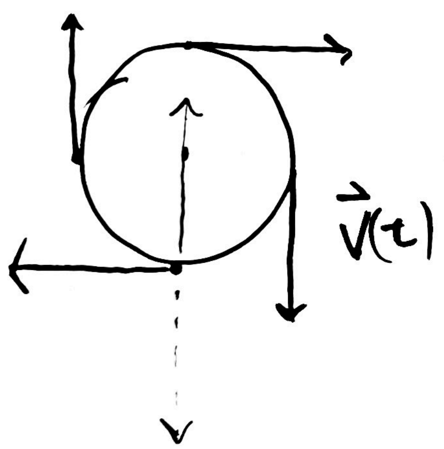
14.01.2022 6.9 Akselerasjon og fart.

Hastighet $\vec{V}$ (eng. velocity)

Fart $|\vec{V}|$ (eng. speed)

① 

$|V(t)|$ konstant fart

$\vec{V}$ hastigheten endrer seg.

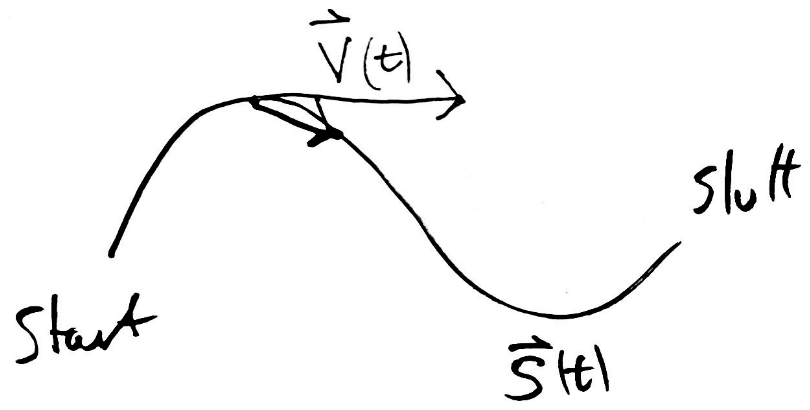
$\vec{S}(t)$ posisjon

gjennomsnittlig hastighet fra t til $t+h$

$$\frac{\vec{S}(t+h) - \vec{S}(t)}{h}$$

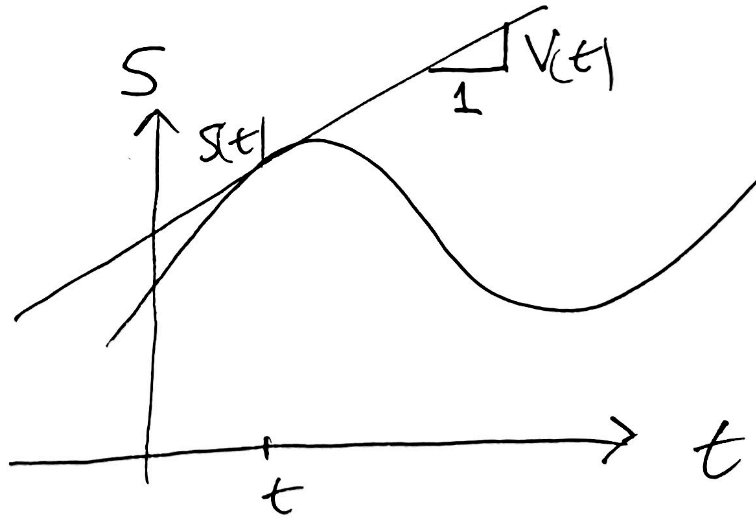
(momentan) haslighet $\lim_{h \to 0} \frac{\vec{s}(t+h) - \vec{s}(t)}{h} = \vec{s}\,'(t)$

(2)



Langs en linje

$s(t)$



Akselerasjon er vekstfarten til hastigheten
(endringsraten)

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$$a(t) = V'(t) = (S'(t))' = S''(t).$$

$$S(t) = t^3 - 2t + 3 \qquad t \in [0, 2]$$

hastigheten $V(t) = S'(t) = 3t^2 - 2 \cdot 1 + 0$

$$= 3t^2 - 2$$

akselerasjon $a(t) = V'(t) = (3t^2 - 2)'$

$$= \underline{6t}$$

Bevegelses likningene

a akselerasjon konstant

(4)

$$V(t) = a \cdot t + V_0 , \qquad V_0 = V(0)$$

$$S(t) = \frac{1}{2} a t^2 + V_0 \cdot t + S_0 , \qquad S_0 = S(0)$$

enheter:

$S(t)$ meter

t sekunder

$$S'(t) = \lim_{h \to 0} \frac{S(t+h) - S(t)}{h} \qquad \frac{(\text{meter})}{(\text{sekunder})}$$

meter/sekund.

