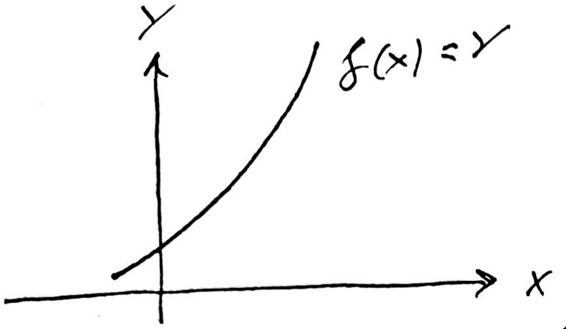
26.01.2022 7.7 Inversfunksjoner



injektiv

én-til-én

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

alternativ formulering:

$x_1 \neq x_2 \in D_f$, da er $f(x_1) \neq f(x_2)$

Injektive funksjoner har inversfunksjoner:

Inversfunksjonen g til f har egenskapen

$g(y) = x$ s.a. $f(x) = y$.

(bare én mulig x siden f er inj.)

$$\boxed{\begin{aligned} g(f(x)) &= x \\ f(g(y)) &= y \end{aligned}}$$

$g \circ f = id$

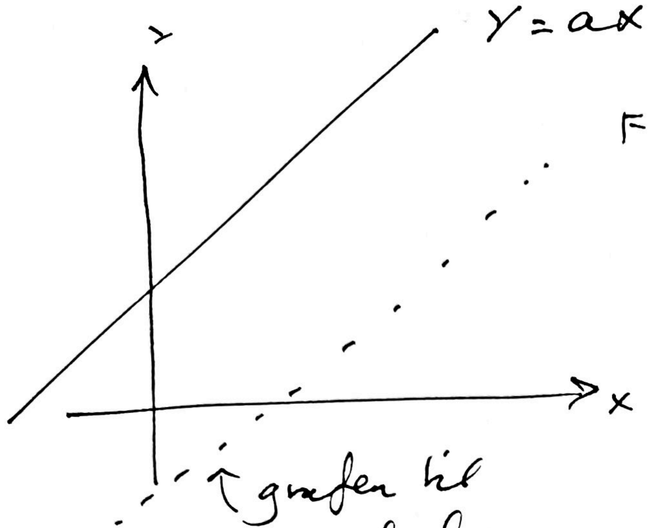
$f \circ g = id$

id er identitetsfunksjonen

$id(x) = x$

Inversfunksjonen til f skrives f^{-1}

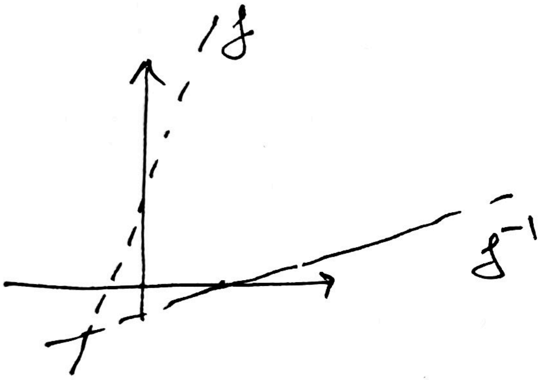
(Advarsel $f^{-1}(x) \neq \underbrace{(f(x))^{-1}}_{= 1/f(x)}$)



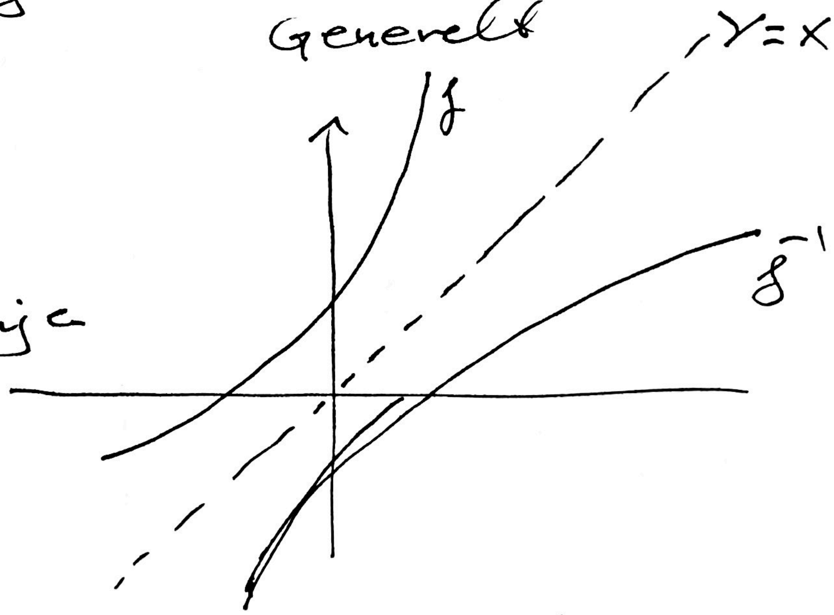
$a \neq 0 \Rightarrow$ injektiv.

