21.01
2022

Potensfunksjoner

1

$f(x) = x^r$ potensfunksjon $\underline{x > 0}$

$r \in \mathbb{R}$

$r = -1$: $\frac{1}{x}$ $x \neq 0$

$r = 1/2$: $\sqrt{x}$ $x \geq 0$

$r = -1/4$: $x^{-1/4} = \frac{1}{\sqrt[4]{x}}$ $x > 0$

$r = \pi$ x^{π} $x > 0$

$r = \frac{1}{3}$: $\sqrt[3]{x}$ def. for alle x

(a^x eksponentfunksjon ...
2^x e^x) senere.

$$(x^r)' = r x^{r-1} \qquad r \in \mathbb{R}, \; x > 0$$

2

Eksempler

$$\frac{1}{x^3} = (x^3)^{-1} = x^{-3}$$

$$\left(\frac{1}{x^3}\right)' = (x^{-3})' = -3x^{-3-1} = -3x^{-4} = -3(x^4)^{-1}$$

$$\left(\frac{1}{x^3}\right)' = \frac{-3}{x^4}$$

$$\sqrt[3]{x^2} = (x^2)^{1/3} = x^{2/3}$$

$$(\sqrt[3]{x^2})' = \frac{2}{3} x^{\frac{2}{3}-1} = \frac{2}{3} x^{-1/3} = \frac{2}{3}(x^{1/3})^{-1}$$

$$= \underline{\frac{2}{3} \cdot \frac{1}{\sqrt[3]{x}}}$$

③

$$(x^{\pi})' = \pi x^{\pi-1}$$

$$(x^{\sqrt{2}})' = \sqrt{2}\, x^{\sqrt{2}-1}$$

$$(x^{4/7})' = \frac{4}{7} x^{\frac{4}{7}-1} = \frac{4}{7} x^{\frac{-3}{7}} = \frac{4}{7} \left((x^3)^{\frac{1}{7}}\right)^{-1}$$

$$= \underline{\frac{4}{7 \cdot \sqrt[7]{x^3}}}$$

$(x^5)' = 5x^4$ men $(x^{-5})' = -5x^{-6}$

opg.

Deriver

1) $x^{2.73}$

2) $x \cdot \sqrt{x}$

3) $\dfrac{4}{\sqrt[5]{x}}$

$$(x^{2.73})' = 2.73 \cdot x^{1.73}.$$

$$(x \cdot \sqrt{x})' = (x \cdot x^{1/2})' = (x^{3/2})' = \frac{3}{2} x^{1/2}$$

$$= \underline{\frac{3}{2}\sqrt{x}}$$

$$4\left(\sqrt[5]{x}\right)^{-1} = 4\left(x^{1/5}\right)^{-1}$$

$$= 4x^{-1/5}$$

$$\left(\frac{4}{\sqrt[5]{x}}\right)' = \left(4x^{-1/5}\right)' = 4\left(\frac{-1}{5}\right)x^{-1/5-1} = \frac{-4}{5} x^{-6/5}$$

$$= \underline{\frac{-4}{5\sqrt[5]{x^6}}}$$

Deriver $\sqrt{x} \cdot \sqrt[3]{x} = x^{1/2} \cdot x^{1/3} = x^{\frac{3+2}{6}}$

$= x^{\frac{1}{2}+\frac{1}{3}}$

$= x^{(5/6)}$

(4)

$(\sqrt{x} \cdot \sqrt[3]{x})' = (x^{5/6})' = \frac{5}{6} x^{\frac{5}{6}-1}$

$= \underline{\frac{5}{6} x^{-1/6}} \quad \left(= \frac{5}{6\sqrt[6]{x}} \right)$

Funksjonsdrøfting

$f(x) = \frac{\sqrt{x}-2}{x} = \frac{\sqrt{x}}{x} - \frac{2}{x}$

$= \frac{x^{1/2}}{x} - \frac{2}{x} = x^{1/2} \cdot x^{-1} - 2x^{-1}$

$= \underline{x^{-1/2} - 2x^{-1}}$

$f'(x) = \left(\frac{-1}{2} x^{-3/2}\right) - 2\left(\frac{-1}{x^2}\right)$

$= \frac{-1}{2x^{3/2}} + \frac{2}{x^2} = \frac{-1}{2x^{3/2}} \cdot \frac{x^{1/2}}{x^{1/2}} + \frac{2}{x^2} \cdot \frac{2}{2}$

$= \frac{-\sqrt{x} + 4}{2x^2}$

$f'(x) = 0$ når $-\sqrt{x} + 4 = 0$

$\sqrt{x} = 4 \overset{\Leftrightarrow}{\text{kvadrere}} \quad x = 4^2 = \underline{16}$

5

$\sqrt{x}$ økende så $f'(x) > 0$ $\langle 0, 16 \rangle$

$f'(x) < 0$ $\langle 16, \infty \rangle$

så toppunkt i $(16, f(16)) = \left(16, \frac{\sqrt{16} - 2}{16}\right)$

$\underline{\left(16, \frac{1}{8}\right)}$

Oppg. $g(x) = 4x + \frac{1}{x}$ Finn topp- og bunnpunkt.