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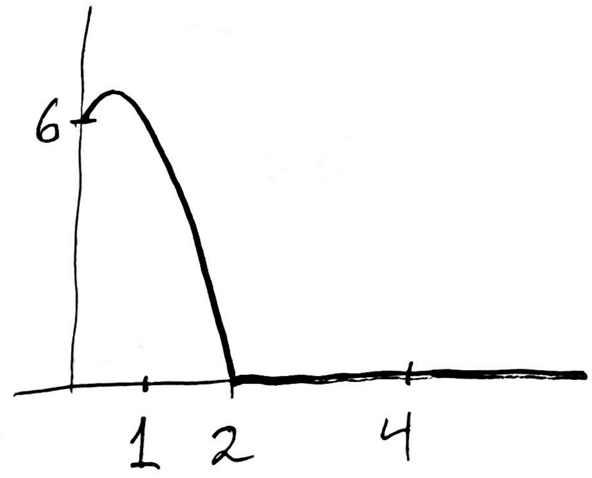
$$S(t) = \begin{cases} -t^3 + t + 6 & 0 \le t \le 2 \\ 0 & t > 2 \end{cases}$$



Finn $V(t)$ og $a(t)$.

$$V(t) = S'(t) = \begin{cases} -3t^2 + 1 & 0 \le t < 2 \\ 0 & 2 < t \end{cases}$$

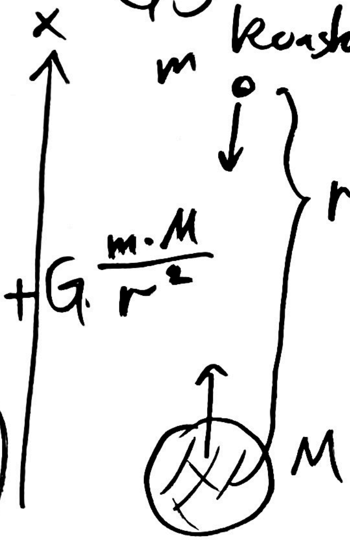
ikke deriverbar i $t = 2$.

$$a(t) = V'(t) = \begin{cases} -6t & 0 \le t < 2 \\ 0 & 2 < t \end{cases}$$

Potensiell energi: $P(x)$

$$P'(x) = F(x)$$

Energi = Kraft · vei (i kraftens retning)



G gravitasjonskonstant

$+G \cdot \frac{m \cdot M}{r^2}$

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Nye deriverte $\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$ $\quad x \neq 0$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$ $\quad x > 0$

$\frac{1}{x} = x^{-1}$ så $(x^{-1})' = (-1)\,x^{-1-1}$

$= \frac{-1}{x^2}$

$\sqrt{x} = x^{1/2}$ $\quad (x^{1/2})' = \frac{1}{2} \cdot x^{\frac{1}{2}-1}$

$= \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} (x^{1/2})^{-1}$

$= \frac{1}{2 x^{1/2}}$

$(x^n)' = n x^{n-1}$ også for $n = -1$

og $n = 1/2$

eks $\left(2\sqrt{x} - 5/x\right)' = 2\left(\sqrt{x}\right)' - 5\left(\frac{1}{x}\right)'$

$= 2\left(\frac{1}{2\sqrt{x}}\right) - 5\left(-1 \cdot \frac{1}{x^2}\right)$

$= \underline{\frac{1}{\sqrt{x}} + \frac{5}{x^2}}$

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opg. $\left(\sqrt{4x} + \frac{1+3x}{x}\right)'$

$\left(\sqrt{4} \cdot \sqrt{x} + \frac{1}{x} + \frac{3x}{x}\right)' = \left(2\sqrt{x} + \frac{1}{x} + 3\right)'$

