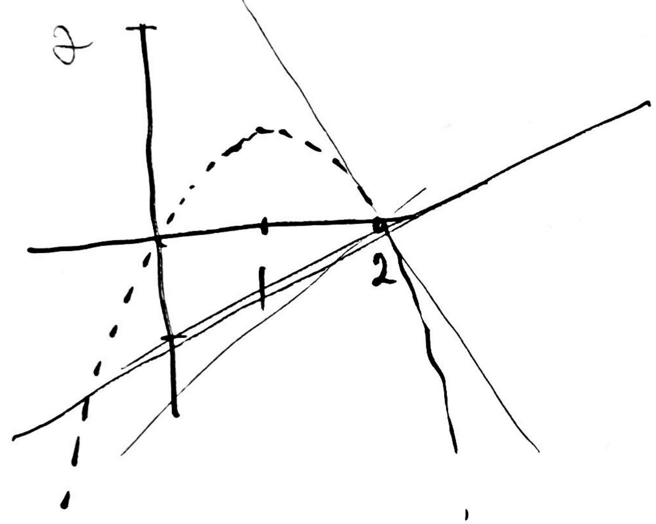
6.85 **Øvinger**

3.01.2022

$$f(x) = -x^2 + 2x$$
$$= x(2-x)$$

$$\left(-(x^2-2x) = -((x-1)^2-1) \right.$$
$$\left. = -(x-1)^2 + 1 \right)$$

a)



b) Tangent og normal i (2,0)

$$f'(x) = (-x^2+2x)'$$
$$= -1(x^2)' + 2(x)'$$
$$\underline{f'(x) = -2x+2}$$

$$f'(2) = -2\cdot 2+2 = -2$$

Tangentlinjen har stigningstall $f'(2) = -2$

ettpunktsformelen: Tangentlinjen

$$y = f'(2)(x-2) + f(2)$$
$$y = -2(x-2) + 0$$
$$= \underline{-2x+4}$$

Normallinjen har stigningstall $\frac{-1}{f'(2)} = \frac{-\cancel{1}}{-2} = \frac{1}{2}$

$$y = \frac{1}{2}(x-2) + 0 = \underline{\frac{1}{2}x - 1}$$

d) Finn tangent & normal linje til i $(1,1)$

$$f(x) = -x^2 + 2x$$
$$f'(x) = -2x + 2$$
$$f'(1) = 0$$

Tangentlinjen

$$y = f'(1)(x-1) + f(1)$$
$$0 \cdot (x-1) + 1$$
$$\underline{y = 1}$$

Normallinjen: er vertikal (siden tangentlinjen er horisontal)

$$\underline{x = 1}$$

$$(x^n)' = n x^{n-1}$$

Derivasjon er linear.

Dette lar oss derivere alle polynomer.

$$\begin{aligned}
(3x^8 - 13x^3)' &= (3x^8)' + (-13x^3)' \\
&= 3(x^8)' - 13(x^3)' \\
&= 3 \cdot 8 \cdot x^7 - 13 \cdot 3x^2 \\
&= 24x^7 - 39x^2
\end{aligned}$$

$$\begin{aligned}
(3x^8 - 13x^3)'' &= ((3x^8 - 13x^3)')' \\
&= (24x^7 - 39x^2)' \\
&= 24(x^7)' - 39(x^2)' \\
&= 24 \cdot (7 \cdot x^6) - 39(2x) \\
&= \underline{168 \cdot x^6 - 78x}
\end{aligned}$$

Opg Finn alle deriverte til $f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 4(x^3)' = 4 \cdot 3x^2 = 12x^2$

$f'''(x) = f^{(3)}(x) = 12(2x) = 24x$

$f^{(4)} = 24$

$f^{(5)} = 0$

$\underline{f^{(n)}(x) = 0 \quad n \geq 5}$

spørsmål

$g = 3x^3 + 2x^2$

$g' = 3(3x^2) + 2(2x)$

$= 9x^2 + 4x$

$g'' = 9(2x) + 4(1)$

$= 18x + 4$

$g''' = 18 \cdot 1 + 0 = 18$

$g'''' = g^{(4)} = 0$

$g^{(n)} = 0$

for alle $n \geq 4$.