Finner inversfunksjonen

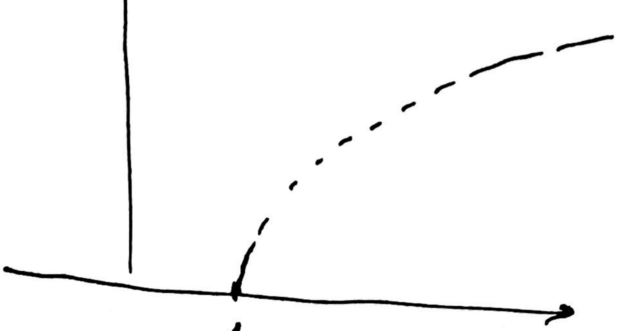
$$y = ax+b$$
$$y-b = ax$$
$$\frac{y-b}{a} = x$$
$$\underline{x = \frac{1}{a}\cdot y - \frac{b}{a}}$$



Generelt



grafen til f speglet om linja $x=y$ gir grafen til f^{-1}.



$f(x) = \sqrt{x-1}$ økende ($f'(x)>0$) så injektiv

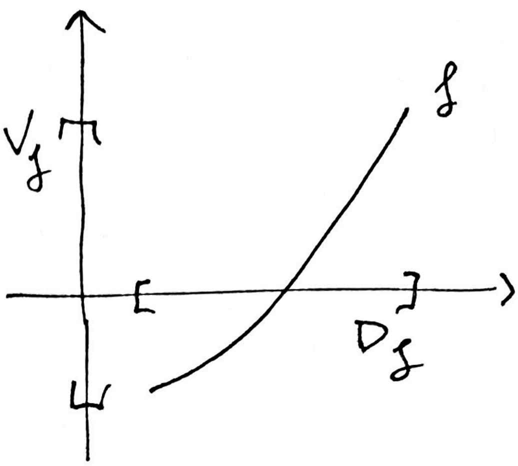
Inversfunksjon:

$$y = \sqrt{x-1} \quad \text{kvadrerer}$$
$$y^2 = x-1$$
$$\underline{x = y^2+1} \qquad y \geq 0$$

