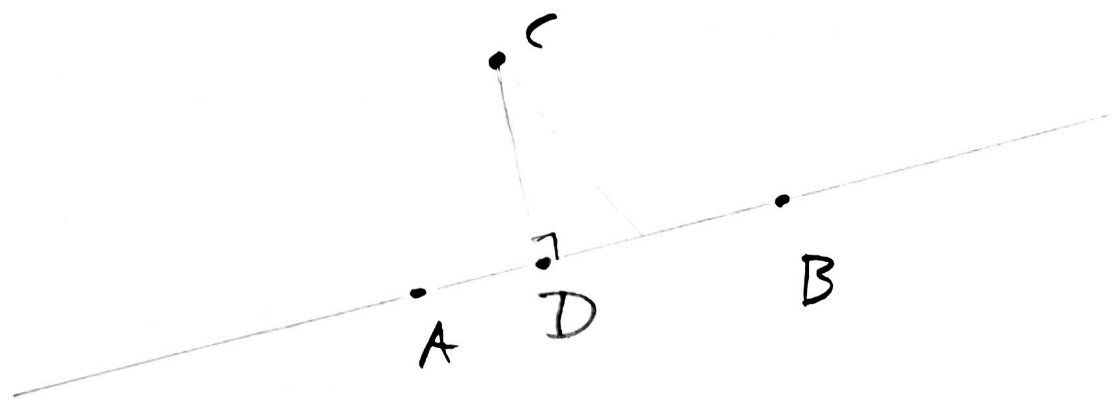
$$t = \frac{\vec{AC}\cdot\vec{AB}}{|AB|^2} \qquad \left(\vec{AD} = t\,\vec{AB}\right)$$

oppg.

$A(1,-1,2)$ $B(3,4,3)$

$C = (1,1,1)$

Finn punktet D på linjen gjennom A og B som er nærmest C.



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$$\vec{AB} = \vec{OB} - \vec{OA} = [3,4,3] - [1,-1,2]$$
$$= [2,5,1].$$

$$\vec{AC} = \vec{OC} - \vec{OA} = [1,1,1] - [1,-1,2]$$
$$= [0,2,-1].$$

$$\vec{AD} = t\vec{AB}$$ hvor

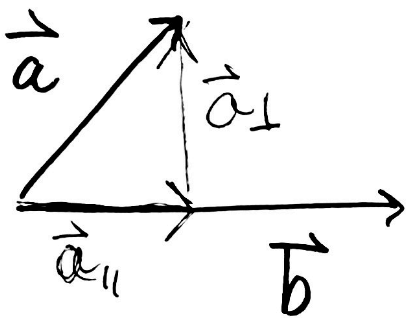
$$t = \frac{\vec{AC} \cdot \vec{AB}}{|\vec{AB}|^2} = \frac{[0,2,-1] \cdot [2,5,1]}{|[2,5,1]|^2}$$

$$t = \frac{10-1}{(4+25+1)} = \underline{\frac{9}{30}} = \frac{3}{10}$$

$$\vec{OD} = \vec{OA} + \vec{AD}$$
$$= [1,-1,2] + \frac{3}{10}[2,5,1] = [1,-1,2] + [0.6,1.5,0.3]$$
$$= [1.6, 0.5, 2.3]$$

$$\underline{D(1.6, 0.5, 2.3)}$$

(12)



