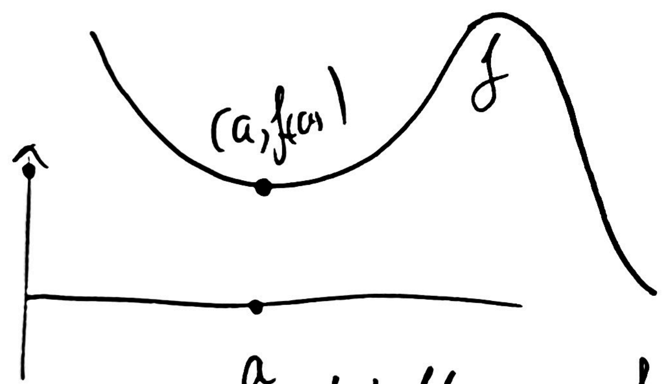
18. jan 2022.

Kap 7

Fysisk undervisning man. ons. tors.

7.1 Funksjonsdrøfting.



$(a, f(a))$ bunnpunkt
$f(a)$ minimalverdi
a minimalpunkt.

Lokalt bunnpunkt: $f(x) \geq f(a)$ for x nær a

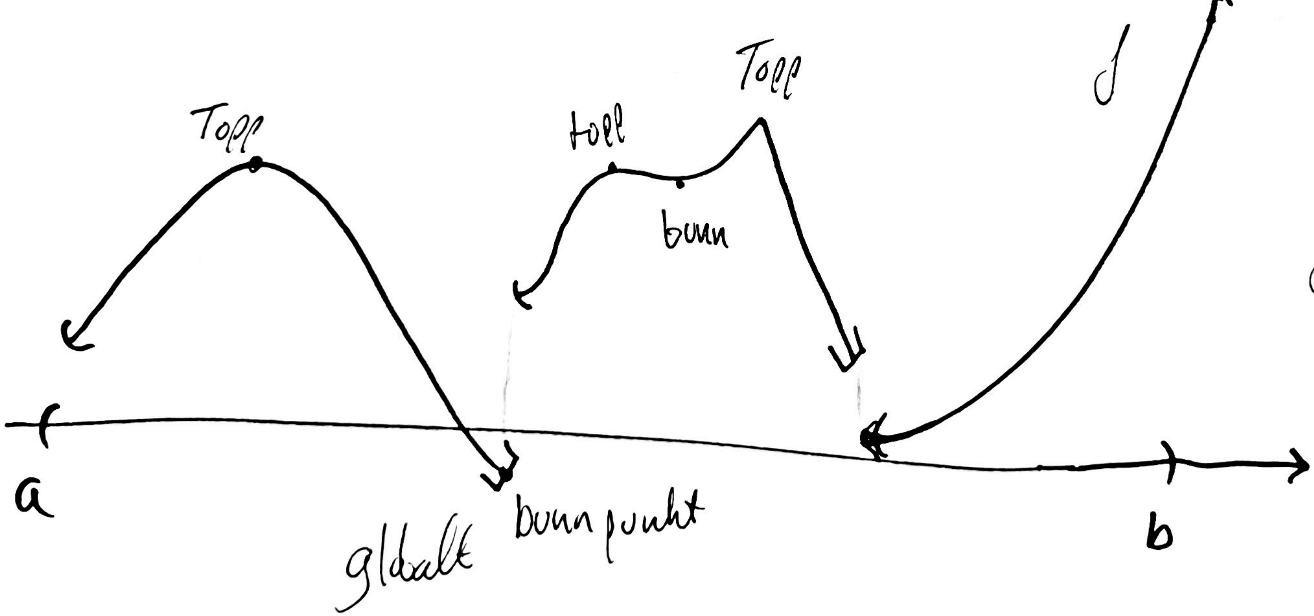
$(a, f(a))$ Globale bunnpunkt: $f(x) \geq f(a)$ for alle $x \in D_f$ definisjonsmengden til f

Fellesbetegnelse for topp og bunnpunkt er ekstremalpunkt.