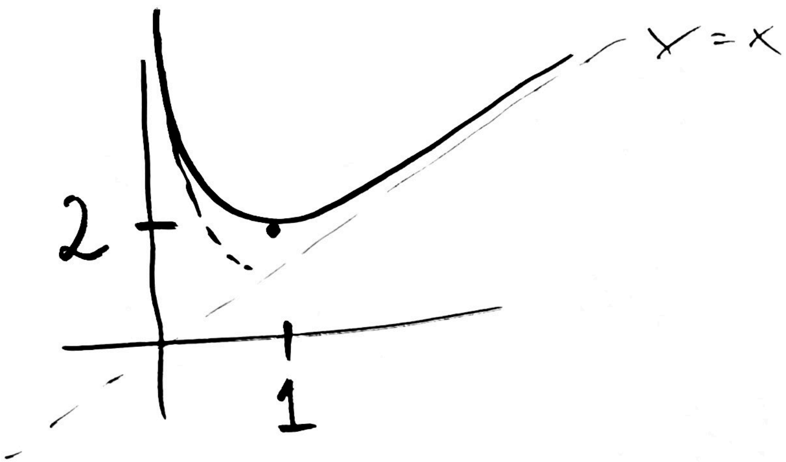
$= 2\left(\sqrt{x}\right)' + \left(\frac{1}{x}\right)' + 3' = \underline{\frac{1}{\sqrt{x}} - \frac{1}{x^2}}$

Når er $S(x) = x + \frac{1}{x}$ minst? $x > 0$

$S\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{1/2} = \frac{1}{2} + 2 = 2.5$

$S(1) = 1 + 1 = \underline{2}$

$S(10) = 10 + \frac{1}{10} = 10.1$



$$S'(x) = \left(x + \frac{1}{x}\right)'$$
$$= 1 + \frac{-1}{x^2}$$
$$= \frac{x^2 - 1}{x^2}.$$

(8) $S'(x) = 0$ for $x = 1$ $(x > 0)$

$S(x)$ er derfor minst for $x = 1$.

$x + \frac{1}{x} \geq 2$ for alle $x > 0$

Likhet for $x = 1$.

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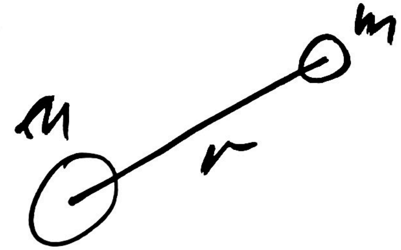
Gravitasjons kraft $F(r) = G\dfrac{Mm}{r^2}$



Potensiale $P(r)$

$$P'(r) = F(r) = G\frac{Mm}{r^2}.$$

siden $\left(\dfrac{-1}{r}\right)' = \dfrac{d}{dr}\left(\dfrac{-1}{r}\right) = -1\cdot\left(\dfrac{1}{r}\right)' = \dfrac{1}{r^2}.$

Så $P(r) = \underset{\text{konst.}}{E} \underline{- GMm\dfrac{1}{r}}$

"$p(\infty) = 0$" betyr $\lim_{r\to\infty} P(r) = 0$ gir $E = 0$.

$$\underline{P(r) = -\frac{GMm}{r}}$$

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Hvor stor hastighet må en rakett ha for å kunne unslippe jordens gravitasjonsfelt

$$\frac{1}{2} m_R V^2 + \frac{-G M m_R}{R_E} = 0$$

m_R masse Rakett

M_E masse jorden

R_E radius –

$$V^2 = \frac{2 G M_E}{R_E}$$

$$V = \sqrt{\frac{2 G M_E}{R_E}}$$

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Viser at $\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$

$$\left(\frac{1}{x}\right)' = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$ Felles nevner

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{x-(x+h)}{(x+h)x}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{-h}{x(x+h)}\right) = \lim_{h\to 0} \frac{-1}{x(x+h)} = \underline{\frac{-1}{x^2}}$$

Vi ser nå på $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$$(\sqrt{x})' = \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h} \cdot \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$(b-a)(b+a)$

$= b^2 - a^2$

gir

$(b-a) = \frac{b^2-a^2}{b+a}$

$$(\sqrt{x})' = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

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Resultat: Hvis $f(x)$ er deriverbar i a så er $f(x)$ kontinuerlig i a.

deriverbar $\Rightarrow$ kontinuerlig.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \text{ eksisterer}$$

$$\Rightarrow \lim_{h \to 0} f(a+h) - f(a) = 0.$$

Lineær kjerneregel

a, b konstanter.

$$\frac{d\,f(ax+b)}{dx} = a\,f'(ax+b)$$

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Sammensatt funksjon $\quad f(ax+b)$

eks $\quad f(z) = z^2 \qquad z = 2x+3$

$$f(2x+3) = (2x+3)^2$$

$$g(x) = \sqrt{x} \qquad g(5-x) = \sqrt{5-x}$$

$$\frac{1}{3x-7} = h(3x-7) \quad \text{hvor} \quad h(x) = \frac{1}{x}$$

$$\left((2x+3)^2\right)' \underset{\text{kjerne regel}}{=} 2 \cdot f'(2x+3)$$
$$= 2 \cdot 2(2x+3)$$
$$= \underline{8x+12}$$