f inversfunksjon f^{-1}

$D_{f^{-1}} = V_f$

$V_{f^{-1}} = D_f$



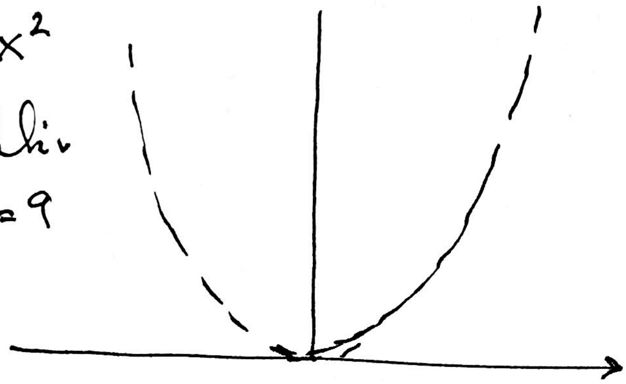
$f(x) = x^2$

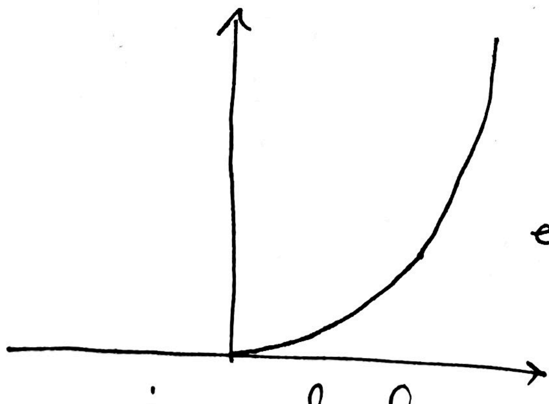
ikkje injektiv

$f(-3) = f(3) = 9$

men $-3 \neq 3$

har ikkje inversfunksjon





$g(x) = x^2$

$D_g = [0, \infty)$

er injektiv

inversfunksjonen er $h(x) = \sqrt{x}$

$g(h(x)) = (\sqrt{x})^2 = x \, , \; x \geq 0$

$h(g(x)) = \sqrt{x^2} = |x|$

$= x$ for $x \geq 0$

x^r $\quad r \neq 0 \quad x > 0$

$(x^r)' = r x^{r-1}$

positiv for $r>0$, voksende
negativ – $r<0$, avtagende
så injektiv

$f(x) = x^r$

$\underline{f^{-1}(x) = x^{1/r}}$

$(x^r)^{1/r} = x^{r \cdot 1/r} = x^1 = x$

$r = 2$ inversfunksjonen til $x^2, x \geq 0$ er $\sqrt{x}$

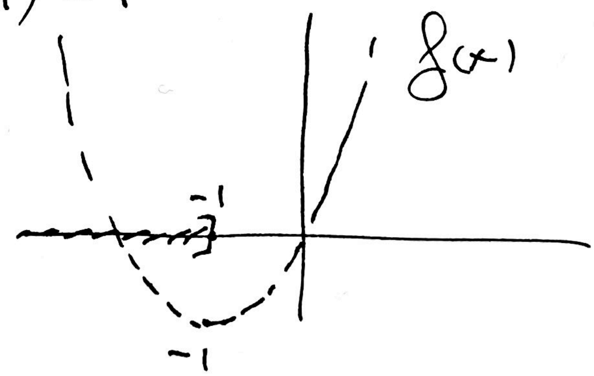
x^n inversfunksjon $\sqrt[n]{x}$.

---

pause

opg. Finn inversfunksjonen til

$$f(x) = x^2 + 2x \qquad D_f = \langle -\infty, -1]$$
$$= (x+1)^2 - 1$$



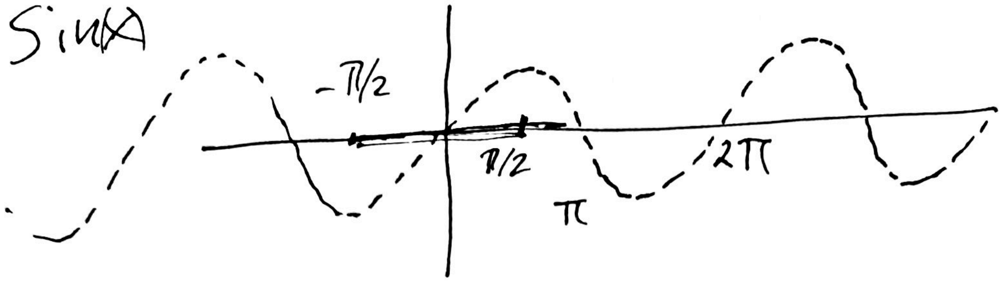
$f'(x) = 2x + 2$
$= 2(x+1) < 0$
for $x < -1$
avtagende.

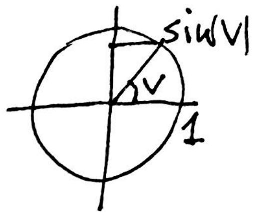
$$y = (x+1)^2 - 1$$
$$(x+1)^2 = y + 1$$
$$x + 1 = \pm\sqrt{y+1} \quad ?$$
$$x = -1 - \sqrt{y+1}$$

minus fordi
$x \leq -1$.

$\underline{f^{-1}(x) = -1 - \sqrt{y+1}}$

Sin(x)



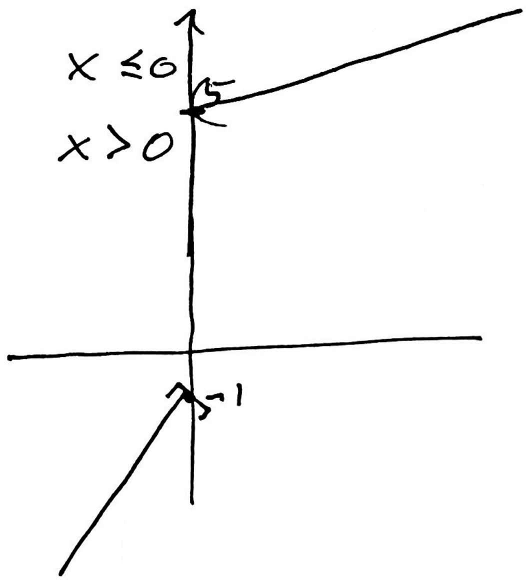


Sin stigende på $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
så injektiv