Tilsvarende

Toppunkt	$(a, f(a))$	
maksimalverdi	$f(a)$	
maksimalpunkt	a	

$$f(x) \le f(a) \begin{cases} x \text{ nær } a & : \text{lokalt} \\ x \in D_f & : \text{globalt.} \end{cases}$$



f har ikkje globalt toppunkt

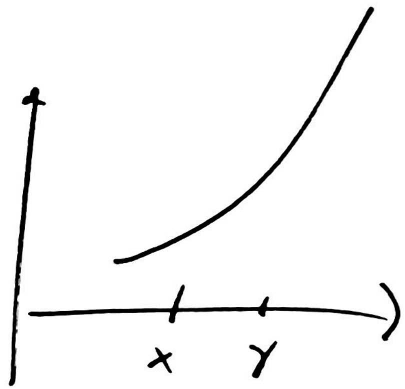
f har globalt bunnpunkt

$D_f = (a, b)$

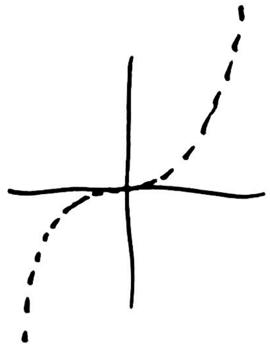
Monotoniegenskapene. (voksende – avtagende)

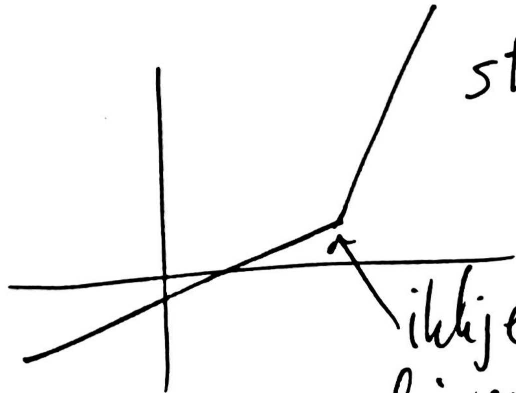
f voksende, økende, stiger i (a,b) hvis $f(y)-f(x)>0$ når $y-x>0$

$\iff$ $f(x)<f(y)$ når $x<y$



Hvis $f'(x)>0$ i (a,b), da stiger f i (a,b).





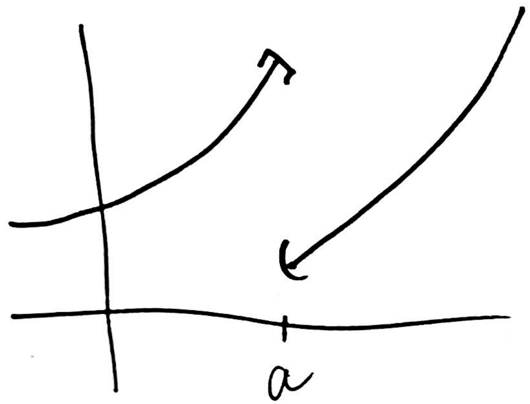
stigende

ikkje deriverbar

x^3 stigende for alle x

$(x^3)'=3x^2>0$ for $x\neq 0$

$=0$ for $x=0$



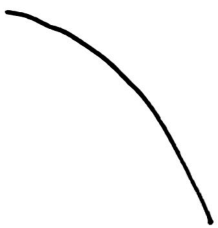
stigende nær a ?

ikkje stigende !

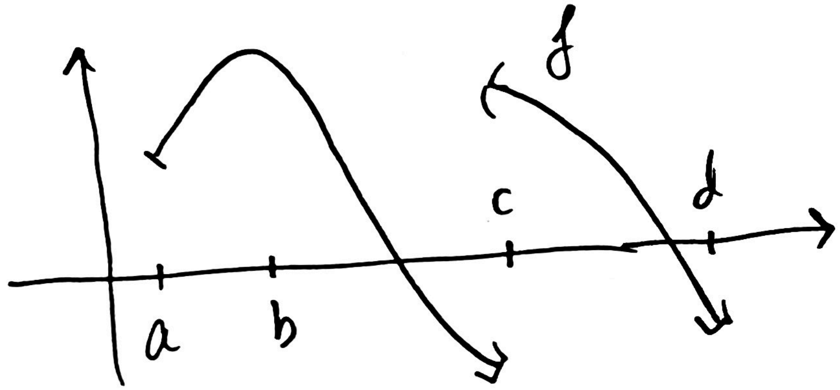
Avtagende, minkende, synkende i (a,b)

$x < y \Rightarrow f(x) > f(y)$

for alle $x, y \in (a,b)$



$D_f = [a, d]$.

