1. feb.
2022

Logaritmer

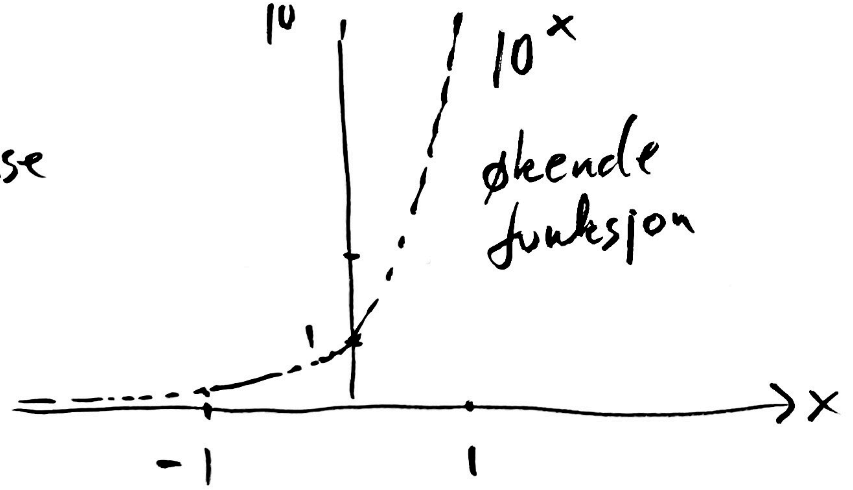
kap 8.

$$10^{?} = b$$

$$10^{\log b} = b$$

Logaritmen av b er eksponenten til 10 som gir b.

Skisse



Eksponenten x til $10^x = b$ er entydig for $b > 0$ (akkurat én løsning)

$10^x = b$ ingen løsning for $b = 0$ eller $b < 0$.

Definisjonsmengden til $\log(x)$ er alle $x > 0$.

Alternativ notasjon $\lg$

$$\lg(x) = \log(x) = \mathrm{Log}(x)$$

Briggske logaritmer

10-er logaritmer

$$\log(1000) = 3$$

$$\log(\sqrt{10}) = 1/2$$

siden $\sqrt{10} = 10^{1/2}$.

$$10^2 = 100 \qquad \log(100) = 2$$

$$10 = 10^1 \qquad \log(10) = 1$$

$$\frac{1}{10} = 10^{-1} \qquad \log(1/10) = -1$$

$$\log(1) = 0$$ siden $10^0 = 1$.

$$2^4 = 16 > 10$$

$$2 > \sqrt[4]{10} = 10^{1/4}$$

$$\log(2)$$

$$2^3 = 8 < 10$$

$$2 < \sqrt[3]{10} = 10^{1/3}$$

$$\frac{1}{4} < \log 2 < \frac{1}{3}$$

$$\mathrm{Log}(2) = 0.301029995\ldots$$

$\lg(60) = \log(60) \sim 1{,}7781512\ldots$

(estimat

$60 < 64 = 2^6$

$10^{0.3} \sim 2$

$10^{0.3 \cdot 6} \sim 2^6 = 64$

$10^{1.8} \sim 64$

$Y = \log(3567)$

$= 3.5523031\ldots$

$10^3 = 1000$

$10^4 = 10000$

Så $3 < Y < 4$.

$\sqrt{10} \sim 3.162\ldots$

$10^3 \cdot \sqrt{10} = 10^3 \cdot 10^{1/2} = 10^{3.5} = 3162 < 3567$

Forventer Y litt over 3.5

$\log(0.005)$ $\quad 10^{-3} = 0.001 < 0.005 < 0.01 = 10^{-2}$

Så $\log(0.005)$ ligger mellom -2 og -3.

$$0.005 = \frac{0.01}{2} = 0.01 \cdot 2^{-1} = 10^{-2} \cdot \left(10^{\log 2}\right)^{-1} = 10^{-2-\log 2}$$

Så $\log(0.005) = -2 - \log 2 = \underline{-2.30102\ldots}$

Eks

1000kr. 10% årlig rente. x antall år

Hvor lenge må pengene stå i banken for at de skal forrente seg til 2000kr?

$$\left(1.+10\%\right)^x = (1.1)^x = 2$$

$$\left(10^{\log 1.1}\right)^x = 2$$

$$10^{(\log 1.1)\cdot x} = 2 = 10^{\log 2}$$

$$\log(1.1)\cdot x = \log 2$$

$$x = \frac{\log 2}{\log(1.1)}$$

$$x \sim \underline{7.27 \text{ år}}$$

Logaritme reglene

$$\boxed{\begin{aligned} \log(a\cdot b) &= \log a + \log b \\ \log(a^r) &= r\log(a) \end{aligned}}$$

$$\log(1) = 0 \qquad \log(10) = 1$$

$$\log\left(10^2 \cdot 10^5\right) = \log\left(10^{2+5}\right)$$

$$= 2+5 = \log\left(10^2\right) + \log\left(10^5\right)$$

$$\log\left(\left(10^2\right)^7\right) = \log\left(10^{2\cdot 7}\right) = 2\cdot 7$$

$$= 7\left(\log\left(10^2\right)\right)$$

Viser hvordan logaritmereglene følger fra potensreglene

$$10^x \cdot 10^y = 10^{x+y}$$

$$\left(10^x\right)^y = 10^{x\cdot y}$$

$$\log(a\cdot b) = \log\left(10^{\log a} \cdot 10^{\log b}\right) = \log\left(10^{\log a + \log b}\right)$$

$$= \log a + \log b$$

$$\log\left(a^r\right) = \log\left(\left(10^{\log a}\right)^r\right) = \log\left(10^{r\cdot \log a}\right) \underline{= r \log a}$$