$$\vec{a}_\parallel = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$$

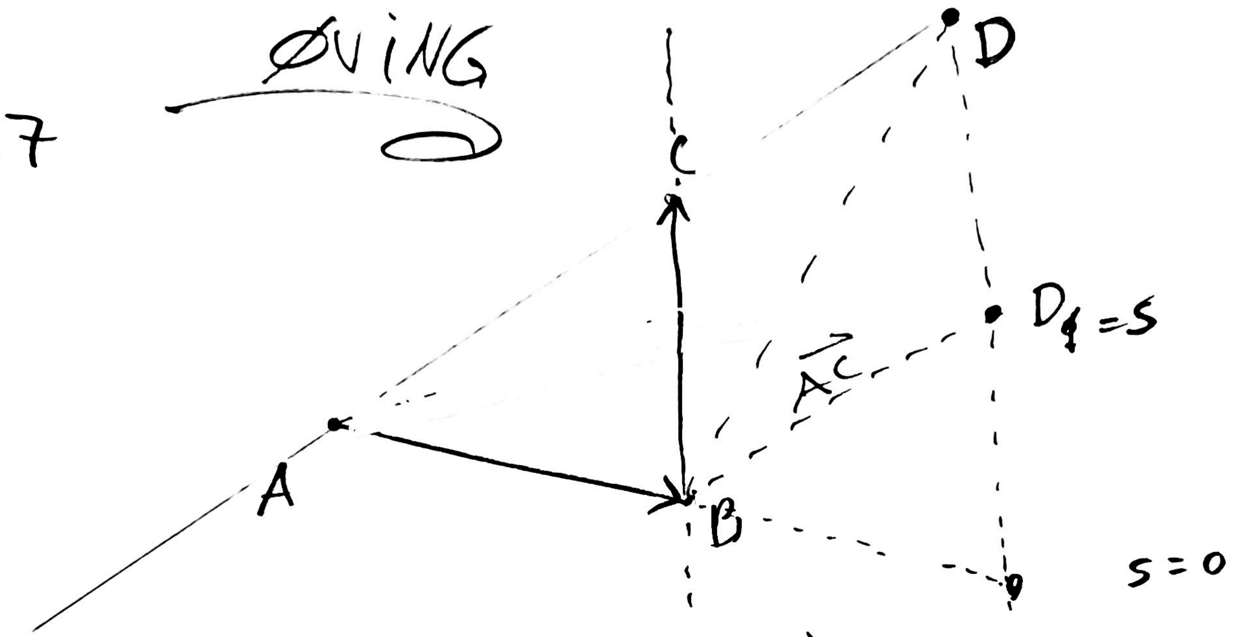
$$\vec{a}_\parallel + \vec{a}_\perp = \vec{a}$$

$$\vec{a}_\perp = \vec{a} - \vec{a}_\parallel$$

oppg 14.27

ØVING

(13)



$\vec{BD} = \vec{AB} + s\vec{BC}$

D på linja gjennom A og C

$\vec{AD} = t\vec{AC}$

$\vec{AD} = \vec{AB} + \vec{BD} = 2\vec{AB} + s\vec{BC}$

Så $2\vec{AB} + s\vec{BC} = t\vec{AC}$

ser at $s=2$ og $t=2$ er en løsning.

observasjon $\underline{s = 2}$:

$2\vec{AB} + 2\vec{BC}$

$= 2(\vec{AB} + \vec{BC}) = 2\vec{AC}$ så $\underline{t=2}$

oblig 2

14 oppg 8.

d) $\sin^2 v + \cos v - 1 = 0$

e) $2\sin v - \tan v = 0$

Hint:

d) Pytagoras $\sin^2 v + \cos^2 v = 1$

$\sin^2 v = 1 - \cos^2 v$

(10.8 og 9 i boka ...)

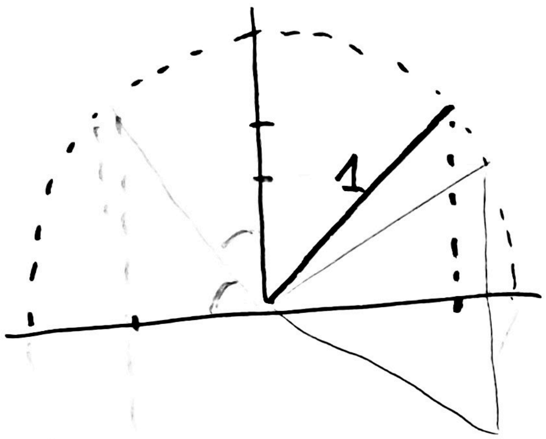
$1 - \cos^2 v + \cos v - 1 = 0$

$-\cos^2 v + \cos v = 0$

$\cos v(-\cos v + 1) = 0$ ...

e) $2\sin v - \frac{\sin v}{\cos v} = 0$

$\sin v(\ldots) = 0$



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$\cos(45°) = \sin(45°) = \frac{1}{\sqrt{2}} \sim 0.707$

$\sin(30°) = \cos(60°) = \frac{1}{2} = 0.5$

$\cos(30°) = \sin(60°) = \frac{\sqrt{3}}{2} = 0.866$

$\cos(0°) = \sin(90°) = 1$

$\cos(90°) = \sin(0°) = 0$

Oppg 6? $\cos(V) = \frac{-1}{2}$

$V = 90° + 30°$
$= \frac{\pi}{2} + \frac{\pi}{6} = \frac{3\pi}{6} + \frac{\pi}{6}$
$= \frac{4\pi}{6} = \underline{\frac{2\pi}{3}}$

og $-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}$.

Løsningene $\underline{\frac{2\pi}{3} \text{ og } \frac{4\pi}{3}}$

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$\sin v = b$

$v = \arcsin b + 2\pi \cdot n$ n heltall

$v = \pi - \arcsin b + 2\pi \cdot n$

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