Øving 14:30

(spørsmål)

20.01
2022

$$(3x)' = 3(x)' = 3 \cdot 1 = 3$$

$$\left(\frac{3x}{4}\right)' = \left(\left(\frac{3}{4}\right) \cdot x\right)' = \left(\frac{3}{4}\right) \cdot (x)' = \underline{\frac{3}{4}}$$

$$\left(\frac{1}{2}x - \frac{x^3}{5}\right)' \underset{\text{lineært}}{=} \frac{1}{2}(x)' + \left(-\frac{1}{5}\right)(x^3)'$$

$$= \underline{\frac{1}{2} - \frac{3x^2}{5}}$$

$$\left(\frac{13 - 7x^3}{5}\right)' = \left(\frac{13}{5} - \frac{7}{5} \cdot x^3\right)'$$

$$= 0 - \frac{7}{5}(x^3)' = -\frac{7 \cdot 3}{5}x^2$$

$$= \underline{\frac{-21}{5}x^2}$$

$$f(x) = \frac{4x^2+1}{x}$$

- monotoni egenskaper, topp- og bunnpunkt.
- Asymptoter.

opg.

$$= \frac{4x^2}{x} + \frac{1}{x}$$

$$= 4x + \frac{1}{x}.$$

$$f'(x) = \left(4x + \frac{1}{x}\right)' = 4(x)' + \left(\frac{1}{x}\right)' = 4 - \frac{1}{x^2} = \cancel{\frac{4-x^2}{x^2}}$$

$$= 4 \cdot 1 + \frac{-1}{x^2} = \frac{4x^2}{x^2} - \frac{1}{x^2} = \frac{4x^2-1}{x^2}$$

$(x \neq 0)$

$$f'(x) = 0 \qquad \frac{4x^2-1}{x^2} = 0 \iff 4x^2 - 1 = 0$$

$$x^2 = \frac{1}{4}$$

$$x = \pm\frac{1}{2}$$

Kritiske punkter $x = \pm\frac{1}{2}$. { ingen endepunkt. $f(x)$ er deriverbar

$$-\frac{1}{2} \qquad 0 \qquad \frac{1}{2}$$

$f'(x)$ ———o- - - - x - - - -o———

Maksimums-punkt

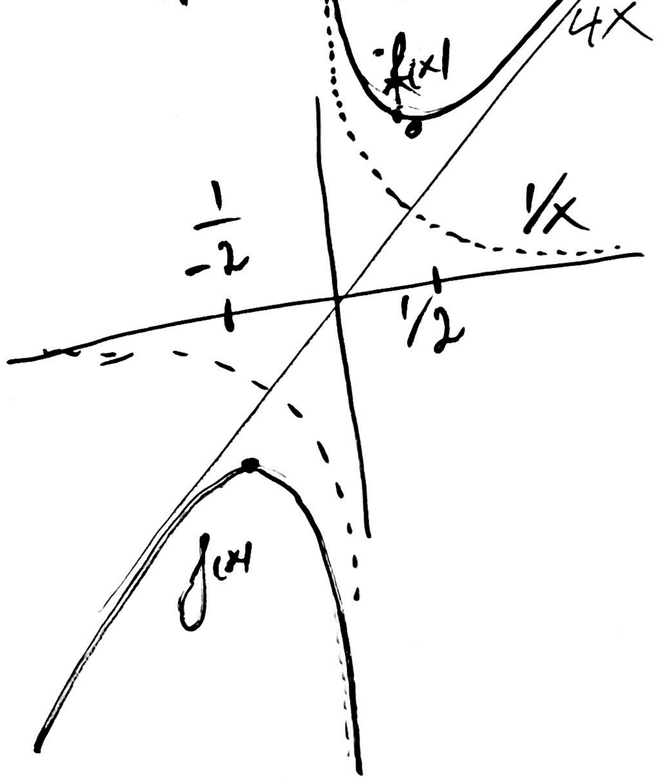
Minimums-punkt

$f(x)$ øker $\langle -\infty, -\frac{1}{2} \rangle$ og $\langle \frac{1}{2}, \infty \rangle$

— avtar $\langle -\frac{1}{2}, 0 \rangle$ og $\langle 0, \frac{1}{2} \rangle$

toppunkt $(\frac{-1}{2}, -4)$.