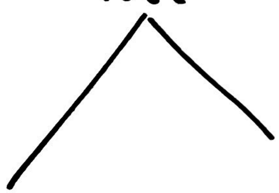


$$\left.\begin{array}{lll} f & \text{stigende} & (a, b) \\ f & \text{synkende} & (b, c) \\ f & \text{synkend} & (c, d). \end{array}\right\} \text{monotoni-egenskapene til } f.$$

$f'(x) < 0$ i (a, b) $\Rightarrow$ $f(x)$ avtagende.

$f(x)$ <u>kont i a</u>. Hvis $f(x)$ vokser til venstre for a
$f(x)$ avtar — høyre for a,
da har $f(x)$ et maksimalpunkt i $x = a$.

topp.



Tilsvarende for bunnpunkt.

Eks $f(x) = x^2 + 4x - 1$

$f'(x) = 2x + 4 = 2(x+2)$

$f'(x) = 0$ for $x = -2$

$f'(x) < 0$ for $x < -2$ avtagende

$f'(x) > 0$ for $x > -2$ økende

$f(x)$ er kontinuerlig i $x = -2$.

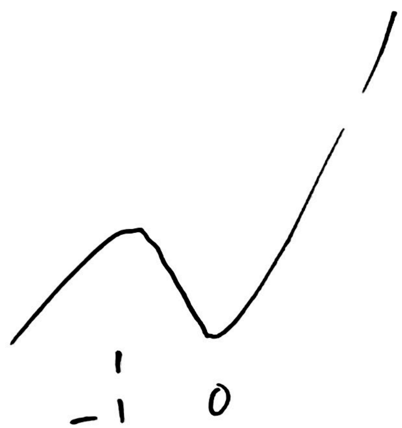
så $(-2, f(-2)) = (-2, -5)$ er et bunnpunkt.

6/4 Hvor er $f(x) = 2x^3 + 3x^2$ stigende/synkende?

$$f'(x) = 2\cdot 3x^2 + 3\cdot 2x = 6(x^2+x) = 6x(x+1)$$

Hva er fortegnet til $6x(x+1)$?

		-1		0	
$6\cdot(x)$	- - - -	- - -	- - -	0	———
$(x+1)$	- - - -	0	———		———
$6x(x+1)$	———	0	- - -	0	———



$f'(x) > 0$ i $(-\infty, -1)$ stigende

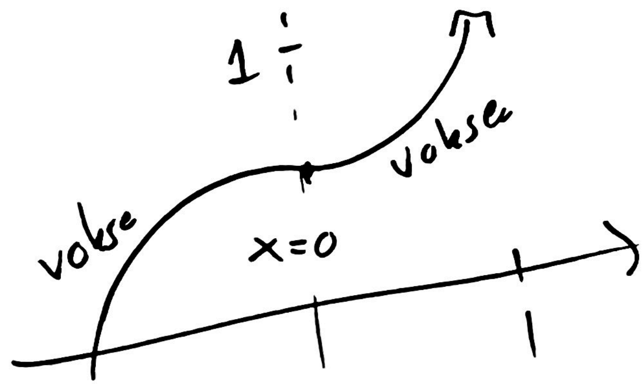
$(0, \infty)$ — " —

$f'(x) < 0$ $(-1, 0)$ avtagende.

$f(x)$ har maksimumspunkt $x = -1$ og minimumspunkt $x = 0$.

$$g(x) = \begin{cases} x^3 & x \leq 1 \\ 2x & x > 1 \end{cases}$$

Hvor vokser/avtar $g(x)$?



$x<1$ $\quad g'(x) = 3x^2$ positivt for $x \neq 0$.

Vokser.

$x>1$ $\quad g'(x) = (2x)' = 2 > 0$

$$\lim_{x\to1^+} g(x) = \lim_{x\to1^+} 2x = 2 > g(1) = 1 \checkmark$$

$g(x)$ er voksende (for alle x)

7.2 Konkavitet

Høyere deriverte.

$(f'(x))' = f''(x)$

$$\left(\left(\left(\left(f'(x)\right)'\right)'\right)'\right)' = \underline{f^{(5)}(x)} \quad \text{5-te deriverte}$$

$f'''''(x)$

$f''(x)$ andrederiverte. (dobbelderivert)