$(a+b)^2 = a^2 + 2ab + b^2$

Deriver $f(x) = (2x - 3x^2)^2$

$= (2x)^2 + 2 \cdot 2x(-3x^2) + (-3x^2)^2$

$f(x) = 4x^2 - 12x^3 + 9x^4$

$f'(x) = 9(x^4)' - 12(x^3)' + 4(x^2)'$

$= 9 \cdot 4x^3 - 12 \cdot 3x^2 + 4 \cdot 2x$

$= \underline{36x^3 - 36x^2 + 8x}$

$g(x) = \frac{2}{3}x - \frac{7x^3}{9} + \sqrt{16x^2}$ Finn $g'(x)$

$= \left(\frac{2}{3}\right)x + \left(-\frac{7}{9}\right)x^3 + 4\sqrt{x^2} = \left(\frac{2}{3}\right)x + \left(\frac{-7}{9}\right)x^3 + 4|x|$

$\underline{g'(x) = \frac{2}{3} \cdot 1 - \frac{7}{9} \cdot 3x^2 + 4 \cdot \begin{cases} -1 & x<0 \\ 1 & x>0 \end{cases}}$ $x \neq 0$

$$g'(x) = \begin{cases} -\frac{7}{3}x^2 + \frac{2}{3} - 4 & x < 0 \\ -\frac{7}{3}x^2 + \frac{2}{3} + 4 & x > 0 \end{cases}$$ ikke def for $x = 0$

oppg 6.186 $\lim\limits_{x \to 2} \frac{2x^2 - x - 6}{x^2 + 2x - 8}$ "type $\frac{0}{0}$"

teller:

$$\begin{array}{l} 2x^2 - x - 6 : x - 2 = 2x + 3 \\ \underline{2x^2 - 4x} \\ \quad 3x - 6 \end{array}$$

så $2x^2 - x - 6 = (x-2)(2x+3)$

nevner:

$$\begin{array}{l} x^2 + 2x - 8 : x - 2 = x + 4 \\ \underline{x^2 - 2x} \\ \quad 4x - 8 \end{array}$$

så $x^2 + 2x - 8 = (x-2)(x+4)$

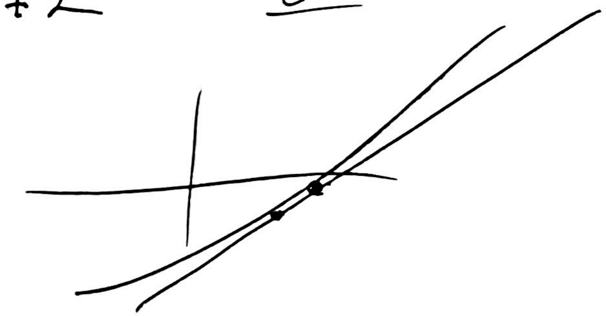
$$\lim_{x \to 2} \frac{(x-2)(2x+3)}{(x-2)(x+4)} = \lim_{x \to 2} \frac{2x+3}{x+4} = \underline{\underline{\frac{7}{6}}}$$

Hvis $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)}$ "type 0/0"

og $\lim\limits_{x \to a} \dfrac{f'(x)}{g'(x)}$ eksistere, da

er $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to a} \dfrac{f'(x)}{g'(x)}$. L'Hopitals.

$$\lim_{x \to 2} \frac{2x^2 - x - 6}{x^2 + 2x - 8} = \lim_{x \to 2} \frac{4x - 1}{2x + 2} = \underline{\frac{7}{6}}$$



$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

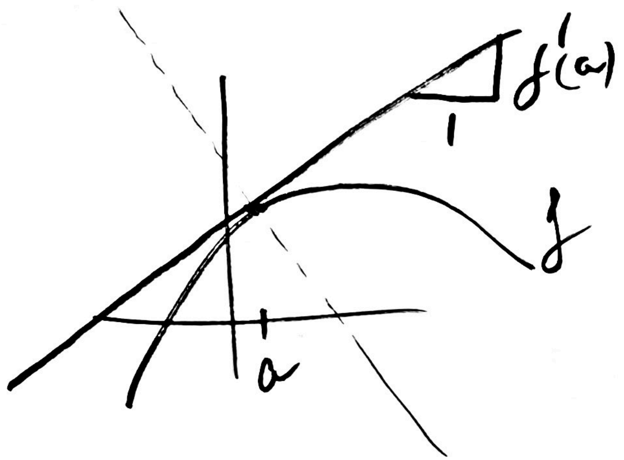
$f(x) - f(a) \sim f'(a)(x-a)$ x nær a.