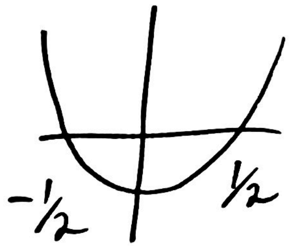
$x \neq 0$ Hvor stiger og synker $g(x)$?

$g'(x) = (4x + x^{-1})' = 4 + (-1)x^{-2} = \frac{4x^2 - 1}{x^2}$

$g'(x) = 0 \Leftrightarrow 4x^2 = 1 \Leftrightarrow x = \pm 1/2.$

$g'(x) > 0$ for $x < -1/2$: $\langle -\infty, -1/2 \rangle$ ($4x^2 - 1$

og for $x > 1/2$: $\langle 1/2, \infty \rangle$.



(6)

toppunkt $\left(-\frac{1}{2}, -4\right)$

bunnpunkt $\left(\frac{1}{2}, 4\right)$

Konkavitet :

$g''(x) = (4 - x^{-2})' = 0 - (-2x^{-3}) = \frac{2}{x^3}$.

$g''(x) > 0$ $x > 0$ konkav opp

$g''(x) < 0$ $x < 0$ konkav ned.

Asymptoter

vertikal asymptote $x = 0$ (y-aksen)

skrå asymptote $y = 4x$

Neste uke:

Produktregel

$$\big(f(x)\cdot g(x)\big)' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$\big((f\cdot g)(x)\big)$$

7

sjekker for potensfunksjoner

$$(x^2\cdot x^5)' = (x^2)'\cdot x^5 + x^2(x^5)'$$
$$= 2x\cdot x^5 + x^2\cdot 5x^4$$
$$= 2x^6 + 5x^6 = 7x^6$$

$$(x^2\cdot x^5)' = (x^7)' = 7x^6$$ ✓ stemmer.

$$(x^r\cdot x^s)' = (x^r)'x^s + x^r(x^s)'$$
$$= \underbrace{rx^{r-1}\cdot x^s}_{rx^{r+s-1}} + \underbrace{x^r\cdot sx^{s-1}}_{sx^{r+s-1}}$$
$$= (r+s)x^{r+s-1}$$

✓ stemmer for alle s og r.

Kjerneregelen $\quad \left(f(k(x))\right)' = f'(k(x)) \cdot k'(x)$

8 $\quad \left((x^r)^s\right)' = \left(x^{r \cdot s}\right)' = r \cdot s\, x^{r \cdot s - 1}$

Kjerneregel $\quad \left((x^r)^s\right)'$

$$= s(x^r)^{s-1} \cdot (x^r)'$$

$$= s\, x^{r(s-1)} \cdot r\, x^{r-1}$$

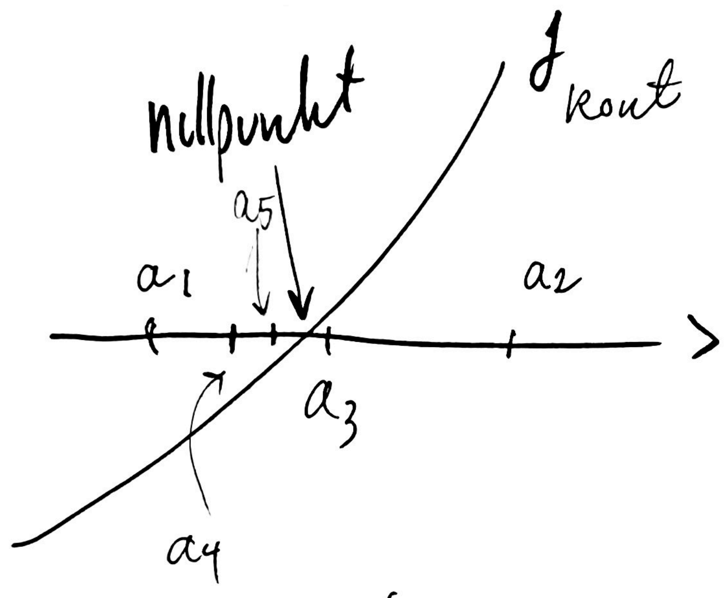
$$= r \cdot s\, x^{r \cdot s - r + r - 1}$$

$$= r \cdot s\, x^{r \cdot s - 1}$$

✓ stemmer!

