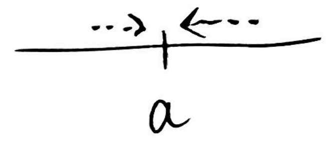
6 jan 2022

Øving

$$\lim_{x \to a} f(x) = L \qquad f(x) \to L \text{ når } x \to a$$



Følge

$a_1, a_2, a_3, \ldots, a_n, \ldots$ vendelig tallfølge.

$$\lim_{n \to \infty} a_n = L$$

For alle $\epsilon > 0$ så finnes det $N \in \mathbb{N}$, slik at $|a_n - L| < \epsilon$ for alle $n \geq N$

følge som en funksjon $\mathbb{N} \longrightarrow \mathbb{R}$

$\{1,2,3,\ldots\} \longrightarrow \mathbb{R}$

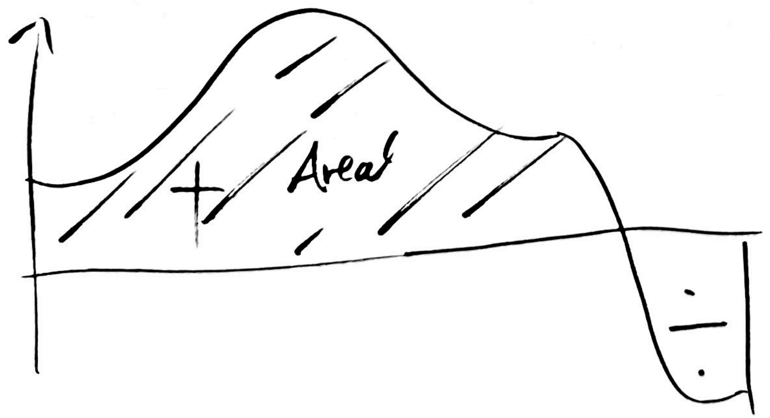
Rekke $a_1 + a_2 + \cdots + a_n + \cdots$

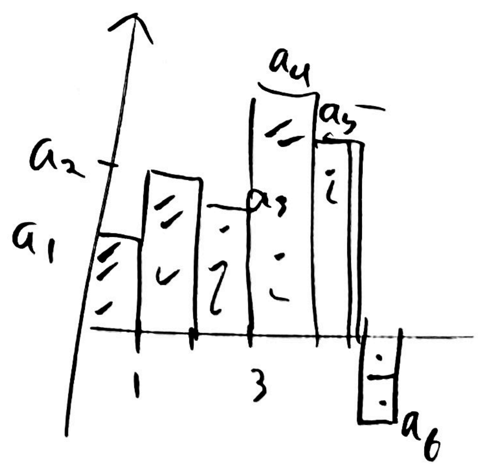
gir en følge av delsummer

$$S_n = a_1 + \cdots + a_n$$

Gir en følge $S_1, S_2, S_3, \ldots$

Hvis følgen av delsummer konvergerer til S, så sier vi at rekken $a_1 + a_2 + \cdots$ konvergerer og har sum S.





$$\lim_{x\to\infty} f(x) = L$$

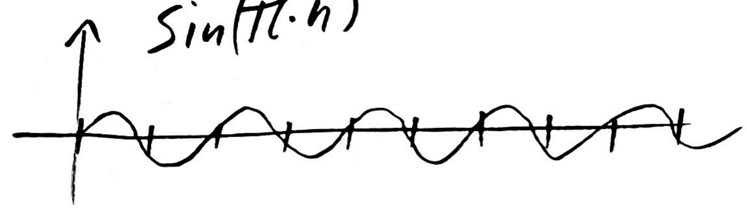
$f(x) \to L$ når x blir stor

$$\lim_{x\to\infty} \frac{x+2}{x} = \lim_{x\to\infty} 1 + \frac{2}{x} = 1$$

$\lim_{x\to\infty} f(x) = L$ så vil følgen $a_n = f(n)$ også konvergere og ha grense L.

Det motsatte er ikkje sant:

$f(x) = \sin(\pi x)$



$\lim_{x\to\infty} f(x)$ eksisterer ikkje

men $\lim_{x\to\infty} f(n) = \lim_{x\to\infty} 0 = 0$

oblig 4 5b) $\sum_{n=1}^{10^9} (-1)^{n+1} n^2 = \underbrace{1^2 - 2^2} + \underbrace{3^2 - 4^2} + \underbrace{5^2 - 6^2} + \dots$

$$\sum_{n=1}^{2M} (-1)^{n+1} n^2 = \sum_{n=1}^{M} \underbrace{\left((2n-1)^2 - (2n)^2\right)}_{(2n)^2 - 2\cdot 2n + (-1)^2 - (2n)^2}$$

