6 jan 2022

Kap 6 Grenser

(1)

$$\lim_{x \to a} f(x) = L$$

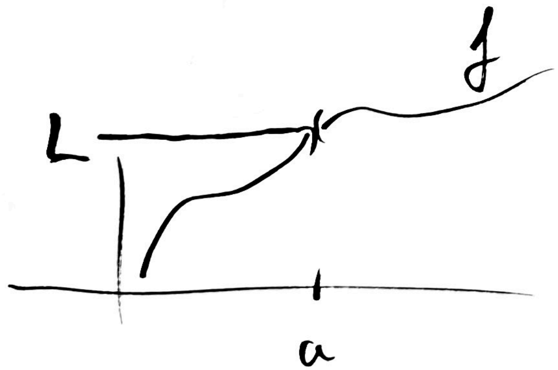
limit (eng.) grense

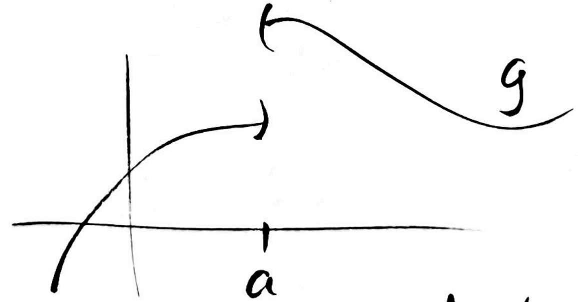
"grensen når x går mot a av $f(x)$ er lik L"

$f(x)$ definert rundt a

behøver ikkje være def. i $x = a$.

(om $f(a)$ finnes, så brukes den ikkje i grensen $x \to a$)





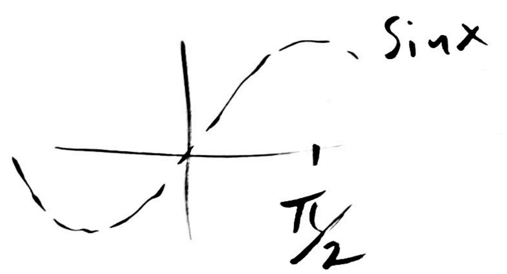
$\lim_{x \to a} g(x)$ eksisterer ikkje.

2 $f(x) = \dfrac{x^2-4}{x+2}$ def $x \neq -2$.

Hva er $\lim\limits_{x\to -2} f(x)$?

$$\frac{x^2-4}{x+2} = \frac{(x+2)(x-2)}{x+2}$$

$$\lim_{x\to -2} \frac{x^2-4}{x+2} = \lim_{x\to -2} \frac{(x+2)(x-2)}{(x+2)} = \lim_{x\to -2} (x-2) = \underline{-4}$$



$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$ (testet numerisk og tegnet grafen i geogebra)

Senere gir vi et geometrisk bevis for grensen.

Radianer.

(brukes grader får vi $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = \dfrac{1}{180/\pi} = \dfrac{\pi}{180}$)

vinkel i grader $= \dfrac{180^\circ}{\pi}$ vinkel i rad.

$10^x - 1 \to 0$ når $x \to 0$

3 $\lim_{x\to 0} \frac{10^x - 1}{x} \sim 2.3025$, $\lim_{x\to 0} \frac{2^x - 1}{x} \sim 0.69314$

Det finnes et tall $e \sim 2.71828 18284\ldots$

slik $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

e Eulertallet. Det er et irrasjonalt tall.

$$\lim_{x \to 2} \frac{x^2+2x-4}{x^2+x-6}$$ eksisterer ikkje, teller nærmer seg 4, nevner nærmer seg 0.

$$\lim_{x \to 2} \frac{x^2+2x-8}{x^2+x-6}$$ ("grense av type $\frac{0}{0}$".)

(4)

Faktoriserer x^2+2x-8

$= (x+1)^2 - 1 - 8 = (x+1)^2 - 3^2$

konjugatsetning gir $(x+1-3)(x+1+3)$

$= (x-2)(x+4)$

$x^2+x-6 = (x+3)(x-2)$

$$\lim_{x \to 2} \frac{x^2+2x-8}{x^2+x-6} = \lim_{x \to 2} \frac{(x-2)(x+4)}{(x-2)(x+3)} = \lim_{x \to 2} \frac{x+4}{x+3} = \frac{6}{5} = \underline{1.2}$$

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1) $\lim\limits_{x \to 0} \dfrac{x + x^2}{3x + x^3}$ 2) $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - x}$ "type $\frac{0}{0}$"

