

2.2
2022

Logaritmer

$$a > 0 \\ \neq 1$$

Logaritmen med base a
"a-logaritmen"

$$\text{Log}_a(x)$$

er eksponenten $a^{\text{Log}_a x} = x$

$$\log_2 16 = 4 \quad \text{fordi: } 2^4 = 16$$

$$\log_2 \left(\frac{1}{4}\right) = -2 \quad \text{fordi: } 2^{-2} = (2^2)^{-1} = \frac{1}{4}$$

$$\log_2(-4) \text{ eksisterer ikke} \quad 2^y > 0 \\ \text{for alle reelle } y.$$

$$\log_{1/3}(9) = -2 \quad \left(\frac{1}{3}\right)^{-2} = \left(\left(\frac{1}{3}\right)^{-1}\right)^2 \\ = 3^2 = 9 \checkmark$$

$$\text{Log}_a(x) = \frac{\text{Log}(x)}{\text{Log}(a)}$$

$$= \left(\frac{1}{\text{Log}(a)}\right) \cdot \text{Log}(x) \\ \uparrow \\ \text{konstant.}$$

$$\left(\begin{array}{l} \text{Log} \\ = \text{Log}_{10} \end{array} \right)$$

$$a^{\text{Log}_a(x)} = x$$

$$a = 10^{\text{Log } a}$$

$$\cancel{10^{\text{Log}_a(x)}} \quad \underbrace{(10^{\text{Log } a})}_{a} \text{Log}_a x = x$$

$$10^{\text{Log}(a) \cdot \text{Log}_a(x)} = x = 10^{\text{Log } x}$$

$$\Leftrightarrow \text{Log}(a) \cdot \text{Log}_a(x) = \text{Log}(x)$$

$$\text{Så} \quad \underline{\underline{\text{Log}_a(x) = \frac{\text{Log}(x)}{\text{Log}(a)}}}}$$

$$2^x = 17$$

$$\log 2^x = \log 17$$

$$x \log 2 = \text{Log } 17$$

$$x = \frac{\text{Log } 17}{\text{Log } 2}$$

$$\sim 4.0875\dots$$

$$3 \cdot 2^x = 25$$

$$3 \cdot (2^x) = 25$$

I deler med 3 på begge sider osv = -tegnet

$$2^x = 25/3$$

$$\text{Log } 2^x = \text{Log}(25/3)$$

$$x \log 2 = \text{Log}(25/3)$$

$$x = \text{Log}(25/3) / \text{Log } 2$$

$$x \sim 3.0588\dots$$

$$\begin{aligned} \text{II} \quad \text{Log}(3 \cdot 2^x) &= \text{Log} 25 \\ \text{Log}(3) + \text{Log} 2^x &= \text{Log} 5^2 \\ \text{Log}(3) + x \text{Log} 2 &= 2 \text{Log} 5 \\ x \text{Log} 2 &= 2 \text{Log} 5 - \text{Log}(3) \\ x &= \frac{2 \text{Log}(5) - \text{Log}(3)}{\text{Log}(2)} \\ x &\sim 3.0588\dots \end{aligned}$$

$$\boxed{\text{Log}\left(\frac{x}{y}\right) = \text{Log}(x) - \text{Log}(y)}$$

Forklaring: $\text{Log}\left(\frac{x}{y}\right) = \text{Log}(x \cdot y^{-1})$

$$\begin{aligned} &= \text{Log}(x) + \underbrace{\text{Log}(y^{-1})}_{(-\text{Log}(y))} \\ &= \text{Log}(x) + (-\text{Log}(y)) \\ \text{så} \quad \text{Log}\left(\frac{x}{y}\right) &= \underline{\text{Log} x - \text{Log} y} \end{aligned}$$

Finne den deriverte til

$$5 \sqrt[4]{4-3x}$$

(funksjon
h
i oblig 5)