$$= \sum_{n=1}^{M} 1 - 4n = \sum_{n=1}^{M} 1 - 4 \sum_{n=1}^{M} n$$

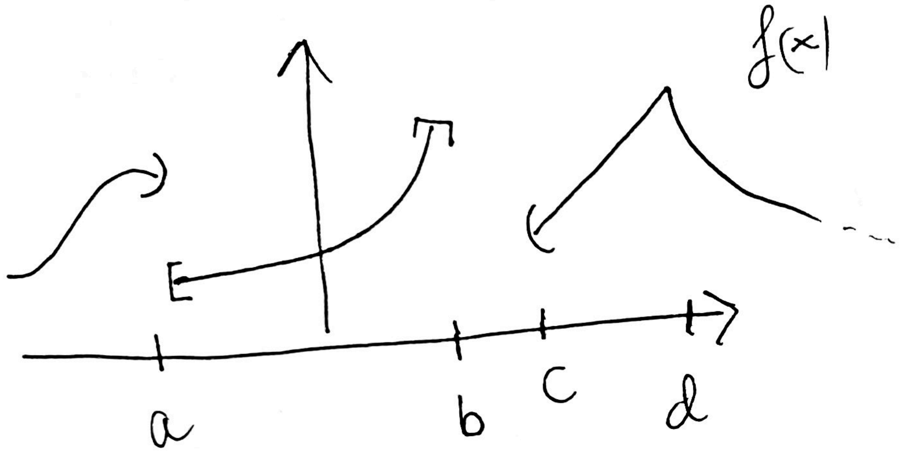
$$= M - 4 \frac{M(M+1)}{2} = M - 2M(M+1)$$

$$= M(1 - 2(M+1))$$

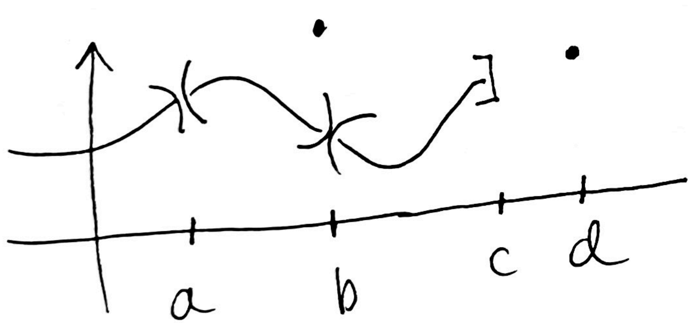
$$= \underline{-M(2M+1)}$$

$$\sum_{n=1}^{10^9} (-1)^{n+1} n^2 = -5\cdot 10^8 (\cdot 10^9 + 1) = \underline{-5\cdot 10^{17} - 5\cdot 10^8}$$

$$M = \frac{10^9}{2} = 5\cdot 10^8$$



$f(x)$ diskontinuerlig i $x = a$

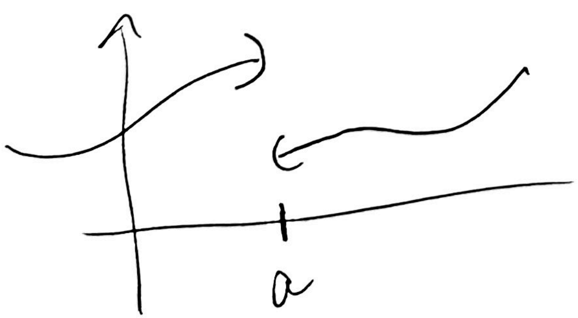


$f(x)$ er <u>diskontinuerlig</u> i $x = b$

$$\lim_{x \to b} f(x) = L \neq f(b)$$

$f(x)$ kont. i c og i d.

$a \notin D_f$



$f(x)$ er ikkje def. i $x = a$.
så ingen diskontinuitet i a.

$$\lim_{x \to 0} g(x) = 1.$$

$$\lim_{x \to 0} 3(g(x))^2$$

$$= 3 \lim_{x \to 0} (g(x))^2 = 3\left(\lim_{x \to 0} g(x)\right)^2$$

$$= 3 \cdot (1)^2 = \underline{3}$$

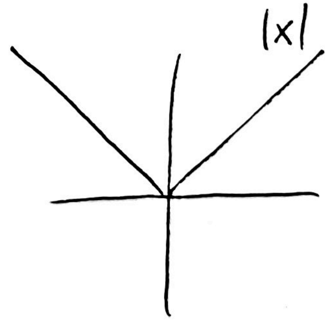
$$\lim_{x \to 0} \frac{2\sin^2 x}{3 \cdot x^2}$$

$$\frac{2}{3} \lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2 = \frac{2}{3}\left(\lim_{x \to 0} \frac{\sin x}{x}\right)^2$$

$$= \underline{\frac{2}{3}.}$$

(minner om $\lim_{x \to 0} \frac{\sin x}{x} = 1$)

$\lim_{x \to 0} \frac{|x|}{x}$ eksisterer ikkje



$$\lim_{x \to 0^+} \frac{|x|}{x} = 1$$

$$\lim_{x \to 0^-} \frac{|x|}{x} = -1$$

Finn grensene

$$\lim_{x\to 1} \frac{x^2-1}{x+1} = \frac{\lim_{x\to 1} x^2-1}{\lim_{x\to 1} x+1} = \frac{0}{2} = \underline{0}$$

$$\lim_{x\to -1} \frac{x^2-1}{x+1}$$

$$= \lim_{x\to -1} \frac{(x+1)(x-1)}{(x+1)} = \lim_{x\to -1} (x-1)$$

$$= -1-1 = \underline{-2}$$

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} \quad \left(\text{type } \frac{0}{0}\right)$$

$$= \lim_{x\to 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1} \frac{((\sqrt{x})^2-1)}{(x-1)(\sqrt{x}+1)}$$

$$= \lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{(\lim_{x\to 1}\sqrt{x})+1} = \frac{1}{1+1} = \underline{\frac{1}{2}}$$

Laning