(14)

(Alternativt kan vi gange ut

$$(2x+3)^2 = (2x)^2 + 2 \cdot 2x \cdot 3 + 3^2$$
$$\left((2x+3)^2\right)' = (4x^2+12x+9)' = 8x+12 \ldots$$
)

$$j(x) = x^{20}$$
$$j'(x) = 20x^{19}$$

$$\left((2x+3)^{20}\right)' = \left(j(2x+3)\right)'$$
$$2 \cdot j'(2x+3) = 2 \cdot 20(2x+3)^{19}$$
$$= \underline{40(2x+3)^{19}}$$

(15)

$$\left(\sqrt{5-x}\right)' = \underset{a}{(-1)} \cdot g'(5-x)$$

$$= -1 \cdot \frac{1}{2\sqrt{5-x}} = \underline{\frac{-1}{2\sqrt{5-x}}}$$

$$\left(\frac{1}{3x-7}\right)' = 3 \cdot h'(3x-7) = 3\,\frac{-1}{(3x-7)^2}$$

$$= \underline{\frac{-3}{(3x-7)^2}}$$

lineære
Viser kjerne regelen:

$$\lim_{h\to 0} \frac{f(a(x+h)+b) - f(ax+b)}{h} \cdot \frac{a}{a} \qquad ah = p$$

$$= a \lim_{h\to 0} \frac{f(ax+b+ah) - f(ax+b)}{(ah)}$$

$$= a \lim_{p\to 0} \frac{f(ax+b+p) - f(ax+b)}{p}$$

$$\big(f(ax+b)\big)' = a\, f'(ax+b).$$

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opg. Deriver

a) $(1-4x)^7 = f(1-4x)$ hvor $f(x) = x^7$

b) $\dfrac{1}{3x+17} = g(3x+17)$ hvor $g(x) = \dfrac{1}{x}$

c) $\sqrt{2x+1} = h(2x+1)$ hvor $h(x) = \sqrt{x}$

a) $\big((1-4x)^7\big)' = -4 \cdot 7(1-4x)^6 = -28(1-4x)^6$

b) $\left(\dfrac{1}{3x+17}\right)' = 3 \cdot \dfrac{-1}{(3x+17)^2} = \dfrac{-3}{(3x+17)^2}$

c) $2 \cdot \dfrac{1}{2\sqrt{2x+1}} = \dfrac{1}{\sqrt{2x+1}}$

6.21 $f(x) = \begin{cases} x+1 & x \le 2 \\ -x+5 & x > 2 \end{cases}$

$f(x)$ er kontinuerlig for $x < 2$ og for $x > 2$.

Undersøker hvor: $x = 2$.

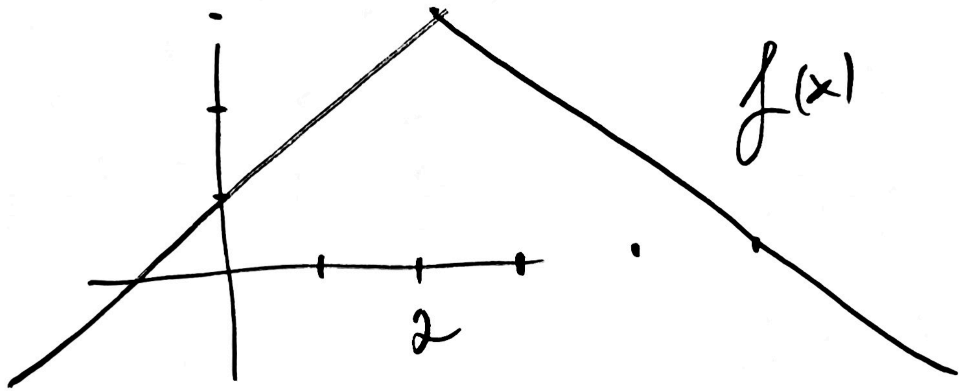
$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x+1 = \underline{3}$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} -x+5 = -2+5 = \underline{3}$$

Så $\lim_{x \to 2} f(x) = 3 = f(2)$

Så $f(x)$ er kontinuerlig (for alle x).