Bunnpunkt $(\frac{1}{2}, 4)$



$$f(x) = \frac{4x^2+1}{x} = 4x + \frac{1}{x}$$

$$\lim_{x \to 0^+} f(x) = \infty$$

$$\lim_{x \to 0^-} f(x) = -\infty$$

Vertikal asymptote $x = 0$

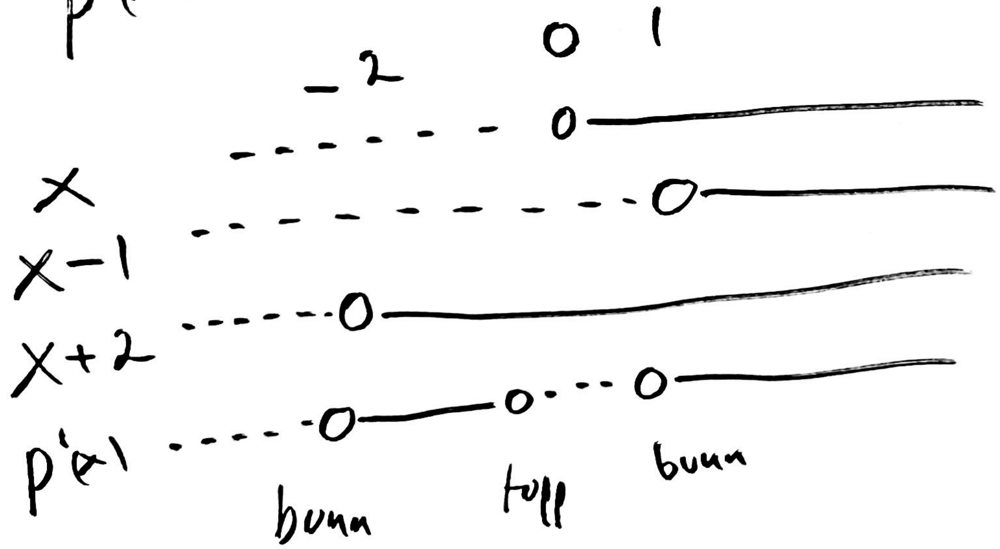
Skrå asymptote $y = 4x$

Funksjonsdrøfting

$$p(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$
$$= x^2\left(\frac{x^2}{4} + \frac{x}{3} - 1\right)$$

$$p'(x) = \frac{1}{4}(x^4)' + \frac{1}{3}(x^3)' - (x^2)' = \frac{1}{4}\cdot 4x^3 + \frac{1}{3}\cdot 3x^2 - 2x$$
$$= x^3 + x^2 - 2x = x(x^2+x-2)$$
$$= x(x-1)(x+2)$$

$p'(x) = 0$ for $x = -2, 0, 1$ ← kritiske punktene



$p(x)$ vokser i intervallene $(-2,0)$ og $(1,\infty)$

$p(x)$ avtar $(-\infty,-2)$ og $(0,1)$

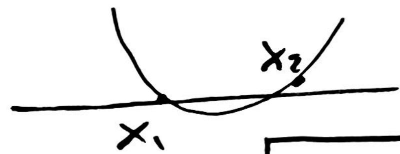
$$p(-2) = \frac{1}{4}(-2)^4 + \frac{1}{3}(-2)^3 - (-2)^2 = -8/3$$

$$p(0) = 0$$

$$p(1) = \frac{1}{4} + \frac{1}{3} - 1 = \frac{3+4-12}{12} = \frac{-5}{12}.$$

Ekstremalpunkt: bunnpunkt $(-2, -8/3)$, $(1, \frac{-5}{12})$

toppunkt $(0, 0)$



$$p''(x) = (x^3 + x^2 - 2x)' = 3x^2 + 2x - 2$$

$$p''(x) = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(3)(-2)}}{2 \cdot 3} = \frac{-2 \pm 2\sqrt{1+6}}{2 \cdot 3}$$

