2.2
2022

Logaritmer

$a > 0$
$\neq 1$

Logaritmen med base a
"a-logaritmen"

$\log_a(x)$

er eksponenten $a^{\log_a x} = x$

$\log_2 16 = 4$ fordi: $2^4 = 16$

$\log_2\left(\frac{1}{4}\right) = -2$ fordi: $2^{-2} = (2^2)^{-1} = \frac{1}{4}$

$\log_2(-4)$ eksisterer ikkje $2^y > 0$ for alle reelle y.

$\log_{(1/3)}(9) = -2$ $\left(\frac{1}{3}\right)^{-2} = \left(\left(\frac{1}{3}\right)^{-1}\right)^2 = 3^2 = 9$ ✓

$$\boxed{\log_a(x) = \frac{\log(x)}{\log(a)}}$$

$= \left(\frac{1}{\log(a)}\right) \cdot \log(x)$

↑
konstant.

$\left(\log = \log_{10}\right)$

$a^{\log_a(x)} = x$, $a = 10^{\log a}$

$\cancel{10^{\log_a(x)}}$ $\quad \Big(\underbrace{10^{\log a}}_{a}\Big)^{\log_a x} = x$

$$10^{\log(a)\cdot\log_a(x)} = x = 10^{\log x}$$

$$\Leftrightarrow \quad \log(a)\cdot\log_a(x) = \log(x)$$

så $\quad \underline{\log_a(x) = \dfrac{\log(x)}{\log(a)}}$

$$2^x = 17$$

$$\log 2^x = \log 17$$

$$x\log 2 = \log 17$$

$$x = \frac{\log 17}{\log 2}$$

$$\sim 4.0875\ldots$$

$$3\cdot 2^x = 25$$

$$3\cdot(2^x) = 25$$

I deler med 3 på begge sider av =-tegnet

$$2^x = 25/3$$

$$\log 2^x = \log(25/3)$$

$$x\log 2 = \log(25/3)$$

$$x = \log(25/3)/\log 2$$

$$x \sim 3.0588\ldots$$

II

$$\log(3 \cdot 2^x) = \log 25$$

$$\log(3) + \log 2^x = \log 5^2$$

$$\log(3) + x\log 2 = 2\log 5$$

$$x \log 2 = 2\log 5 - \log(3)$$

$$x = \frac{2\log(5) - \log(3)}{\log(2)}$$

$$x \sim 3.0588\ldots$$

$$\boxed{\log\left(\frac{x}{y}\right) = \log(x) - \log(y)}$$

Forklaring: $\log\left(\frac{x}{y}\right) = \log(x \cdot y^{-1})$

$$= \log(x) + \underbrace{\log(y^{-1})}$$

$$= \log(x) + (-\log(y))$$

så $\log\left(\frac{x}{y}\right) = \log x - \log y$

Finn den deriverte til

$$5\sqrt[4]{4-3x}$$

(funksjon h i oblig 5)

Hint: $\sqrt[n]{x} = x^{1/n}$

$$\left(x^{1/n}\right)' = \frac{1}{n}x^{\frac{1}{n}-1}$$

$$\left(\sqrt[4]{x}\right)' = \left(x^{1/4}\right)' = \frac{1}{4}x^{\frac{1}{4}-1}$$