lineær tilnærming av $f(x)$ nær $x=a$. $f(x) \sim \underbrace{f'(a)(x-a) + f(a)}_{\text{tangentlinjen.}}$

hvis $f(a) = 0$, $g(a) = 0$ (antakelse)

$$\frac{f(x)}{g(x)} \sim \frac{f'(a)(x-a)}{g'(a)(x-a)} = \frac{f'(a)}{g'(a)}$$

når $g'(a) \neq 0$



Tangentlinjen

$$y = f'(a)(x-a) + f(a).$$

Normallinjen

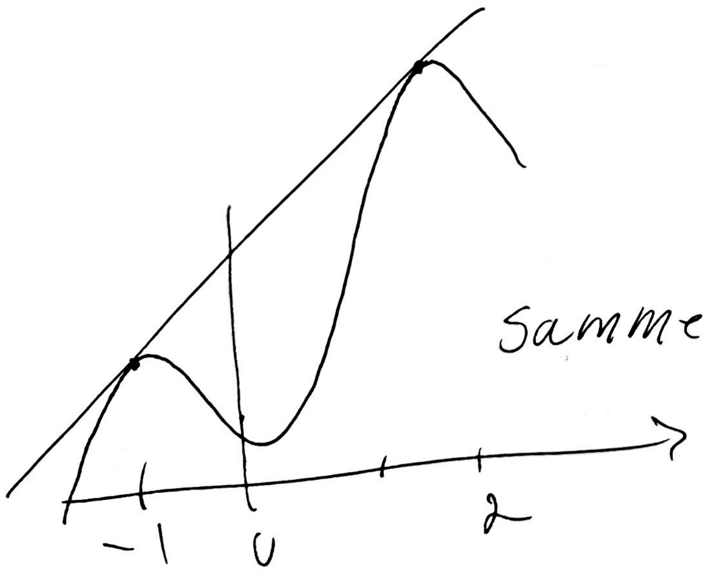
Stigningstall $\frac{-1}{f'(a)}$ $\quad f'(a) \neq 0$

vertikal $\quad f'(a) = 0$

$$y = \frac{-1}{f'(a)}(x-a) + f(a)$$

hvis $f'(a) = 0$ så er normallinjen

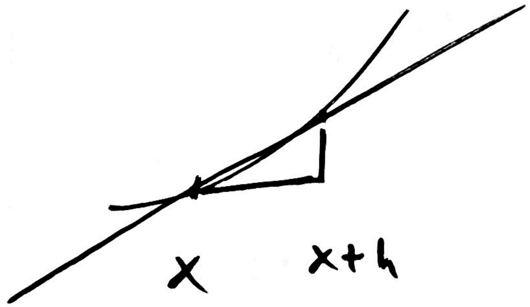
$x = a$ (alle y)



samme tangentlinje tilordnet forskjellige punkt.

6.73. $f(x) = x^3 - 4x$

a) $\dfrac{f(x+h) - f(x)}{h}$



undersøkt vekstfarten for $h = 10^{-n}$ $n = 0, +1, +2, \ldots, 7$

vekstfarten normer seg -1.

Momentan vekstfart: $f'(x) = 3x^2 - 4$

$f'(1) = 3 \cdot 1^2 - 4 = \underline{-1}$

Tangent og normal i $(1, f(1)) = (1, -3)$

Tangentlinjen $y = -1(x-1) + (-3) = \underline{-x - 2}$

Normallinjen $y = \left(\frac{-1}{-1}\right)(x-1) + (-3) = \underline{x - 4}$

$f(x) = 4x^5 - 3x^2 + 1$, $f(1) = 2$

$f'(x) = 4(x^5)' - 3(x^2)' + (1)' = 20x^4 - 6x$

Likning for tangentlinjen gjennom $(1,2)$

Finn tilnærmingen til $f(1,1)$ gitt ved å følge tangentlinjen.