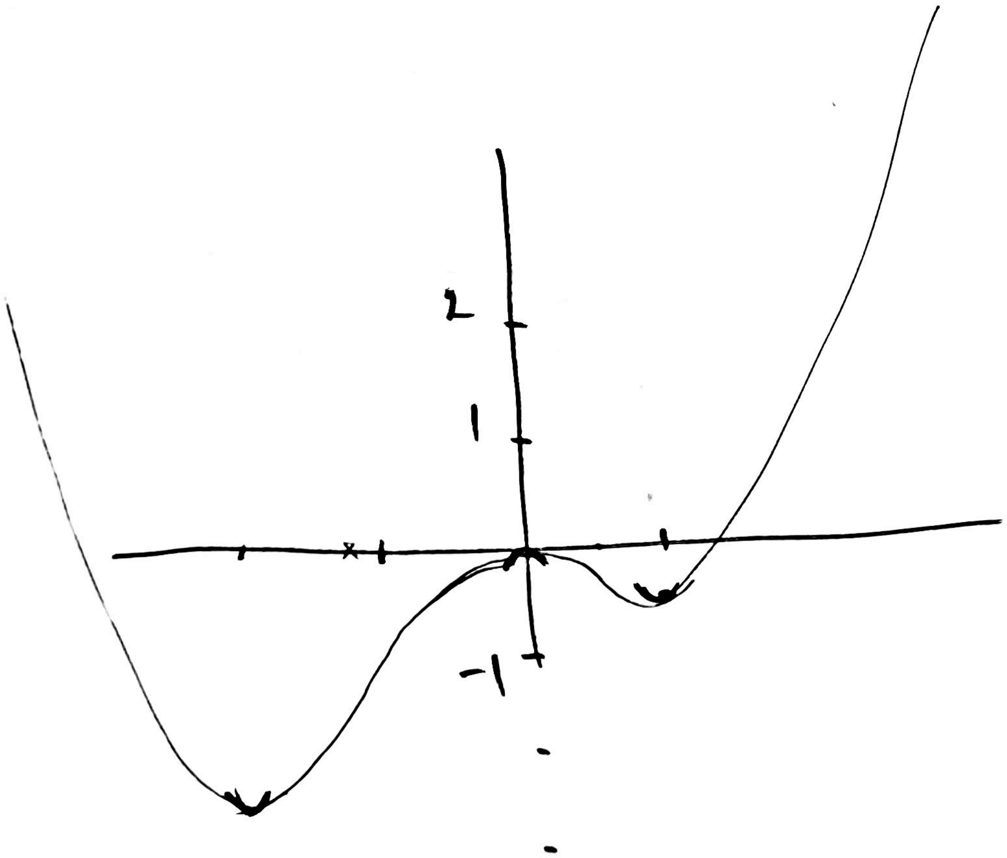
$$= \frac{\cancel{2}(-1 \pm \sqrt{7})}{\cancel{2} \cdot 3} = \frac{-1 \pm \sqrt{7}}{3}.$$

$x_1 = -1.215..$ $\quad x_2 = 0.548...$

p) Konkav ned i (x_1, x_2)

Konkav opp $(-\infty, x_1)$ og (x_2, ∞).

Vendepunkt for x lik x_1 og x_2



$f(x) = 3x^4 - 8x^3 + 6x^2 + 1$ Funksjonsdrøfting.