$\sin^{-1}(x) = \arcsin(x)$ er inversfunksjon
til Sin x avgrenset
til $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Eks

$$g(x) = \begin{cases} 2x - 1 & x \leq 0 \\ (1/3)x + 5 & x > 0 \end{cases}$$

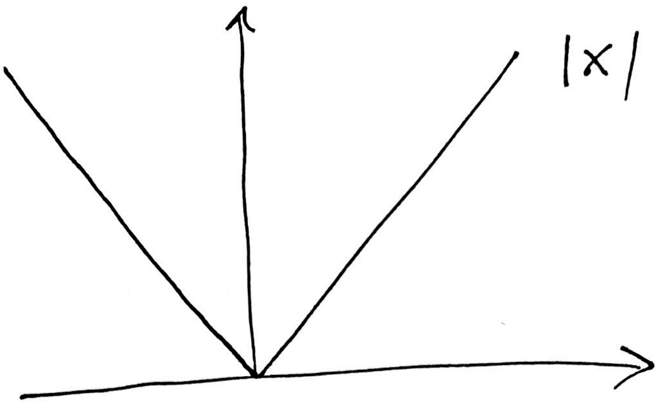


g(x) økende
så injektiv

$y = 2x - 1$ gir $x = \frac{y+1}{2}$

$y = \frac{1}{3}x + 5$ gir $x = 3(y-5)$

$$g^{-1}(x) = \begin{cases} \frac{y+1}{2} & x \leq -1 \\ 3(y-5) & x > 5 \end{cases}$$



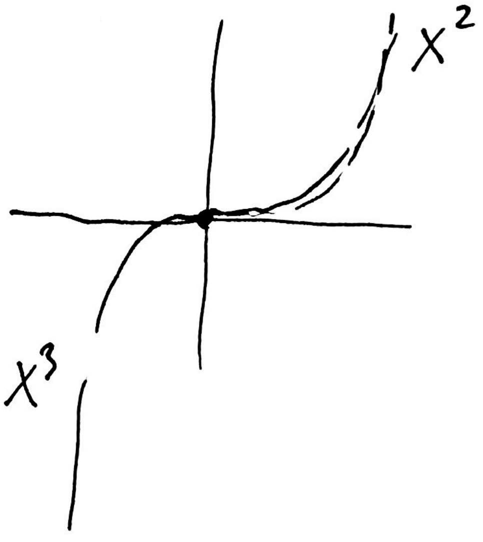
$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$(|x|)' = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

ikkje deriverbar i $x = 0$.

$$f(x) = \begin{cases} x^2 & x \geq 0 \\ x^3 & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 2x & x \geq 0 \\ 3x^2 & x < 0 \end{cases}$$



Deriverbar i origo

$$f(f^{-1}(x)) = x$$

Deriverer og bruker kjerneregeln

$$f'(f^{-1}(x)) \cdot (f^{-1}(x))' = 1$$

$$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$$

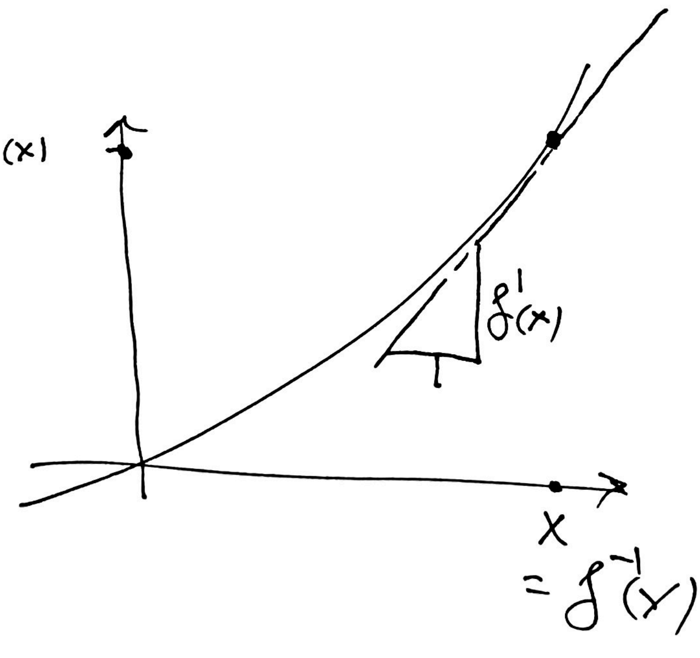
$f(x) = x^2 \qquad D_f = [0, x\rangle$

$f^{-1}(x) = \sqrt{x}$

$$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))} = \frac{1}{2(f^{-1}(x))}$$

$$= \frac{1}{2\sqrt{x}} \quad \checkmark$$

(stemmer : $\frac{1}{2\sqrt{x}} = (\sqrt{x})'$)



$$(f^{-1})'(y) = \frac{1}{f'(x)} = \frac{1}{f'(f^{-1}(y))}$$