8.2 Eksponentlikning

$3^x = 5$ tar log på begge sider

$\log(3^x) = \log 5$ | $\log 5 \sim 0.69897\ldots$

$x \log 3 = \log 5$ | $\log 3 \sim 0.47712\ldots$

$x = \dfrac{\log 5}{\log 3} \sim \underline{1.46497\ldots}$

$\log 2 + \log 5 = \log(2 \cdot 5) = \log 10 = 1$

generelt $a \cdot b = 10 \Rightarrow \log a = 1 - \log b$

opg. 1) $\left(\frac{1}{2}\right)^x = 7$ 2) $3^{2x-1} = 5$

$\log(a) + \log\left(\frac{1}{a}\right) = \log\left(a \cdot \frac{1}{a}\right) = \log 1 = 0$

Alternativt $\underline{\log\left(\frac{1}{a}\right)} = \log(a^{-1}) = \underline{-\log(a)}$

$\left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x} = 7$

$2^2 = 4$
$2^3 = 8.$

$x \log\left(\frac{1}{2}\right) = \log 7$

$x = \frac{\log 7}{\log(1/2)} = -\frac{\log 7}{\log 2} = -2.80735\ldots$

$3^{2x-1} = 5$

$\log 3^{2x-1} = \log 5 \iff (2x-1)\log 3 = \log 5$

$2x-1 = \frac{\log 5}{\log 3} \sim 1.46497\ldots$

$x = \frac{1}{2}\left(1 + \frac{\log 5}{\log 3}\right) \sim 1.232486\ldots$

Opg

$4^{3x-1} = 12$

$(3x-1)\log 4 = \log 12 \;(= \log 3 \cdot 4 = \log 3 + \log 4)$

$3x-1 = \frac{\log 12}{\log 4}$ gir $x = \frac{1}{3}\left(1 + \frac{\log 12}{\log 4}\right)$

$\sim 0.930827\ldots$

$$3^{2x} - 4 \cdot 3^x - 12 = 0$$

$$(3^x)^2 - 4 \cdot 3^x - 12 = 0$$

$$y^2 - 4 \cdot y - 12 = 0$$

$$(y-6)(y+2) = 0$$

$y = 6$ eller $y = -2$.

$3^x = -2$ ingen løsning

$$3^x = 6$$

$$x \log 3 = \log 6$$

$$x = \log 6 / \log 3 = \underline{1.6309\ldots}$$

$3^x = y$

benytter

$3^{2 \cdot x} = (3^x)^2$

$(a^r)^s = a^{r \cdot s}$

opg

$$4^x - 2^{x+1} - 3 = 0$$

$$2^{2 \cdot x} - 2^1 \cdot 2^x - 3 = 0$$

$$(2^x)^2 - 2 \cdot 2^x - 3 = 0$$

2.gradslikning i 2^x

$$(2^x - 3)(2^x + 1) = 0$$

$2^x = 3$ og $2^x = -1$

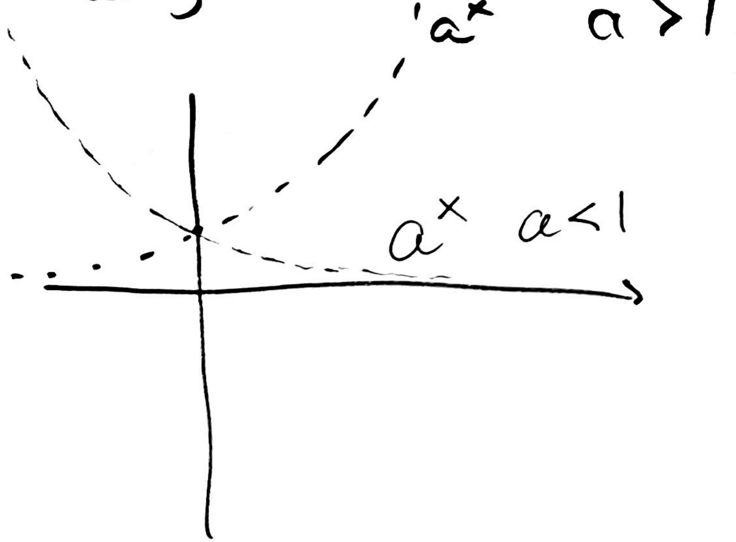
ingen løsning

$$x \log 2 = \log 3$$

$$x = \frac{\log 3}{\log 2} \sim 1.58496\ldots$$

Logaritmer med basis a $\quad a > 0$, $a \neq 1$

$a^0 = 1$



$\text{Log}_a(x)$ tallet slik at:

$$a^{\text{Log}_a(x)} = x$$

$2^3 = 8$ så $\text{Log}_2(8) = 3$

$\text{Log}_2(10) = ?$

$2^x = 10$

$x \log 2 = \log 10 = 1$ så $x = \frac{1}{\log 2}$

$\text{Log}_2(10) = \frac{1}{\log 2} = 3.321928\ldots$

$\text{Log}_a\, x = \frac{\log x}{\log(a)}$ (viser det neste gang)

eks $\log x^3 + 2 \log x^2 - 14 = 0$

$3 \log x + 2 \cdot 2 \log x - 14 = 0$

$7 \log x = 14$

$\log x = 2$

$x = 10^{\log x} = 10^2 = 100.$