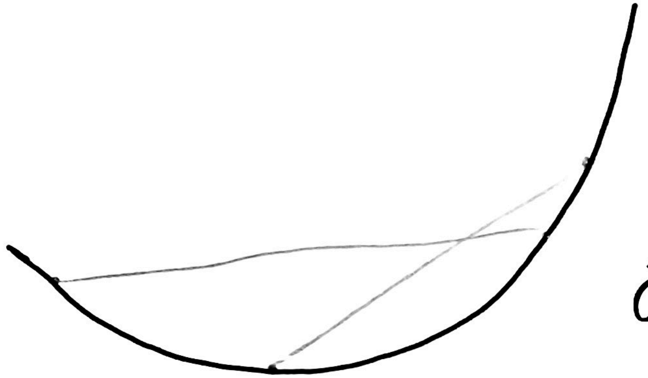
$(x^7)' = 7x^6$

$(x^7)'' = (7x^6)' = 7 \cdot 6x^5 = 42x^4$

$p(x)$ polynom av grad n

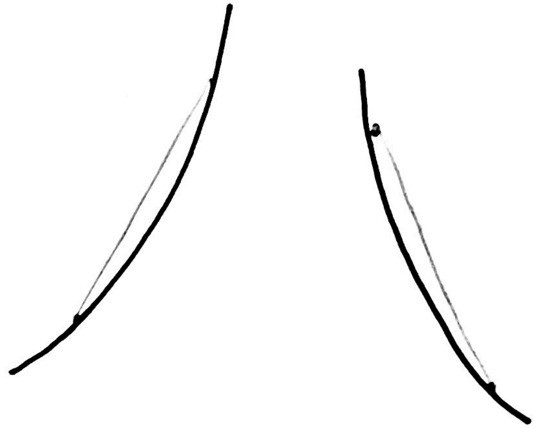
$p^{(m)}(x)$ polynom

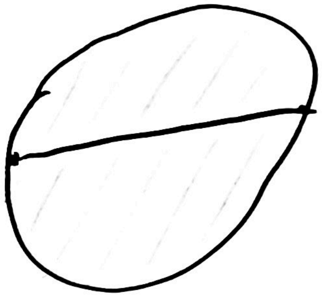
$$\begin{cases} \text{av grad } n-m & m \leq n \\ \text{0-polynomet} & m > n \end{cases}$$



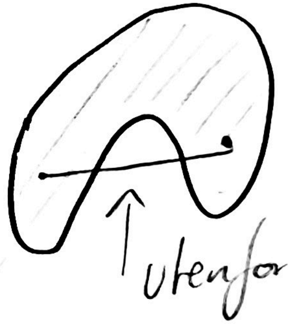
$f'(x)$ stiger

$f''(x) > 0$





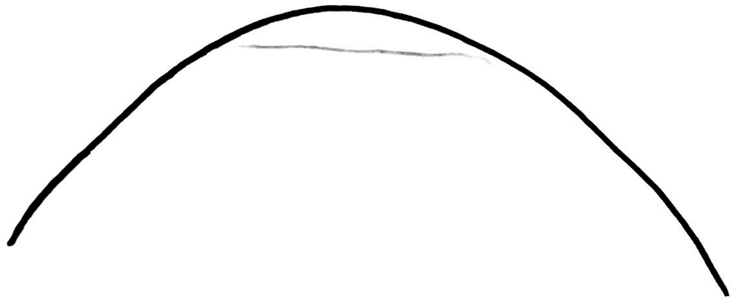
konkav
linjer mellom
to punkt i figuren
ligger selv i figuren

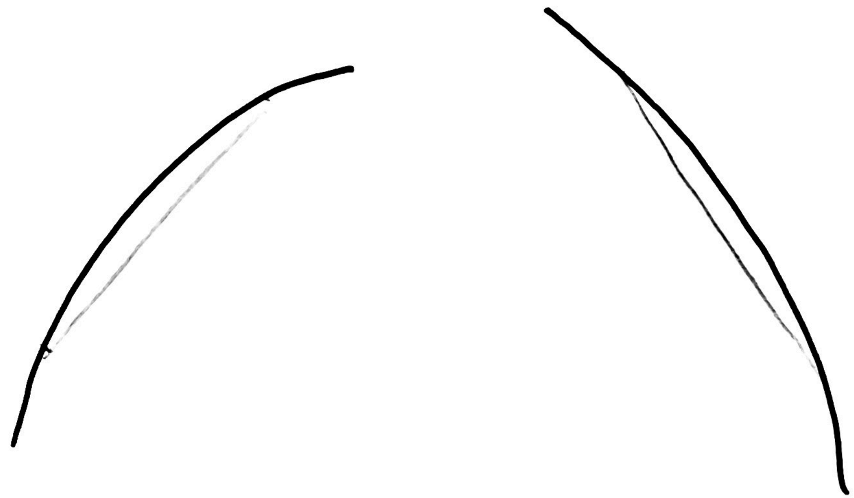


ikkje konkav

$f''(x) > 0$ i (a,b) da er $f(x)$ konkav opp i (a,b)
området over grafen til $f(x)$ er konkavt.