$$= \frac{1}{4}x^{-3/4} = \frac{1}{4}\cdot\frac{1}{x^{3/4}}$$

$$= \frac{1}{4\left(x^3\right)^{1/4}}$$

$$= \frac{1}{4\sqrt[4]{x^3}}$$

alternativt $= \dfrac{1}{4\left(x^{1/4}\right)^3}$

$$= \frac{1}{4\left(\sqrt[4]{x}\right)^3}$$

$$\left(5\sqrt[4]{4-3x}\right)' = 5\left(\left(4-3x\right)^{1/4}\right)'$$

$$= 5\cdot\frac{1}{4}\left(4-3x\right)^{\frac{1}{4}-1}\cdot\left(4-3x\right)'$$

$$= \frac{5\,(-3)}{4\sqrt[4]{(4-3x)^3}} = \frac{-15}{4\sqrt[4]{(4-3x)^3}}$$

$$\mathrm{Log}(x) = 1{,}5$$

$$x = 10^{\mathrm{Log}\,x} = 10^{1{,}5}$$

$$= 10^{1+\frac{1}{2}} = 10^{1} \cdot 10^{1/2}$$

$$= 10\sqrt{10}$$

$$\sim 31.622\ldots$$

$$\mathrm{Log}_2(x) = 3$$

$$x = 2^{\mathrm{Log}_2\,x} = 2^3 = 8$$

alternativt $\mathrm{Log}_2(x) = \dfrac{\mathrm{Log}(x)}{\mathrm{Log}(2)}$

$\mathrm{Log}(x) = 3 \cdot \mathrm{Log}(2)$ etc...

$$\mathrm{Log}(x^3) = 5$$

I $x^3 = 10^{\mathrm{Log}(x^3)} = 10^5 = 100\ 000$

så $x = \sqrt[3]{100\ 000}$

$$= \sqrt[3]{100} \cdot \sqrt[3]{1000}$$

$$= \underline{10 \cdot \sqrt[3]{100}}$$

$$\log(x^3) = 3\log x$$

Så $3\log(x) = 5$

og da $\log(x) = \frac{5}{3}$.

$$x = 10^{\log x} = 10^{5/3} = 10^{1+2/3}$$

$$= 10^1 \cdot (10^2)^{1/3}$$

$$= 10 \cdot \sqrt[3]{10^2}$$

$$= 10 \cdot \sqrt[3]{100}$$

$$\sim \underline{46{,}4158\ldots}$$

$$\log_5(x) = -1$$

$$\log_5(5^y) = y$$

$$x = 5^{\log_5(x)} = 5^{-1} = \underline{\frac{1}{5}}$$

$$\log_4(x) = 2$$

$$x = 4^{\log_4(x)} = 4^2 = \underline{16}$$

$$\log(x^5) - 2\underbrace{\log(x\sqrt{x})}_{\log(x^{3/2})} = 5$$

$$5\log(x) - 2\left(\tfrac{3}{2}\log x\right) = 5$$

$$\left(5 - 2\cdot\tfrac{3}{2}\right)(\log(x))$$

$$2\cdot\log(x) = 5$$

$$\log(x) = 5/2$$

$$x = 10^{\log(x)} = 10^{5/2}$$

$$= 10^{2+1/2} = 10^2\cdot 10^{1/2}$$

$$= 100\cdot\sqrt{10}$$

$$\sim 316{,}22776602\ldots$$

$$(\log x)^2 - \log(x) - 2 = 0$$

$$u^2 - u - 2 = 0 \qquad u = \log x$$

$$(u-2)(u+1) = 0$$

$$\log(x) = -1 \qquad x = 10^{\log(x)} = 10^{-1} = \underline{1/10}$$

$$\log(x) = 2$$

$$x = 10^{\log(x)} = 10^2 = \underline{100}$$

Løsningene er $\underline{\frac{1}{10} \text{ og } 100}$.

$$\log_2 x + \log_3(x) = 5$$

$$\frac{\log(x)}{\log(2)} + \frac{\log(x)}{\log(3)} = 5$$

$$\log(x)\left(\frac{1}{\log 2} + \frac{1}{\log 3}\right) = 5$$

$$\log(x) = \frac{5}{\left(\frac{1}{\log 2} + \frac{1}{\log 3}\right)}$$

$$x = 10^{\log(x)} = 10^{5/\left(\frac{1}{\log(2)} + \frac{1}{\log(3)}\right)}$$

$$= 10^{5/\left(\frac{\log(3)+\log(2)}{\log(2)\cdot\log(3)}\right)}$$

$$= 10^{5\log(2)\cdot\log(3)/(\log(2)+\log(3))}$$