$f'(x) = 12x^3 - 24x^2 + 12x$

$= 12x(x^2 - 2x + 1) = 12x(x-1)^2$

$f''(x) = 12 \cdot 3x^2 - 24 \cdot 2x + 12 = 12(3x^2 - 4x + 1)$

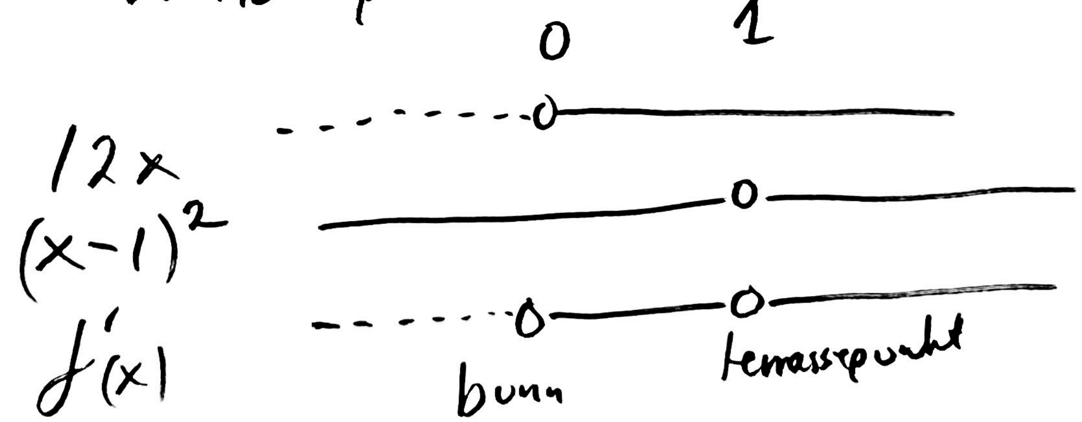
$= 12(x-1)(3x-1)$

ingen endepunkt

$f(x)$ deriverbar

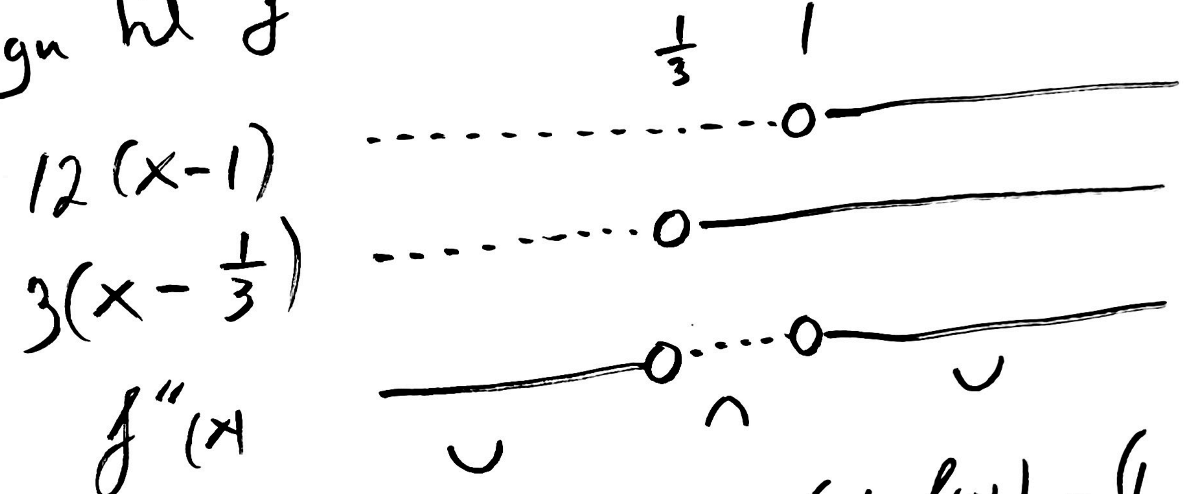
$f'(x) = 0$ for $\underbrace{x = 0, \quad x = 1.}_{\text{kritiske punkt}}$

Fortegn til f'



$f(0) = 1$ Bunnpunkt i $\underline{(0,1)}$.

Konkavitet: Fortegn til f''



Vendepunkt i $(\frac{1}{3}, f(\frac{1}{3}))$ og i $(1, f(1)) = (1,2)$

Konkav opp i $(-\infty, \frac{1}{3})$ og $(1, \infty)$

Konkav ned i $(\frac{1}{3}, 1)$.