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1) $\lim\limits_{x \to 0} \dfrac{x(1+x)}{x(3+x^2)} = \lim\limits_{x \to 0} \dfrac{1+x}{3+x^2} = \underline{\dfrac{1}{3}}$

2) $\lim\limits_{x \to 1} \dfrac{(x-1)(x+2)}{x(x-1)} = \lim\limits_{x \to 1} \dfrac{x+2}{x} = \dfrac{3}{1} = \underline{3}$

(type $\frac{0}{0}$)

Eks $\lim\limits_{x \to -2} \dfrac{x^2 + 2x}{x^2 + 4x + 4}$

$= \lim\limits_{x \to -2} \dfrac{x(x+2)}{(x+2)^2} = \lim\limits_{x \to -2} \dfrac{x}{x+2}$ grensen eksisterer ikkje.

6

Grensesetninger $\lim_{x \to a} f(x) = L$ og $\lim_{x \to a} g(x) = K$

$$\lim_{x \to a} \left(f(x) + g(x) \right) = L + K$$

$$\lim_{x \to a} \left(k f(x) \right) = k \cdot L \qquad k \text{ konstant}$$

$$\lim_{x \to a} \left(f(x)\, g(x) \right) = K \cdot L$$

$$\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right) = \frac{L}{K} \quad \text{når } K \neq 0.$$

ϵ-δ-definisjon av grense.

For alle $\epsilon > 0$ så finnes det en $\delta > 0$ s.a.

$|f(x) - L| < \epsilon$ for alle $0 < \underbrace{|x-a|}_{} < \delta$ (d.v.s $x \neq a$)

7

$$\lim_{x \to a} f(x) = L \qquad \lim_{z \to a} f(z) = L$$

$$\lim_{x \to h} f(x) = L \qquad \lim_{x \to 0} \frac{10^x - 1}{x} = \lim_{h \to 0} \frac{10^h - 1}{h}$$

Hvis $\lim_{x \to 2} f(x) = 3$ og $\lim_{x \to 2} g(x) = -2$

Hva er 1) $\lim_{x \to 2} \left(f(x) + 2 \cdot g(x) \right)$? $= \lim_{x \to 2} f(x) + 2 \lim_{x \to 2} g(x) = 3 + 2(-2)$

$= \underline{-1}$

$$\lim_{x \to 2} \frac{(f(x))^2}{g(x)} \ ? = \frac{\lim_{x \to 2} (f(x))^2}{\lim_{x \to 2} g(x)}$$

$$= \frac{\left(\lim_{x \to 2} f(x) \right)^2}{\lim_{x \to 2} g(x)} = \frac{3^2}{-2} = \frac{9}{-2} = \underline{-4.5}$$

p(x) polynom $\quad p(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$

$\text{grad}(p(x)) = n \quad (\text{når } a_n \neq 0)$

(8)

$$\lim_{x \to h} x^n = \left(\lim_{x \to h} x\right)^n = (h)^n = h^n \quad \text{grensesetning}$$

$$\lim_{x \to h} p(x) = a_0 + a_1\left(\lim_{x \to h} x\right) + a_2\left(\lim_{x \to h} x^2\right) + \dots + a_n\left(\lim_{x \to h} x^n\right)$$

$$= p(h)$$

$$\lim_{x \to h} \frac{p(x)}{q(x)} = \frac{\lim_{x \to h} p(x)}{\lim_{x \to h} q(x)} = \frac{p(h)}{q(h)}, \quad \lim_{x \to h} q(x) \neq 0$$