Øving

spørsmål om numerisk løsning av likninger



Halveringsmetoden (M1000)

$f(a_1) < 0$

$f(a_2) > 0$

$a_3 = \frac{a_1 + a_2}{2}$

$f(a_3) > 0$ $\quad (a_1, a_3)$

$a_4 = \frac{a_1 + a_3}{2}$

$f(a_4) < 0$ bruker (a_4, a_3)

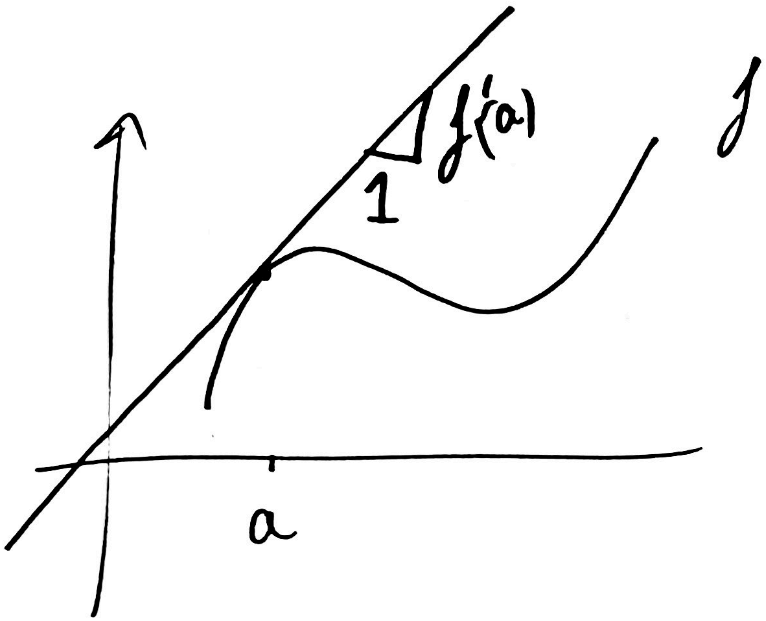
$\underline{a_5} = \frac{a_3 + a_4}{2}$

$f(a_5) < 0$ bruker (a_5, a_3)

gjentar . . .

nullpunkt i $(-, -)$

lengden på intervallet halveres i hvert steg.



Tangentlinjen

går gjennom $(a, f(a))$

stigningstall $f'(a)$

$$y = f'(a)(x-a) + f(a).$$

Opg 7.61 e) $\left(\frac{1}{x^2+1}\right)'$?

$\frac{1}{x^2+1}$ sammensatt funksjon

$\frac{1}{u}$, $u = x^2+1$,

ytrefunksjon kjernen

Benytter kjerneregelen

$$\left(\frac{1}{u}\right)'_{u=x^2+1} \cdot (x^2+1)'$$

$$= \frac{-1}{(x^2+1)^2} \cdot 2x = \frac{-2x}{(x^2+1)^2}$$

$$\left(\frac{1}{x^2-1}\right)' = \frac{-2x}{(x^2-1)^2}$$ (tilsvarende)

Alternativt:

$$\frac{1}{(x-1)(x+1)} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

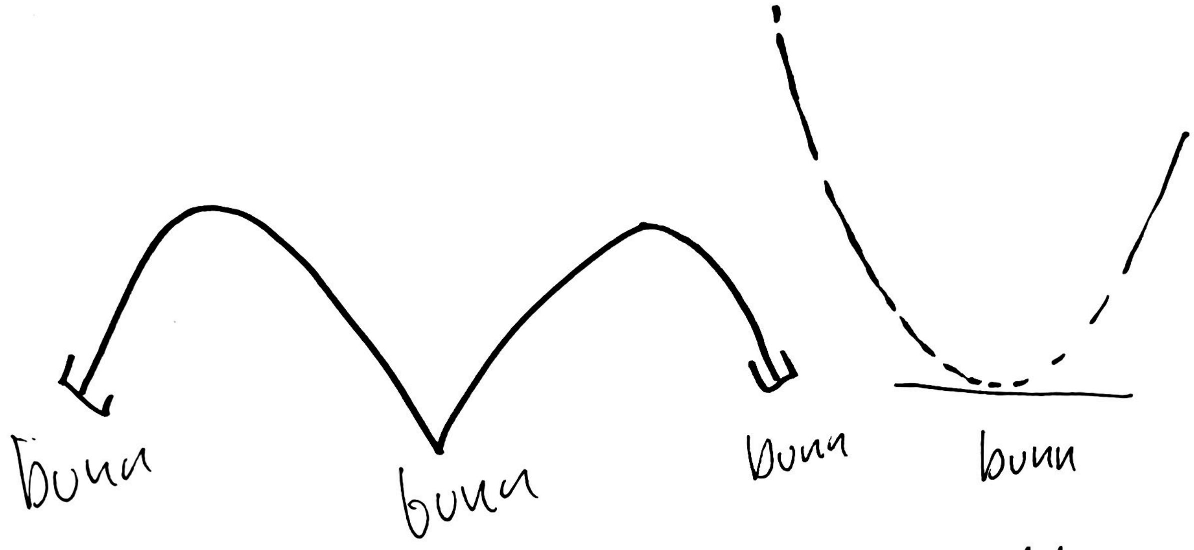
$$\left(\frac{1}{x^2-1}\right)' = \frac{1}{2}\left(\frac{-1}{(x-1)^2}\cdot(1) - \frac{-1}{(x+1)^2}(1)\right)$$ Felles nevner