$f'(1) = 20 - 6 = \underline{14}$

$y(x) = 14(x-1) + 2 = 14x - 12$

$y(1.1) = 14(1.1-1) + 2 = \underline{3.4}$

Faktisk verdi $f(1.1) \sim 3.812..$

17.81 (Rekker)

a) Konvergensområdet til $1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \cdots$

b) Summen $S(x)$

c) Grafen til $S(x)$

d) Finnes det x s.a. $S(x) = 2$, $S(x) = 1/3$?

$1 + k + k^2 + \cdots = \frac{1}{1-k}$

konvergerer $\Leftrightarrow |k| < 1$

La $k = \frac{1}{x}$

$1 + \frac{1}{x} + \cdots + = \frac{1}{1 - 1/x} = \frac{x}{x-1}$

$\left|\frac{1}{x}\right| < 1$

$\frac{1}{|x|} < 1$ siden $|x| \geq 0$

$\Leftrightarrow \underline{1 < |x|}$

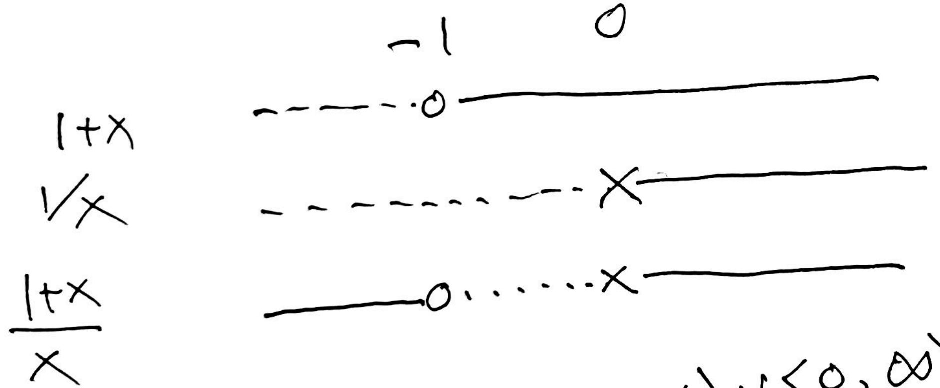
$\Leftrightarrow x \in \underline{\langle -\infty, -1\rangle \cup \langle 1, \infty\rangle}$

$$\frac{1}{|x|} < 1 \iff -1 < \frac{1}{x} < 1$$

$$\iff -1 < \frac{1}{x} \quad \text{og} \quad \frac{1}{x} < 1$$

$$\Updownarrow$$

$$0 < \frac{1}{x} + 1 = \frac{1+x}{x}$$



$$-1 < \frac{1}{x} \iff x \in \langle -\infty, -1\rangle \cup \langle 0, \infty\rangle$$

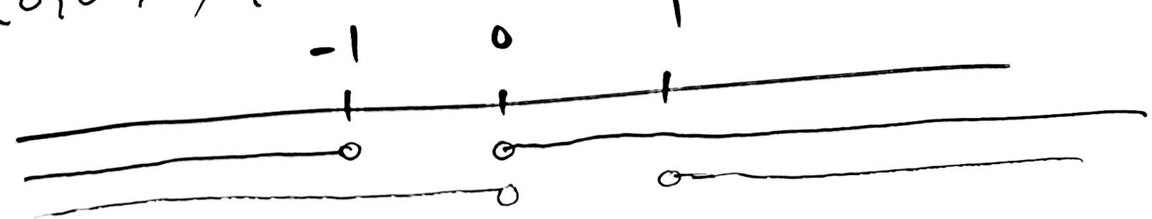
tilsvarende

ekvivalent til

$\langle -\infty, 0\rangle \cup \langle 1, \infty\rangle$

begge er oppfylt i snittet deres

$$\left(\langle -\infty, -1\rangle \cup \langle 0, \infty\rangle\right) \cap \left(\langle -\infty, 0\rangle \cup \langle 1, \infty\rangle\right)$$



Snittet er $\underline{\langle -\infty, -1 \rangle \cup \langle 1, \infty \rangle}$

(som vist tidligere)