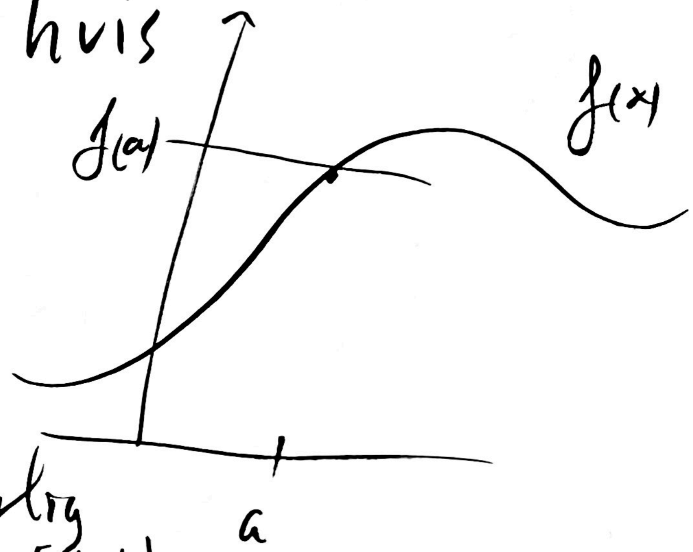
rasjonalt uttrykk

6.2 Kontinuitet

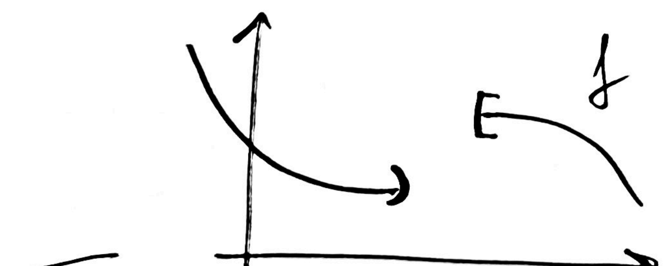
9. $f(x)$ definert i $x = a$.

$f(x)$ er kontinuerlig i $x = a$ hvis

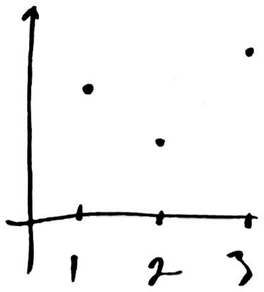
$$\lim_{x \to a} f(x) = f(a)$$

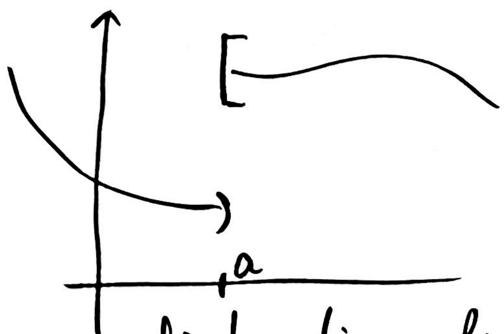


$f(x)$ er kontinuerlig, hvis den er kontinuerlig i alle $x \in D_f$ (definisjonsmengden til f)



3 verdier.





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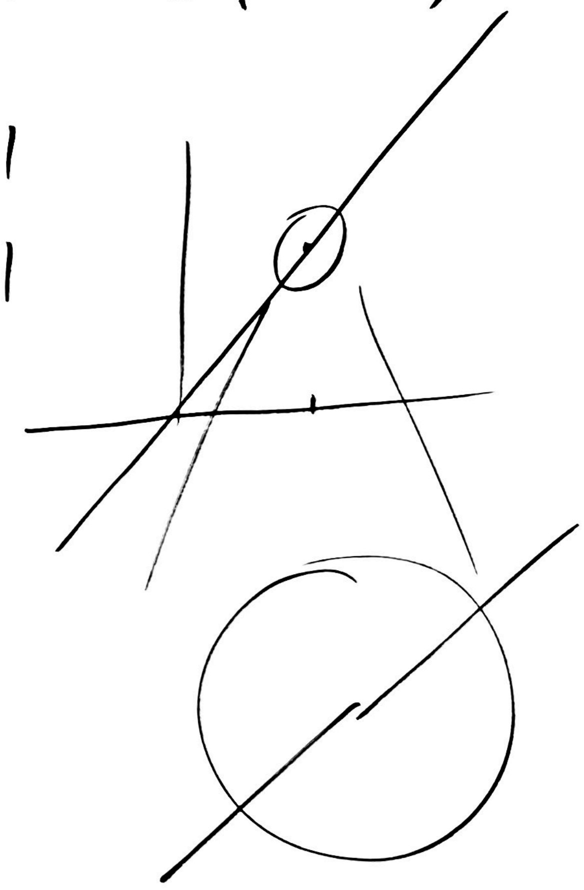
$p(x)$ polynom er kontinuerlige.

$r(x) = \frac{p(x)}{q(x)}$ rasjonale funksjoner er kontinuerlige i sin naturlige def. mengde $(q(x) \neq 0)$.

Delt forskrift

$$f(x) = \begin{cases} \pi x & x \leq 1 \\ 3.14x & x > 1 \end{cases}$$

diskontinuerlig i $x = 1$



$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} \pi x = \pi$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} 3.14x = 3.14 \ (\neq \pi) \ f(1).$$