$$= \frac{1}{2}\cdot\frac{-(x+1)^2+(x-1)^2}{(x-1)^2(x+1)^2} = \frac{1}{2}\,\frac{-(x^2+2x+1)+x^2-2x+1}{(x^2-1)^2}$$

$$= \frac{-4x}{2(x^2-1)^2} = \underline{\frac{-2x}{(x^2-1)^2}}$$

Deriverbar $\Rightarrow$ kontinuerlig.

Så ikkje kontinuerlig $\Rightarrow$ ikkje deriverbar

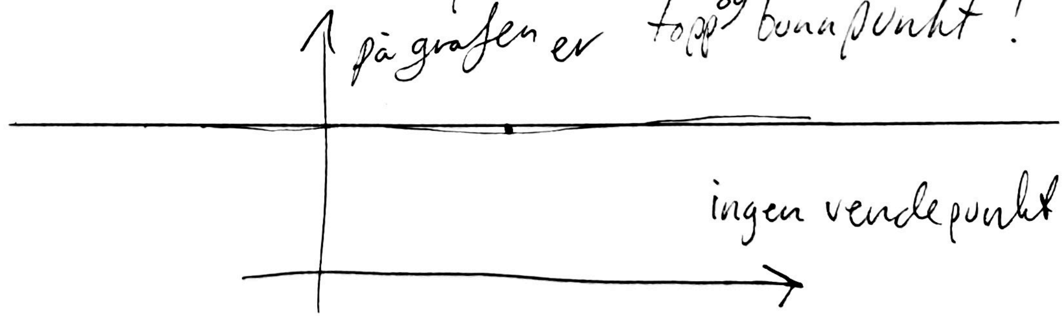


Kritiske punkt $\supset$ Ekstremalpunkt

Endepunkt
stasjonære punkt $f'(x) = 0$
ikkje deriverbar.

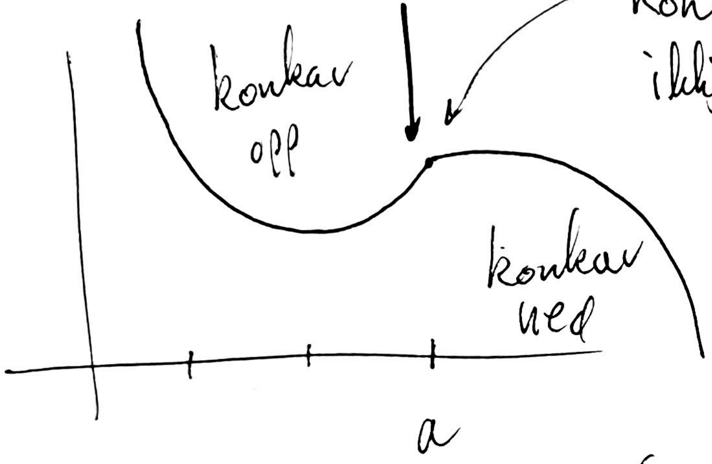
$f(x) = 2$

Allepunkt på grafen er topp og bunnpunkt!



ingen vendepunkt.

Vendepunkt (selv om f'' ikkje er 0)



kont. men ikkje deriverbar i $x=a$.

$f''(a)$ eksisterer ikkje.

Eksempel:

$$f(x) = \begin{cases} (x-2)^2+1 & x \leq 3 \\ -(x-3)^2+2 & x > 3 \end{cases} \quad \text{kont.}$$

$$f'(x) = \begin{cases} 2(x-2) & x < 3 \\ -2(x-3) & x > 3 \end{cases}$$

$$f''(x) = \begin{cases} 2 & x < 3 \\ -2 & x > 3 \end{cases}$$