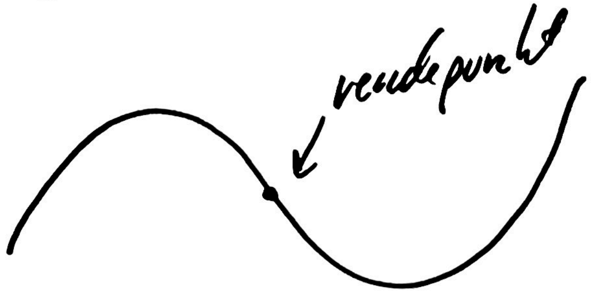
$f''(x) < 0 \quad i\ (a,b)$

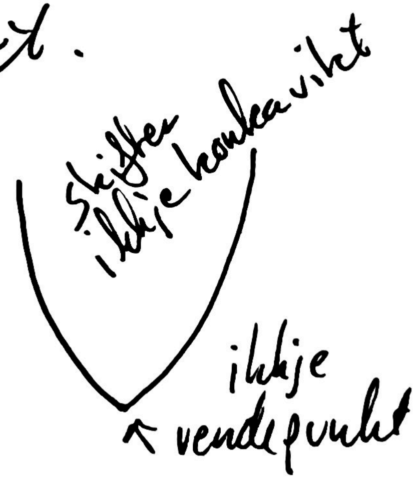


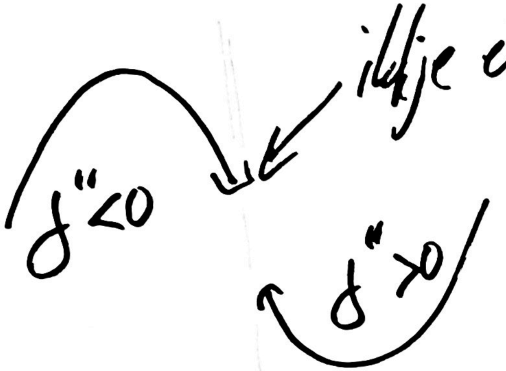


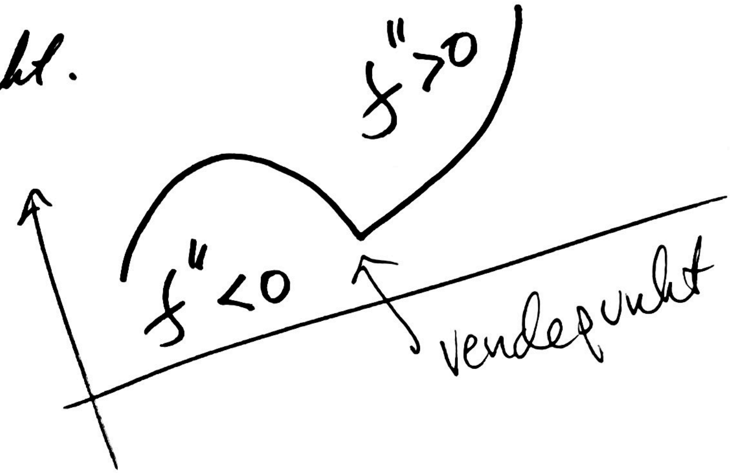
$f''(x) < 0 \quad i\ (a,b)$ $\quad f(x)$ er konkav ned i (a,b)

$(a, f(a))$ er et vendepunkt hvis f er kontinuerlig i $(a, f(a))$ og vi shifter konkavitet.









skifter konkavitet,
men er ikkje kontinuerlig

$f(x) = 2x^3 + 3x^2$

$f'(x) = 6(x^2 + x)$

$f''(x) = 6(2x+1) = 12(x + \frac{1}{2})$

$f''(x) < 0$ når $x < -\frac{1}{2}$ konkav ned

$f''(x) > 0$ når $x > -1/2$ — opp.

tegnet grafen...

Vendepunkt $\left(-\frac{1}{2}, f\left(-\frac{1}{2}\right)\right) = \underline{\left(-\frac{1}{2}, \frac{1}{2}\right)}$

Vendetangent : tangentlinje i et vendepunkt.
(tilnærmer funksjonen til 2. orden)