$f'(x) = \begin{cases} 1 & x < 2 \\ -1 & x > 2 \end{cases}$ ikke deriverbar i $x=2$ (knekkpunkt)



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Undersøk om $f(x) = \begin{cases} x^3 & x < 1 \\ 3x - 2 & x \geq 1 \end{cases}$

er kontinuerlig, deriverbar (i $x = 1$)

kont: $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^3 = 1^3 = 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 3x - 2 = 3 - 2 = 1 = f(1)$

$\lim_{x \to 1} f(x) = f(1)$

Så $f(x)$ er kont i $x = 1$.

$$f'(x) = \begin{cases} 3x^2 & x < 1 \\ 3 & x > 1 \end{cases}$$

Hva er $f'(1)$? $\quad \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

siden f er kont i 1

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$$\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^-} \frac{(1+h)^3 - 1^3}{h}$$

$$= (x^3)' \text{ i } x = 1 \quad = 3x^2 \big|_{x=1} = \underline{3}$$

$$\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(3(1+h)-2) - (3 \cdot 1 - 2)}{h}$$

$$= (3x-2)'\big|_{x=1} = 3.$$

så $f(x)$ er deriverbar i $x=1$

$$f'(1)=3.$$

$$f'(x) = \begin{cases} 3x^2 & x<1 \\ 3 & x\geq 1 \end{cases}$$

(20)

$$g(x) = \begin{cases} x^3 & x<1 \\ 3x & x\geq 1 \end{cases}$$

$$g'(x) = \begin{cases} 3x^2 & x<1 \\ 3 & x>1 \end{cases}$$

ikkje deriverbar i $x=1$

$g(x)$ er ikkje kont. i $x=1$

$$\lim_{h\to 0^-} \frac{g(1+h) - g(1)}{h}$$

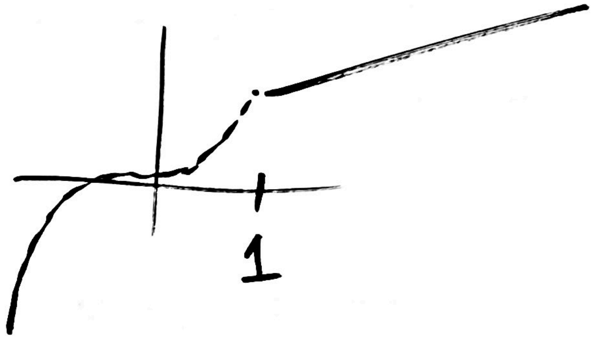
$$= \lim_{h\to 0^-} \frac{(1+h)^3 - 3}{h} \quad \cancel{x}\ 1^3 \quad \text{eksisterer ikkje!}$$

$$h(x) = \begin{cases} x^3 & x < 1 \\ x & x \geq 1 \end{cases}$$

kont. i $x = 1$

$$h'(x) = \begin{cases} 3x^2 & x < 1 \\ 1 & x > 1 \end{cases}$$

ikkje derivebar i $x = 1$



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Oppg. $\left(-3x^2\left(x^3-\frac{1}{x^3}\right)\right)'$

$= \left(-3x^5 + 3\frac{1}{x}\right)' = -3(x^5)' + 3\left(\frac{1}{x}\right)'$

$= -3(5x^4) + 3\left(\frac{-1}{x^2}\right) = \underline{-15x^4 - 3\frac{1}{x^2}}$

(22)

$\left(\frac{x^2}{x-1}\right)'$

$= \left(x+1+\frac{1}{x-1}\right)'$

$= 1+0+1\cdot\frac{-1}{(x-1)^2}$

$= \underline{1-\frac{1}{(x-1)^2}}$

$\Bigg(x^2 : x-1 = x+1+\frac{1}{x-1}$

$\underline{x^2-x}$

x

$\underline{x-1}$

$1 \Bigg)$

$\left[\left(\frac{1}{x-1}\right)' = g(x-1), \quad g(u) = \frac{1}{u}\right.$

$g'(u) = \frac{-1}{u^2}$

$\left(\frac{1}{x-1}\right)' = 1\cdot g'(x-1) = 1\cdot\frac{-1}{(x-1)^2}$

$\left.= \frac{-1}{(x-1)^2}\right]$

23 $\left((2-x)^4\right)'$

$= (-1)\cdot 4(2-x)^3 = \underline{-4(2-x)^3}$

koeff. til x
i $2-x$
$= -x+2$

$(2-x)^4 = f(2-x)$
hvor $f(u) = u^4$
$f'(u) = 4u^3$.