Hint:

$$\sqrt[n]{x} = x^{1/n}$$

$$(x^{1/n})' = \frac{1}{n} x^{\frac{1}{n}-1}$$

$$(\sqrt[4]{x})' = (x^{1/4})' = \frac{1}{4} x^{\frac{1}{4}-1}$$
$$= \frac{1}{4} x^{-3/4} = \frac{1}{4} \cdot \frac{1}{x^{3/4}}$$

$$= \frac{1}{4 (x^3)^{1/4}}$$

$$= \frac{1}{4 \sqrt[4]{x^3}}$$

alternativt

$$= \frac{1}{4 (x^{1/4})^3}$$

$$= \frac{1}{4 (\sqrt[4]{x})^3}$$

$$(5 \sqrt[4]{4-3x})' = 5 ((4-3x)^{1/4})'$$

$$= 5 \cdot \frac{1}{4} (4-3x)^{\frac{1}{4}-1} \cdot (4-3x)'$$

$$= \frac{5(-3)}{4 \sqrt[4]{(4-3x)^3}} = \frac{-15}{4 \sqrt[4]{(4-3x)^3}}$$

$$\text{Log}(x) = 1.5$$

$$\begin{aligned}x &= 10^{\text{Log } x} = 10^{1.5} \\&= 10^{1+\frac{1}{2}} = 10^1 \cdot 10^{1/2} \\&= 10\sqrt{10} \\&\sim 31.622\dots\end{aligned}$$

$$\text{Log}_2(x) = 3$$

$$x = 2^{\text{Log}_2 x} = 2^3 = 8$$

alternativt $\text{Log}_2(x) = \frac{\text{Log}(x)}{\text{Log}(2)}$

$$\text{Log}(x) = 3 \cdot \text{Log}(2) \quad \text{etc...}$$

$$\text{Log}(x^3) = 5$$

$$\text{I } x^3 = 10^{\text{Log}(x^3)} = 10^5 = 100\,000$$

$$\begin{aligned}\text{så } x &= \sqrt[3]{100\,000} \\&= \sqrt[3]{100} \cdot \sqrt[3]{1000} \\&= \underline{10 \cdot \sqrt[3]{100}}\end{aligned}$$

$$\text{Log}(x^3) = 3 \text{Log } x$$

Så $3 \log(x) = 5$

og da $\log(x) = \frac{5}{3}$

$$x = 10^{\log x} = 10^{5/3} = 10^{1+2/3}$$

$$= 10^1 \cdot (10^2)^{1/3}$$

$$= 10 \cdot \sqrt[3]{10^2}$$

$$= 10 \cdot \sqrt[3]{100}$$

$$\sim \underline{\underline{46,4158\dots}}$$

$$\text{Log}_5(x) = -1$$

$$\text{Log}_5(5^y) = y$$

$$x = 5^{\text{Log}_5(x)} = 5^{-1} = \underline{\underline{\frac{1}{5}}}$$

$$\text{Log}_4(x) = 2$$

$$x = 4^{\text{Log}_4(x)} = 4^2 = \underline{\underline{16}}$$

$$\log(x^5) - 2 \underbrace{\log(x\sqrt{x})}_{\log(x^{3/2})} = 5$$

$$5 \log(x) - 2 \left(\frac{3}{2} \log x \right) = 5$$

$$\left(5 - 2 \cdot \frac{3}{2} \right) (\log(x))$$

$$2 \cdot \log(x) = 5$$

$$\log(x) = 5/2$$

$$x = 10^{\log(x)} = 10^{5/2}$$

$$= 10^{2+1/2} = 10^2 \cdot 10^{1/2}$$

$$= 100 \cdot \sqrt{10}$$

$$\sim 316,22776602\dots$$

$$(\log x)^2 - \log(x) - 2 = 0$$

$$u^2 - u - 2 = 0 \quad u = \log x$$

$$(u-2)(u+1) = 0$$

$$\log(x) = -1 \quad x = 10^{\log(x)} = 10^{-1} = \underline{\underline{1/10}}$$

$$\log(x) = 2 \quad x = 10^{\log(x)} = 10^2 = \underline{\underline{100}}$$

Løsningene er 1/10 og 100.

$$\log_2 x + \log_3(x) = 5$$

$$\frac{\log(x)}{\log(2)} + \frac{\log(x)}{\log(3)} = 5$$

$$\log(x) \left(\frac{1}{\log 2} + \frac{1}{\log 3} \right) = 5$$

$$\log(x) = \frac{5}{\left(\frac{1}{\log 2} + \frac{1}{\log 3} \right)}$$

$$x = 10^{\log(x)} = 10^{5 / \left(\frac{1}{\log(2)} + \frac{1}{\log(3)} \right)}$$

$$\left(\begin{aligned} &= 10^{5 / \left(\frac{\log(3) + \log(2)}{\log(2) \cdot \log(3)} \right)} \\ &= 10^{5 \log(2) \cdot \log(3) / (\log(2) + \log(3))} \end{aligned} \right)$$